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Introduction

The dawn of the twentieth century brought us the birth of quantum mechanics and a deeper understanding of the microscopic world. The new theory describing photons, atomic spectra, and many other physical processes postulates that the microscopic world is, in its truest essence, probabilistic. Particles such as electrons or photons move or “propagate” as probability waves to be detected at a given position, or in a given state. However, those waves or wavefunctions are very different from a mere representation of our ignorance about the world.

Quantum mechanics is the mathematical language that we use to describe the microscopic world. Quantum mechanical states – our *wavefunctions* – go beyond our classical understanding of the world, accepting the possibility of a system to coexist in a *quantum superposition* of two distinguishable configurations – i.e., an atom in two positions, a photon in two polarizations, or a neutron moving in two opposite directions. Only when we measure the state of the quantum mechanical object, it collapses to a well-defined configuration, which may be different on each realization of the experiment. Even more dramatically, multiple particles may be in a collective superposition or *entangled state*, allowing arbitrary measurements on these particles to produce random but perfectly correlated outcomes, irrespective of the space and time separation between those measurements. These and other predictions of quantum mechanics made some physicists such as Albert Einstein deem the theory as “incomplete” or even inconsistent.

The end of the twentieth century and the beginning of the twenty-first have witnessed an incredible collective effort to challenge the wildest predictions of quantum mechanics in the broadest variety of experimental systems possible, from photons, to atoms, all the way to macroscopic solid-state devices and circuits. Great experimentalists of our time have revealed quantum properties in objects that we can see with the naked eye, created complex quantum states of tens to thousands of particles, and entangled photons and spread them to different points of the planet,

to test their quantum mechanical correlations. In the process, not only has quantum mechanics been validated, but we have become incredibly good at controlling quantum systems.

Trapped ions were one of the first systems to enter this controlled quantum regime (Leibfried et al., 2003). Using electromagnetic traps and laser cooling, experimentalists may isolate one or more charged particles, cooling some of their properties close to the absolute. This makes it possible to observe the discrete or *quantized* nature of the atomic excitations, not only at the level of the electronic states, but also with respect to quanta of energy stored in the motional states of the trapped atom.

If instead of charged particles we use neutral atoms, we can trap more particles and enter the regime of mesoscopic quantum systems. Experiments in 1995 showed how to trap and evaporatively cool a cloud of 10^6 alkali atoms in an almost perfect vacuum down to few nanokelvins. At these temperatures, the ultracold atoms form a singular state of matter called Bose–Einstein condensate (BEC) (Townsend et al., 1997), in which all bosonic atoms are described by the same wave function, $\phi(x)$, whose dynamics and quantum properties can be engineered at will.

Starting from a perfectly controlled state of matter, such as the condensate, we can build extremely sophisticated quantum states and physical devices. We can split the atoms of a BEC into the pockets of an optical lattice, to recreate a crystal of bosons and simulate quantum magnetism and even Luttinger liquid physics. Bose–Einstein condensates can be used to cool down fermionic atoms and create Fermi seas, study Bardeen–Cooper–Schrieffer (BCS) superconductivity or even implement the Hubbard model for electrons in a solid. And using time-dependent controls, we can create large amounts of entanglement and squeezing that can be used for sensing or for foundational experiments in quantum information.

The beauty of atomic, molecular and optical (AMO) physics' bottom-up approach can hardly be exported to solid-state devices. Condensed-matter objects are simply too large. We cannot control all atoms in a piece of metal and convince them to adopt a perfect state. Moreover, solid devices rarely exist in perfectly isolated environments. They are always in contact with other elements – contacts, substrates, measurement apparatuses – at higher temperatures than the AMO setups we just discussed.

In a way, once we overcome the barriers associated to cooling and trapping, it is not surprising that we can reveal quantum mechanical phenomena in large atomic setups – it just confirms that the rules of quantum mechanics extend to very large composite systems. The million dollar question, however, was whether those rules extend all the way to solid-state and condensed matter systems!

This was indeed the great challenge for condensed matter systems around the 1990s. Can we prepare a perfect solid-state quantum system? Can we build a simple, atomic-like device, with a perfectly controlled state? The approach to

this problem is subtly different from AMO. Starting from a macroscopic object – a gated or self-assembled quantum dot, impurities in diamond, nanoresonators, or superconducting circuits – we select one or two degrees of freedom in which we seek evidence of quantum phenomena. The candidate for a quantum degree of freedom may be the charge of a small superconducting island, the electrical current along a metallic loop, or the wavefunction of electrons in a dot or a color center, for example. Experimentalists focus on one property and work toward isolating it from their environments, cooling them down, and engineering better and better quantum states.

This challenge, the quest for an *artificial atom* – what we now call a *qubit*; see Chapter 6 – was still ongoing when I first learned about the field of superconducting circuits. The review article by Makhlin et al. (2001) showed some promising candidates in the forms of charge qubits (Section 6.2) or three-Josephson junction qubits (Section 4.8), but quantum superpositions were fragile and easily killed by the environment.

The surprise came in year 2004, when the Yale team (Wallraff et al., 2004) radically improved the lifetime of charge qubits by placing them inside microwave resonators. That work, and subsequent works at various groups in the University of California, Santa Barbara (UCSB), Saclay, Karlsruhe Institute of Technology (KIT), etc., sparked the beginning of a productive intersection between superconducting circuit technology, quantum optics, and quantum information, which we now call *circuit-QED*.

We live in the *second quantum revolution* (Dowling and Milburn, 2003), in which quantum mechanics becomes a tool for industrial applications and practical devices. These applications are generally known as *quantum technologies*, because they use the unique properties of quantum mechanics – quantum superpositions, entanglement, noncontextuality, etc. – to create actual technologies for *quantum communication*, *quantum cryptography*, *quantum sensing*, *quantum simulation*, and *quantum computing*.

Superconducting circuits are at the center the quantum revolution. Superconductors are the basis for the most accurate single-photon detectors, which are used in advanced setups for quantum communication. We rely on superconducting qubits for some of the most powerful and accurate quantum computers to date (see Chapter 8), and also to simulate large and complex problems from quantum magnetism (Chapter 9). Superconducting circuits have also found their way, as controls and interfaces for other microwave-based quantum technologies, from quantum dots to nanomechanical resonators. Understanding how these circuits work, how they are designed, and how they are operated is a natural path for both theoreticians and experimentalists alike. This book is a self-contained undergraduate manual designed to help you gain this understanding.

1.1 The Book

The book is structured in three parts. The first part introduces the platform, superconducting circuits, and a common language, formed by quantum mechanics and circuit quantization. The second part introduces the two fundamental objects we build using superconductors: these are photonic devices and artificial atoms. They are studied separately, and then combined to develop the theory of *circuit quantum electrodynamics*, or circuit-QED. The third part concerns the physics that can be explored and the technologies that can be built using superconducting circuits. Along separate chapters and sections, we learn about how to build quantum computers, quantum simulators, quantum optimizers, and other quantum technologies using circuits or hybrid setups.

The structure and the level of the book also allow different applications. This textbook was born and has been used as an introductory course in quantum optics. Using qubits and microwave photons, it is possible to review the basic concepts in light–matter interaction, the Rabi, Jaynes–Cummings and Dicke model, superradiance, and subradiance, and even understand sophisticated models of low-dimensional photonics. But the same book can be repurposed to focus on the design and construction of practical quantum computers or to learn about applications in the field of quantum simulation and many-body physics.

The book is mostly self-contained. It assumes familiarity with the formalism of quantum mechanics, including key concepts such as pure states, density matrices, observables, and the Schrödinger equation, but these concepts are reviewed as they are used. At different stages, we will also introduce minimal concepts in second quantization and field theory, such as the notion of modes, creation and annihilation operators, commutation relations, bosonic statistics, etc. We also review important concepts from quantum information theory, such as qubits, entanglement, quantum gates, etc. The most important ideas are reinforced through selected exercises that students are encouraged to do.

Building on these prerequisites, the reading order of this book would be the following one. In the first part of the book, Chapter 2 provides an overview of quantum mechanics, focusing on concepts that are used throughout the book. In Chapter 3, we introduce a minimal understanding of the theory of superconductivity, with a focus on London’s mesoscopic models and the electrical properties of superconducting circuits. Chapter 4 introduces the theory of quantum circuits. This is a set of rules that allow us to find the mathematical model that describes a given superconducting circuit. We derive these rules using as practical examples all the elementary circuits that we will later study, such as qubits, microwave resonators, or superconducting quantum interference devices (SQUIDs).

In the second part of the book, Chapter 5 focuses on the linear models of superconducting circuits, such as microwave LC resonators and waveguides (the equivalent of coaxial cables), and showing how the quantization of the circuit gives birth to microwave photons that can be created, manipulated, and measured. Chapter 6 studies superconducting circuits in the highly nonlinear regime, in which the circuit acts as an artificial atom or qubit. We review the most popular qubit designs, such as the transmon or the flux qubit, and develop tools to prepare and characterize those qubits. Chapter 7 combines artificial atoms with microwave photons, developing the theory of light–matter interaction. This chapter is a primer on concepts from quantum optics – models for low-dimensional atom–light interaction, the physics of spontaneous emission, photon absorption, and open quantum systems. We then focus on the simpler setup of qubit–resonator interactions, introducing the Rabi and Jaynes–Cummings models, and showing how qubits can control the state of light and vice versa, cavities can be used to control and measure qubits.

The third part of the book focuses on the applications of superconducting circuits to various quantum technologies. We begin in Chapter 8 studying universal quantum computers in the circuit model. We offer a checklist of ingredients that are needed to build a practical quantum computer, and investigate how those elements are built and characterized in the superconducting platform. We also discuss the roadmap toward fault-tolerant, error-corrected, and fully scalable quantum computers, and what we can do with their small-scale, faulty versions in the near term. Finally, Chapter 9 builds on the previous formal developments to introduce a design for an adiabatic quantum computer, also known as *quantum annealer*. This chapter starts with a formal description of this computational model, where a physical system is adiabatically coerced into a configuration that represents the solution to a mathematical problem. It then moves on to the actual superconducting architecture for a quantum annealer using superconducting flux qubits, along the line of the D-Wave machine, but referencing other later designs.

If the book is to be used with a strong focus on quantum optics or quantum circuits, the student or teacher should at least cover Chapters 1–7. This should give students an overview of the most relevant experiments in the field from 2004 to this day, providing them with the language and tools to access more complex literature and explore their own ideas. Chapters 8 and 9 in the third part of the book are independent from each other, but require a good understanding of the first part of the book. A minimal set of exercises is provided within each chapter. Solutions and errata will be posted on the author’s webpage.¹

¹ <http://juanjose.garciaripoll.com>.

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And finally, this book exists because of you, the reader. I hope it will help you approach this field, learn about its beauty and its challenges, and inspire you to be a better “quantum mechanic.”