The Control of Helicopter Rotor Vibration

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DR G S HISLOP (*Chairman of the Executive Council*) *occupying the Chair*

INTRODUCTION BY THE CHAIRMAN

The CHAIRMAN said the subject was of paramount importance to all helicopter designers, manufacturers and operators The Association had not previously had a lecture which dealt specifically with the problem and the importance of the subject meant that such a lecture was long overdue They knew that, in spite of the claims made from time to time by sundry manufacturers, helicopters did vibrate, some to a very great extent Indeed, one eminent fixed wing designer thought that by virtue of this vibration helicopters were rather more grandiose fatigue testing machines than the conventional test equipment for the purpose

Mr Payne had started working on helicopters in 1949, when he joined the Bristol Aeroplane Helicopter Department as an aerodynamicist Later he had become project engineer, moving in this capacity in 1952 to the Saunders-Roe Helicopter Division In 1953 he had joined Auster Aircraft Ltd as chief project engineer, and there he had been responsible for all the work on helicopters He was now with the engine division of the Bristol Aeroplane Company As would be appreciated from the firms by whom he had been employed in the last six years, Mr Payne had had extensive experience

The CHAIRMAN explained that when the Association invited a lecturer to give a Paper, that did not necessarily mean that the Association agreed with the views expressed in the Paper Those were the views of the lecturer himself This was a matter in which they followed the precedent of other learned societies

MR P R PAYNE

SUMMARY

Helicopter structural vibration caused by the periodic displacement of the rotor hub is discussed under the two headings of vibration which is fundamental to a balanced rotor, and vibration which is due to unbalance The fundamental vibration is unavoidable but can be minimised by increasing both tip speed and the number of blades above present-day values Factors which also help are a high disc loading and blade weight, elimination of drag hinges, high flapping pin off-set, and the use of two or more rotors The use of drag hinges leads to a general increase in magnitude of all in-plane vibration harmonics except the first relative to rotor axes which causes second harmonic structural vibration with a two bladed rotor It is shown that first or higher harmonics of force fluctuation on the blade can be balanced out by suitable inclination of the mechanical axis, and it is suggested that this remedy, coupled with the use of moderately high off-set flapping hinges is preferable to the technique of off-loading the rotor (s) on to wing (s)

An elementary theory of rotor unbalance is introduced to demonstrate the physical causes of first harmonic structural vibration, and the results are used to build up a rational method of rotor balancing

In the second half of the paper the general theory of control vibration is considered, particular attention being paid to the system in which a Hafner torque bar is used in place of the conventional torsion bearing Consideration of basic causes of vibration enables a method of eliminating them to be established, and various modifications to present techniques are suggested

PART I

INTRODUCTION

Vibration in helicopters can be divided under two separate headings , general vibration problems which are commonly met in all aircraft, and problems peculiar to helicopters As the title of this lecture indicates it is concerned with the second category

It is evident that vibration peculiar to the helicopter must emanate from the rotor and must be felt as structural vibration or control vibration It is fortunate that these two manifestations can be treated separately if control vibration is balanced out and for clarity the lecture is divided into two parts

Part 1, deals with structural vibration and methods of reducing it to reasonable proportions

Part 2, deals with balancing the flying controls

As we go along we shall find that a third division of the subject is occurring and that the various forms of vibration can be classified as " Fundamental " or " Unbalance " vibration This is a most important division and fundamental vibration can only be dealt with effectively in the early project stages Since most British helicopters currently flying vibrate well above any acceptable limit, it will be realised that Designers should devote a great deal of attention to this aspect on future helicopters The quantitative assessment of fundamental vibration is rather a complex business, and very little published work is available on the subject No attempt has been made to give detailed results or equations for this aspect, partly because of the limited time available and partly because it is rarely possible to write down

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an accurate equation in under half a page because of the number of flapping harmonics which must be considered Attention is therefore focussed on the more immediate problems of unbalance which are being met with every day, and the sections dealing with the fundamental vibration of a balanced rotor are only included to assist in the identification of a given vibration It is obviously impossible to trim out a vibration which is fundamental to a balanced rotor !

No attempt has been made to obtain completeness and this lecture ought really to have been entitled " Some random thoughts on rotor balancing " For this reason I hope no-one will charge me with sins of omission because a very large book could be written on the subject of drag balancing alone I have also tried to avoid controversial statements of the type " off-loading the rotor does not materially reduce vibration," not because I am averse to controversy but because much of the lecture concerns " Tracking," upon which nearly everyone seems to hold different views Hafner once observed that " the paths of all true investigators are asymptotic to the truth " If this is so we are still a very long way from the truth of tracking and in part this is the fault of the technicians for regarding it as a subject rather beneath their notice It is actually a very exacting subject because it concerns a large number of mutually inter-related effects and it is not surprising that the practical engineers who study these effects at first hand established rules of thumb which are mutually conflicting

Vertical Vibration of a Balanced Rotor

The fundamental cause of nearly all balanced rotor vibration is forward flight For example, the velocity of a blade element relative to the air is its rotational velocity (ωr) to which is added a component of the forward speed of the aircraft Putting this into symbols the relative velocity U_T is

$$U_T = \omega r + V \sin \psi$$

where ψ is the azimuth angle of the blade Now aerodynamic force varies as the square of the velocity, so that

$$\text{Force} \propto (\omega r)^2 + 2\,\omega r V \sin \psi + \frac{V^2}{2} - \frac{V^2}{2} \cos 2\,\psi$$

Thus on this simple criterion alone we have first and second harmonic force fluctuations and Tables 1 and 2 show that on a two-bladed rotor there will be resultant second harmonic vertical and in-plane vibrations at the hub

In practice rotor flapping and feathering also introduce force variations and significant force fluctuations are experienced up to at least the tenth harmonic order Qualitative assessment is usually a difficult matter because of the complexity of the equations In Ref 2, Stewart gives the moment of force fluctuation in an equation which is accurate up to the sixth harmonic only for a simple untwisted untapered blade, and the equation contains one hundred terms although it only relates to aerodynamic forces It is obvious therefore that most practical problems must be solved by the application of general principles only, unless first class computing facilities are available

The first and most important general principle for a rotor which is not resonating is that the only harmonic components which appear as vertical vibration are those whose order is a multiple of the number of blades in the rotor Thus a balanced three-bladed rotor is subject to vibration frequencies of 3/rev, 6/rev, 9/rev, and so on This theorem can be neatly expressed in an equation for the amplitude of vibration (z) of a rigid helicopter

$$z = \frac{g}{W b \omega^2}\left(C_b \cos b\psi + \frac{C_{2b}}{4}\cos 2b\psi + \frac{C_{3b}}{9}\cos 3b\psi \quad + \frac{C_{nb}}{n_2}\cos nb\psi\right)$$

where C_b = the amplitude of the cosine component of the b — order harmonic on the individual blade, *i e*, Thrust/blade = A + Σ $C_n \cos n\psi$

b = the number of blades in the rotor

W = the aircraft weight

ω = the rotational speed in radians/sec

The individual blade harmonic amplitude C_n will generally decrease as the harmonic order increases, so that the first vibration frequency in equation 2, is by far the most important one

This equation is presented in Fig 1 in terms which are more down to earth The ratio of the vibrating force to the aircraft weight (F/W) replaces bC_{nb}/W in equation above This diagram was first presented in Ref 1 and is put forward as the most practical way of assessing the importance of vibratory forces Whether or not a particular vibration is serious can be judged by its position on the diagram relative to the threshold of feeling on the one hand, and the limit of uncomfortable vibration on the other

A point of considerable interest is the effect of helicopter size on vibration For a given rotor configuration the ratio F/W is independent of physical size, whilst frequency, which is keyed to rotor speed, diminishes with increasing rotor size Thus on Fig 1 a given (F/W) ratio results in a vibration which becomes increasingly uncomfortable as aircraft size increases It follows that the use of four or five bladed rotors on new and larger helicopters is not just a design improvement, but is necessary if present vibration levels are not to be exceeded In general it may be said that the design variables

TABLE 1

VERTICAL FORCE AT EACH FLAPPING HINGE	TOTAL VERTICAL FORCE ON HUB		
	2 blades	3 blades	4 blades
Fo	2 Fo	3 Fo	4 Fo
Fo sinψ	0	0	0
Fo cosψ	0	0	0
Fo sin 2 ψ	2 Fo sin 2 ψ	0	0
Fo cos 2 ψ	2 Fo cos 2 ψ	0	0
Fo sin 3 ψ	0	3 Fo sin 3 ψ	0
Fo cos 3 ψ	0	3 Fo cos 3 ψ	0
Fo sin 4 ψ	2 Fo sin 4 ψ	0	4 Fo sin 4 ψ
Fo cos 4 ψ	2 Fo cos 4 ψ	0	4 Fo cos 4 ψ
Fo sin 5 ψ	0	0	0
Fo cos 5 ψ	0	0	0
Fo sin 6 ψ	2 Fo sin 6 ψ	3 Fo sin 6 ψ	0
Fo cos 6 ψ	2 Fo cos 6 ψ	3 Fo cos 6 ψ	0

N B Table 1 is for a balanced rotor which is not in resonance with exciting air forces If resonance occurs any harmonic order may appear at the hub

TABLE 2

Force Transmitted by Blade In-plane Forces to Rotor Hub
(Balanced Rotor)

TWO BLADES

In-plane tangential force on one blade	Lateral force at hub	Longitudinal force at hub
Fo	0	0
Fo $\sin \psi$	Fo $\sin 2\psi$	Fo-Fo $\cos 2\psi$
Fo $\cos \psi$	Fo + Fo $\cos 2\psi$	Fo $\sin 2\psi$
Fo $\sin 2\psi$	0	0
Fo $\cos 2\psi$	0	0
Fo $\sin 3\psi$	Fo $\sin 2\psi$ Fo $\sin 4\psi$	Fo $\cos 2\psi$ —Fo $\cos 4\psi$
Fo $\cos 3\psi$	Fo $\cos 2\psi$ Fo $\cos 4\psi$	Fo $\sin 2\psi$ Fo $\sin 4\psi$
Fo $\sin 4\psi$	0	0
Fo $\cos 4\psi$	0	0
Fo $\sin 5\psi$	0	0
Fo $\cos 5\psi$	0	0

THREE BLADES

In-plane tangential force on one blade	Lateral force at hub	Longitudinal force at hub
Fo	0	0
Fo $\sin \psi$	0	3/2 Fo
Fo $\cos \psi$	3/2 Fo	0
Fo $\sin 2\psi$	3/2 Fo $\sin 3\psi$	—3/2 Fo $\cos 3\psi$
Fo $\cos 2\psi$	3/2 Fo $\cos 3\psi$	3/2 Fo $\sin 3\psi$
Fo $\sin 3\psi$	0	0
Fo $\cos 3\psi$	0	0
Fo $\sin 4\psi$	3/2 Fo $\sin 3\psi$	3/2 Fo $\cos 3\psi$
Fo $\cos 4\psi$	3/2 Fo $\cos 3\psi$	3/2 Fo $\sin 3\psi$
Fo $\sin 5\psi$	3/2 Fo $\sin 6\psi$	—3/2 Fo $\cos 6\psi$
Fo $\cos 5\psi$	3/2 Fo $\cos 6\psi$	3/2 Fo $\sin 6\psi$

FOUR BLADES

In-plane tangential force on one blade	Lateral force at hub	Longitudinal force at hub
Fo	0	0
Fo $\sin \psi$	0	2 Fo
Fo $\cos \psi$	2 Fo	0
Fo $\sin 2\psi$	0	0
Fo $\cos 2\psi$	0	0
Fo $\sin 3\psi$	2 Fo $\sin 4\psi$	—2 Fo $\cos 4\psi$
Fo $\cos 3\psi$	2 Fo $\cos 4\psi$	2 Fo $\sin 4\psi$
Fo $\sin 4\psi$	0	0
Fo $\cos 4\psi$	0	0
Fo $\sin 5\psi$	2 Fo $\sin 4\psi$	2 Fo $\cos 4\psi$
Fo $\cos 5\psi$	2 Fo $\cos 4\psi$	2 Fo $\sin 4\psi$

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tending to reduce vibration on a given size of helicopters are, in order of importance,

Increasing the number of blades per rotor
Increasing the number of rotors
Increasing the tip speed
Increasing the disc loading

If a fuselage vibration is experienced which is not a multiple of the number of blades it is generally due to in-plane rotor resonance with the fluctuating air forces This is dangerous if the rotor has no drag hinges

It may be asked why helicopter designers do not make more effort to isolate rotor vibrations from the fuselage by putting a spring of some form between the two The answer is, that to make the fuselage seismic the spring would have to be impractically large Never-the-less it is possible to isolate higher rotor frequencies in this way, and there is much to be said for always mounting the hub in rubber, and in a shaft drive helicopter, dividing the drive in this manner as well

The spring rate k of the hub mounting would have to be zero for absolute insulation between the rotor and the fuselage, so that in practice a mounting must be designed for a desired attenuation of a certain frequency If Ω is the frequency of a sinusoidally oscillating force on the rotor hub ($F \sin\Omega t$), the equivalent hub and rotor mass is m_1, and the equivalent fuselage mass

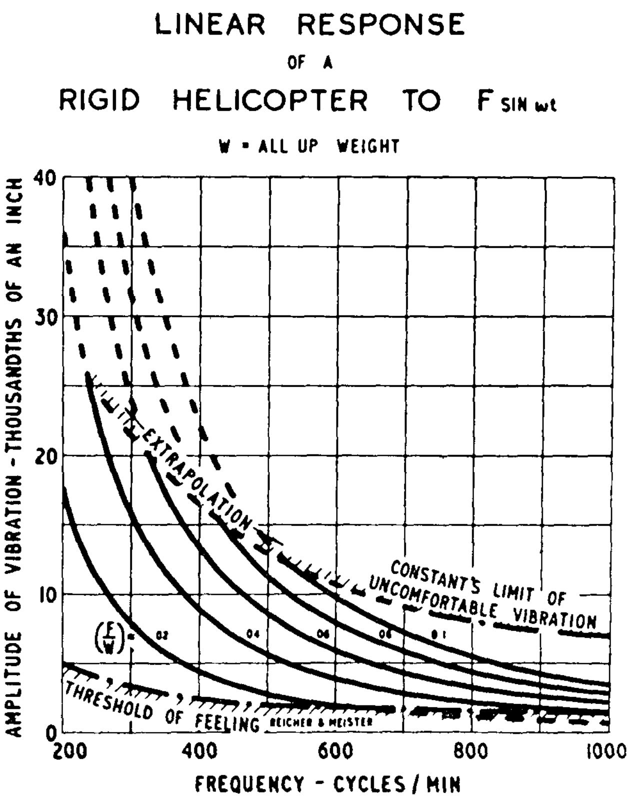

Fig 1

is m_2, then the equations of motion, taking x as the displacement, will be

$$m_1 \frac{d^2x_1}{dt^2} + k(x_1 - x_2) = F \sin \Omega t$$

$$m_2 \frac{d^2x_2}{dt_2} + k(x_2 - x_1) = O$$

In the absence of damping the motion of the two masses will be either in phase with the exciting force, or out of phase by the angle π radians, the frequency in both cases being Ω

writing $x_1 = a_1 \sin \Omega t$

$x_2 = a_2 \sin \Omega t$

we derive equations for the amplitude of each mass

$$a_2 = \frac{F}{\frac{m_1 m_2 \Omega^4}{k} - (m_1 + m_2)\Omega^2} \text{ (fuselage amplitude)}$$

$$a_1 = \frac{F}{(k - m_1 \Omega^2) - \frac{k^2}{(k - m_2 \Omega^2)}} \text{ (rotor amplitude)}$$

This equation can be checked in a number of ways For example, $a_1 = 0$ when $\Omega^2 = k/m_2$ This is the principle of the dynamic absorber used in mechanical engineering Also when $k = oo$

$$a_2 = \frac{-F}{(m_1 + m_2)\Omega^2} = a_1$$

— which is the same as the equation for a rigid body

Taking this as the datum fuselage amplitude, the amplitude with a flexible hub mounting will be $F/\Phi (m_1 + m_2)\Omega^2$ where Φ is the attenuation factor (if $\Phi = 2\ 0$ the vibration is halved) To achieve a specific atteruation the spring rate must be

$$k = \frac{2}{(\Phi - 1)\left(\frac{1}{m_2} + \frac{1}{m_1}\right)}$$

This result is of considerable interest, and leads to several conclusions

(*a*) The spring rate increases as the square of the exciting frequency and therefore directly as tip speed squared and disc loading, but inversely as the all-up weight Thus, for an equal number of blades it is easier to isolate vibration on a small ramjet driven helicopter than on a large shaft or pressure jet driven one, to take specific examples

(*b*) The lighter the rotor is, in relation to the fuselage mass, the easier it is to isolate its vibration

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(*c*) Isolation is most difficult with the Fitzwilliam's configuration, where jet engines are mounted on or in the rotor blades

(*d*) Although not directly obvious from the last equation, the hub can be made a node by attaching to it an auxiliary dynamic system, known as a dynamic absorber This comprises a " spring " and a mass, the fundamental natural frequency of the auxiliary system being equal to the frequency which is to be " absorbed " from the hub If the " spring " stiffness is due to centrifugal force, as in an offset pendulum, any one rotor order can be suppressed throughout the R P M range

In principle, therefore, it is theoretically possible to achieve a high measure of insulation between the rotor hub and fuselage High frequency vibrations will be absorbed by the flexible hub mounting, and any important low frequency vibration (such as once or twice rotor) which is not attenuated by this means can be absorbed by a universally hinged pendulum There is, of course, a critical size below which a pendulum will attenuate but not entirely absorb a vibration, the critical size being a function of the energy content of the vibration

Periodic Blade Flexing

The foregoing conclusions are tacitly based on the assumption of a rigid rotor blade Although considerable blade flexing is known to occur, as described by Hafner in Ref 3 for instance, it is found in practice that a line drawn from the flapping hinge centre through the blade datum at the so-called " standard " or " aerodynamic " radius of 0 7R executes a flapping motion which is adequately described by rigid rotor theory It seems likely (and here there is much room for controversy) that the forces acting on a rotor blade are also given by rigid rotor theory with reasonable accuracy and that blade flexing does not seriously affect theoretical vibration estimates for the lower harmonics

Whilst blade flexing can occur at any frequency and with any mode of deformation, there are of course natural frequencies, in both flapping and drag planes, and in torsion In flight the spanwise load distribution along the blade varies periodically, and the harmonics of this fluctuation can cause resonance when they occur at a natural frequency of the blade In resonance, by definition, the spring and mass forces in the blade are in equilibrium, and therefore the exciting forces are only opposed by the damping forces , hysteresis damping in the blade material and aerodynamic damping It follows that very large amplitudes of blade flexing can occur in resonance, producing internal stresses which are much greater than could be obtained by applying the blade loads statically One of the primary aims of modern rotor blade design is to avoid this resonance at normal operational rotor speeds, but in cases where it does occur we often find that the amplitude of the appropriate harmonic order at the hub is very much higher than simple theory indicates

Excitement of a torsional natural frequency of a blade can also cause additional vertical vibration, the harmonic order being the same as that of the exciting harmonic

Blade Stalling

The vertical vibration due to blade stalling is a severe one of b/rev frequency, coupled with an increase in rotor noise, but is of rather academic interest in design because it is not an operating condition A simple method of estimating stalling boundaries was given in Ref 4 and provided that operation is confined within these limits, no stalling should be experienced What has given a false importance to blade stalling is its popular appeal as a whipping boy for all vibration troubles In this respect it should be noted that the only sure way of determining whether a high speed vibration is due to stalling is to tuft the blades and photograph the tufts in flight It is often found that the vibration attributed to stalling is due to some other cause entirely

In-Plane Vibration of a Balanced Rotor

Ground Resonance

In-plane vibration is usually the most important, since much larger forces can arise in this sense The most extreme example is of course that of Ground Resonance, a divergent and often destructive oscillation which is self excited, and which usually occurs whilst the helicopter is in contact with the ground Ground resonance is not a pure rotor phenomenon, since it must be coupled with at least one degree of translational movement of the hub rotational axis The analysis of such a coupled system leads to equations which require a great deal of computation, which must in turn be based on a considerable experimental programme, if reasonable accuracy is to be achieved A good idea of the work involved is given by Howarth and Jones in Ref 5, who also give an introduction to the theory

Ground resonance can only be effectively controlled in one of two ways by the formation of a skilled and adequately equipped department as an off-shoot of the design team, which specialises in this problem and nothing else , or by the elimination of drag hinges, real and virtual A third alternative for small helicopters, put forward by my late colleague, R M Sieley, is the use of an undercarriage, with an infinite spring rate, which is still fully shock absorbing A practical appraisal of the last approach has been completed, and it offers considerable promise not only of eliminating ground resonance but also of reducing undercarriage and structure weight

The design of rotor blades without drag hinges is quite straight forward for small and medium sized helicopters, and the increase in weight is small if modern methods of construction are used In any case, the saving of this weight by the use of drag hinges introduces hinge dampers and complications to the undercarriage Drag hinges were introduced by Cierva in what Hafner has called the " backyard " days (Ref 6) of helicopters but there seems to be no case at all for their retention today

Coriolis Forces

When a rotor blade flaps it changes its moment of inertia about the rotational axis, and from the law of conservation of angular momentum we shall expect either an accelerating or retarding force on the blade elements, depending on whether the blade has flapped nearer to the axis (usually up) or further away (usually down) These forces appear as vibration at the blade root, and for every harmonic of flapping there is an appropriate in-plane Coriolis vibration In Ref 1 it is shown that the Coriolis vibration at the blade root is, for zero flapping pin offset,

$$F_{cor} = -2\ M_m\ \omega\beta s\ \frac{d\beta s}{dt}$$

where M_m = the first moment of mass about hub centre

ω = the rotational speed of the rotor

βs = the blade angle to the plane of rotation

So far as first harmonic vibration is concerned, this equation shows that if the mechanical axis is normal to the tip path plane $d\beta s/dt$ must be zero and therefore there will be no first harmonic force fluctuation In itself this result has little practical utility, but it is obvious that if we can adjust the mechanical axis to eliminate the vibration, we can also adjust it to give any specified first harmonic blade vibration By so setting it that the Coriolis vibration cancels out the first harmonic blade vibration due to other causes we achieve an elegant solution to the problem of the in-plane vibration of a two bladed rotor without drag hinges It is shown below that this " dynamic balancing " can also be applied to higher harmonics

When a rotor has drag hinges the blade is relatively free to move under the influence of first harmonic Coriolis forces, thus avoiding the high bending stresses at the root The magnitude of the in-plane forces then depends on the drag hinge offset and the drag hinge dampers If all dampers work correctly the resultant hub forces due to first harmonic Coriolis are diminished but if one damper increases or reduces its damping rate, considerable vibration can arise (see for example Ref 7)

A second method of minimising Coriolis forces is the " tilting hub " of which the " floating hub " is a varient With this type of rotor the flapping of the blades relative to the mechanical axis is small except during a control movement The case of the control being moved rapidly through its extreme travel imposes severer in plane bending stresses on the blade structure than are present in the conventional fixed axis designs (in the absence of drag hinges) but the steady Coriolis vibration level is much lower Transmission problems make this system unsatisfactory on shaft driven helicopters, but it is favoured by some designers for tip-driven rotors, particularly with high inertia blades (ramjet rotors for example) where the phasing between flapping and feathering tends to 90°, and where the flapping hinges are centrally located

If we define blade flapping with respect to the mechanical axis in the usual way, the Coriolis force at the root of a blade is of the form, relative to the blade axes, and for the case of no drag hinges

$$\begin{aligned} F_{cor} = -2\,\omega^2 M_m \Big[& a_o a_{1s} \sin\psi - a_o b_{1s} \cos\psi - a_o b_{2s} \cos 2\psi \\ & + \left\{a_o a_{2s} - \tfrac{1}{2}\left(a_{1s}^2 - b_{1s}^2\right)\right\} \sin 2\psi \\ & - \left\{a_o b_{3s} - \left(a_{1s} b_{2s} - b_{1s} a_{2s}\right)\right\} \cos 3\psi \\ & + \left\{a_o a_{3s} - a_{1s}\left(a_{2s} + a_{4s}\right) + b_{1s}\left(b_{2s} + b_{4s}\right)\right\} \sin 3\psi \Big] \end{aligned}$$

Where M_m = first moment of blade mass about hub

This equation brings out a number of important points It is evident that even when the tip path plane is normal to the mechanical axis and the coning angle is zero, higher harmonic force fluctuations still occur If the small coning angle is obtained by off loading onto a low aspect ratio wing

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set directly underneath the rotor (as on the Fairey Rotodyne for instance) it is suggested that the curved flow field above the wing can magnify the higher harmonics of rotor flapping to such an extent that hub vibration is worse than if the wing had never been fitted, neglecting considerations of blade stalling

On a four bladed rotor the most troublesome blade vibration is the third harmonic, and the equation for F_{cor} offers hope of balancing the aerodynamic forces in the same way as was suggested for a two bladed rotor This balance can be maintained throughout a large control range if the helicopter is equipped with aerodynamic stabilising surfaces, such as an adjustable tailplane The same is true for a rotor with any number of blades, but the reliable calculation of aerodynamic force fluctuations above third harmonic is not feasible with the small design effort at present available in this country It should be remembered that Stewart's classic demonstration of the rapid fall-off in blade amplitude with harmonic number is not necessarily true, and that if Hafners pulsating flow field is shown to be significant it may well be that increasing the number of blades will increase the amplitude of some higher harmonics of flapping, and hence hub vibration

Transmission of blade vibration to the fuselage

In Ref 1 the following theorem was proved for fully articulated rotors

For any finite drag hinge offset a non-harmonically varying force acting on the blade is repeated at the drag hinge in the following manner

(1) The steady component is magnified by the multiplier $(1 + \frac{r_F}{l})$ Thus the amplification can be as high as thirty times the original force

(2) Harmonically varying forces are multiplied by the factor

$$\left[1 + \frac{(n^2-1)}{n^2}\,\frac{M_{mD}\,r_F}{I_D}\right]$$

* This equation has been corrected in the written contribution received from Mr Brotherhood appearing later in the discussion

In these expressions

r_F = radius of applied force

l = drag hinge offset

n = harmonic number

M_{mD} = first moment of blade mass about drag hinge

I_D = second moment of blade mass about drag hinge

The general rule for in-plane vibration is that a vibration on the blade of frequency n per revolution produces hub vibration at the two frequencies of n — 1 and n + 1 per revolution As for vertical imputs, only frequencies which are integral multiples of the number of blades appear in the form of structural vibration These conclusions are summarised in Table 2 for clarity

As an example, a three bladed rotor without drag hinges will have the usual higher flapping harmonics, the amplitude of each harmonic being about 1/12 of the preceding one, as demonstrated by Stewart in Ref 2 in the absence of blade flexing or resonance, and in a constant induced velocity

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field These flapping movements will induce corresponding Coriolis forces of the next harmonic order Thus first harmonic flapping yields second harmonic Coriolis vibration, which on our three bladed example gives third harmonic hub vibration Second harmonic flapping results in a third harmonic Coriolis vibration, which cancels out (see Table 2) Third harmonic flapping yields fourth harmonic Coriolis, which adds to the third harmonic hub vibration already caused by first harmonic flapping , and so on Since flapping amplitudes decrease with increasing harmonic number, the higher frequencies diminish in importance unless they excite blade in-plane resonance, or unless blade flexing substantially modifies the simple theory

The diminuation of harmonic amplitude with increasing order is of fundamental importance in helicopter design and points the fact that the only sure way to reduce forward flight vibration is to increase the number of rotor blades The fitting of a wing to off load the rotor, for example, will not materially reduce the magnitude of the higher flapping harmonics, even if all the lift is taken on the wing, and may even increase them, as pointed out above

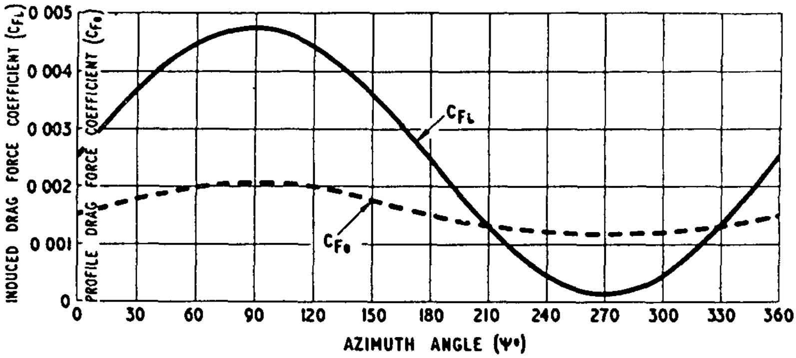
Considering vibration as a whole and neglecting the induced flow field of the wing, one would expect *some* reduction by offloading, because of the reduction of coning angle which this gives , the reduction would be very marked on a two bladed rotor, less on a three blader, and on a four blader one would expect no appreciable benefit at all until reaching speeds associated with blade stalling

In the special case of tandem helicopter it is difficult to see how there could ever be a case for the use of stub wings All their beneficial effects can be obtained in greater measure by simply off-setting the flapping hinges or using stiff hinges (see Ref 15) for a weight penalty which is negligible by comparison with the weight of stub wings The only alteration necessary to blade design is a reduction in the solidity of the rear rotor if the longitudinal stability of the basic tandem is to be improved without a loss in aerodynamic efficiency

Aerodynamic Forces

As we saw in the section Vertical Vibration of a Balanced Rotor, the simultaneous variations of airspeed, pitch angle, inflow and flapping result in very complex force fluctuations on a rotor blade, resulting in both vertical and in-plane hub vibrations The rules given in the preceding section apply for all in-plane vibrations, but although it would be most profitable to examine the fundamental causes of these aerodynamic vibrations, it is not physically possible in the time available In Ref 1 the fundamental mathematics are presented as simply as possible, in order to outline the basic physics A more valuable reference, when it appears, will be a paper which I understand Mr P Brotherhood has prepared at the R A E

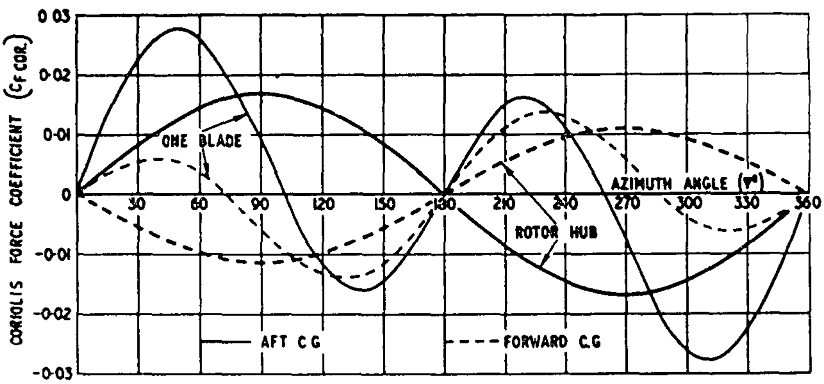
A general idea of the problem is given by Figs 2–5, which are part of routine design calculations for a small two bladed helicopter of about 1600 lb A U W , which has no drag hinges In Fig 2 is shown the variation of the coefficients of induced drag C_{F_i} and profile drag C_{F_0} with azimuth position of the blade Note the induced drag, which includes the effect of disc tilt, is much more powerful than profile drag It should be pointed out that



VARIATION OF INDUCED AND PROFILE DRAG FORCE COEFFICIENTS
WITH
AZIMUTH ANGLE FOR ONE BLADE AT 70 KNOTS
SEA LEVEL I C A N CONDITIONS

$a_0 = 0\ 9225°$ $a_1 = 2\ 4926°$; $b_1 = 0\ 1916°$ $\mu = 0\ 1427$ $\lambda = 0{\cdot}0309$ $\delta_0 = 0\ 0106$ $C_T - 0\ 0551$ V_T 800 FT/SEC

Fig 2



VARIATION OF CORIOLIS FORCE COEFFICIENT WITH AZIMUTH ANGLE
FOR
SINGLE BLADE AND FOR ROTOR HUB AT TWO C.G. POSITIONS
70 KNOTS IN I.C.A.N SEA LEVEL CONDITIONS

AFT C.G. a_{1s} 4°47 b_{1s} 0°
FWD C G a_{1s} +3°·12 b_{1s} = 0°

a_0 0·9225° $a_1 = 2\ 4926°$ $b_1 = 0{\cdot}1916$ $\mu = 0{\cdot}1477$ $M_B = 11{\cdot}29$ SLUGS FEET λ 0 0309
A.U.W. = 1550 LB.

Fig 3

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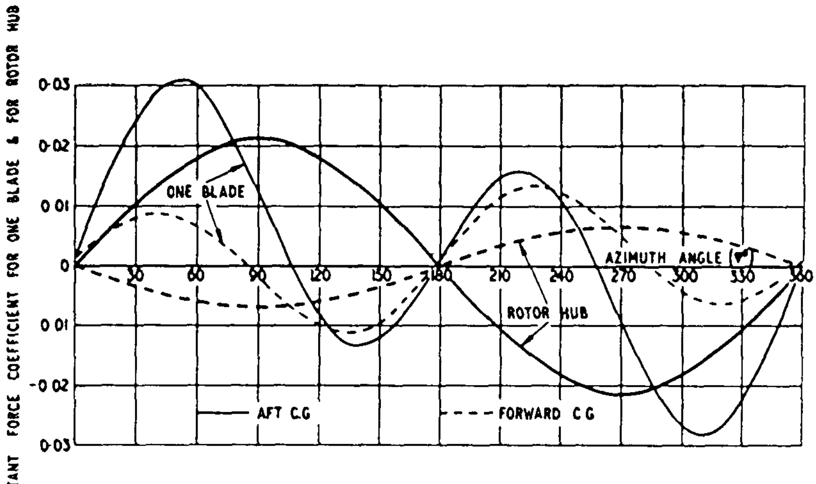
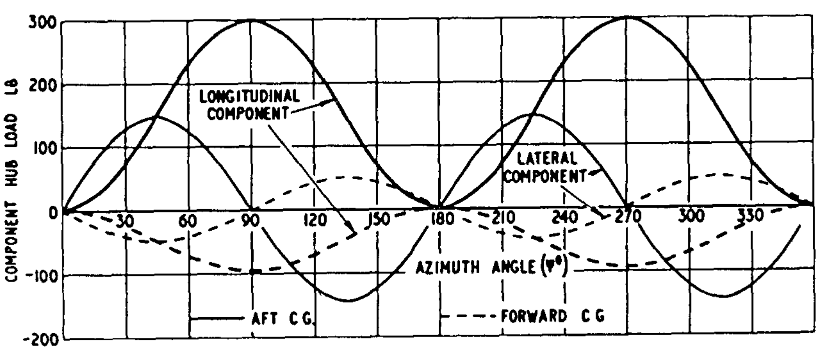


VARIATION OF RESULTANT FORCE COEFFICIENT WITH AZIMUTH ANGLE
FOR
SINGLE BLADE AND FOR ROTOR HUB AT TWO C G POSITIONS
70 KNOTS IN ICAN SEA LEVEL CONDITIONS

μ 0 1477 V_T 800 FT/SEC AUW 1550 LB

Fig 4



VARIATION OF COMPONENT HUB LOAD WITH AZIMUTH ANGLE
FOR TWO C G POSITIONS
70 KNOTS IN ICAN SEA LEVEL CONDITIONS

AFT C G a_{1s} 4° 47 b_{1s} = 0°
FWD C G a 3°·12 b_{1s} 0°

a_0 = 0·9225° a = 2 4926° b = 0 1916° μ = 0·1477 V_T = 800 FEET/SEC λ 0 0309
AUW = 1550 LB

Fig 5

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this variation bears little relation to the curves given by Hafner in Ref 9 for a typical flight condition of the Bristol 171

In Fig 3 Coriolis forces are plotted for the same blade, for two extreme C G positions, and also the rotating hub force when those of the two blades are added By comparison with Fig 2 it is evident that the aerodynamic force fluctuations at most amount to less than a quarter of the Coriolis force fluctuations It is not surprising therefore that in Fig 4 the total force fluctuation is very similar to that of Fig 3

Finally, in Fig 5 the rotating hub forces are related to aircraft axes to give lateral and longitudinal vibration components It is evident that in the case of a rotor without drag hinges there is a very marked change of vibration level with C G position, and also that Coriolis effects produce a very large steady in-plane force, most of which is due to neglecting radial accelerations relative to the blade axes *

Additional Causes of Vibration

In a balanced rotor there are a number of secondary sources of vibration which should be appreciated before approaching the problems of balancing
Off-set flapping hinges cause rolling and pitching vibrations to be transmitted to the fuselage, in addition to the normal vertical and in-plane forces This effect is treated in Ref 10
Pulsation of Induced Flow Field due to the finite number of blades in the rotor This is self explanatory, but as yet no investigator has actually analysed the problem, although Hafner has stated that the Bristol team are working on it
Operation in a wake, the classic example of which is the operation of the rear rotor of a tandem helicopter in the induced wake of the front rotor
Variable "ground effect" If a blade passes close to the fuselage or some stationary component its circulation will be increased in a manner directly analogous to the well known "ground cushion" effect Thus it experiences an impulse, both vertical and in-plane, which is felt at the hub
Classical flutter can occur if the blade elemental C G line is behind the aerodynamic centre line Theoretical treatment to date has been rather unsatisfactory, but a safe general rule for avoiding it is that the C G should not be aft of the aerodynamic centre, and both should be close to the flexural axis

Flutter is not always as catastrophic in practice as theory predicts, presumably because the equations are modified at large amplitudes, provided it occurs some way below the nominal design rotor speed The writer ran a ramjet rotor last February with the principal axis 0 3 ins (5% of the chord) behind the flexural axis at the tip The aerodynamic centres of the blade lay on the flexural axis but the aerodynamic centre of the ramjet was 2 4″ from its leading edge (11% of the ramjet length) and 110% of the tip chord in front of the blade flexural axis Flutter built up to a maximum amplitude within two seconds and from outside the disc it appeared that the tip path plane had become violently unstable (weaving) with a maximum amplitude of between 12 and 18 inches

The blade suffered no damage from four 5 second periods of this extreme flutter, although it was admittedly of steel and glass reinforced plastic construction During flutter a very considerable vibration was observed in the central support pylon

* *Mr Rogers points out in the Discussion that no steady Coriolis force does in fact exist*

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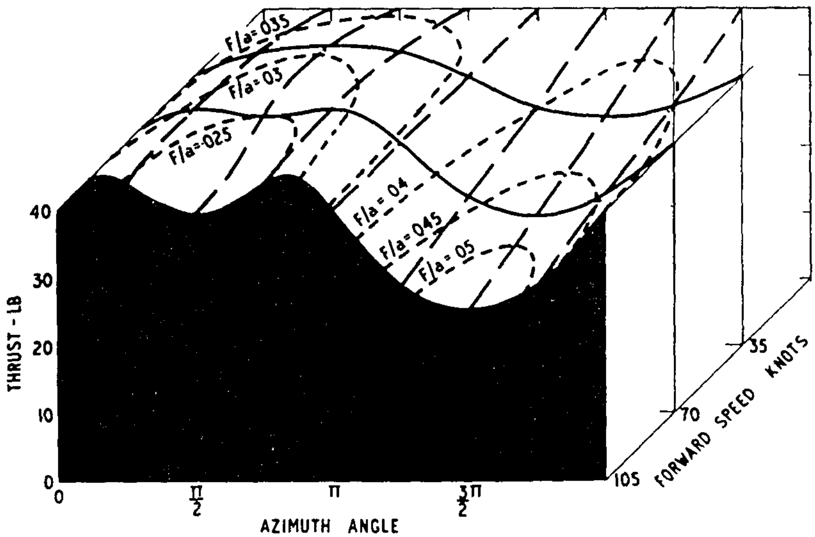
The construction of a trailing edge fairing brought the aerodynamic centre back to the flexural axis, and as would be expected from theoretical considerations, no further flutter took place

Tip mounted power units such as ramjets experience a variation in ram pressure in forward flight, and this results in a thrust pulsation A typical variation for a small helicopter ramjet is given in Fig 6 (taken from Ref 1)



TYPICAL VARIATION OF RAM JET THRUST
WITH AZIMUTH IN FORWARD FLIGHT
I C A N SEA LEVEL
$V_T = 800$ FT/SEC FUEL FLOW $= \frac{50}{\eta_c}$ GALLS/HR

Fig 6

Weaving, a phenomenon experienced on rotors with see-saw flapping hinge, is a form of aerodynamic flutter peculiarised by the fact that it can occur even if the blade elemental C G line is in front of the aerodynamic centre It gains its name from the appearance of the wavy path traced by the blade tips , although conventional flutter can also give this effect Remedies suggested in Ref 11 include decreasing the coning angle of the blade, designing the blades so that their mass is confined to the plane of rotation, increasing the control system stiffness and forward position of the centre of mass, and adding mechanical damping to the rotor system

Operating in the " Vortex Ring " state

At certain rates of descent at low forward speed rather erratic vibration is caused by the unsteady nature of the airflow through the rotor It is not likely that this vibration would be mistaken for anything else, as it is well defined on most helicopters, and limited only to one type of flight path

VERTICAL VIBRATION DUE TO UNBALANCE

A rotor blade is uniquely described by reference to the following properties —

(1) Radial dimensions
(2) Section dimensions
(3) Twist
(4) Root incidence
(5) Surface finish (roughness, wavyness and excrescences)
(6) Weight
(7) First moment of mass
(8) Second moment of mass
(9) Tab size and setting
(10) Position of aerodynamic, flexural and inertia axes

If any one blade differs from its fellows in any one of these particulars, vibration will be caused, in one or all of the three possible ways If we empirically limit our discussion to forms of unbalance that are commonly met with, we may say that the factors most affecting vertical vibration are blade twist, root incidence, and second moment of mass

Blade twist and incidence can be considered together Suppose a simple untwisted untapered blade in a rotor is set at a collective root pitch angle greater than its fellows by an amount $\Delta\theta$ Obviously it will experience a greater thrust, and the excess thrust at any element, assuming constant induced flow, will be

$$d(\Delta T) = a\,\Delta\theta\,\tfrac{1}{2}\,\rho\,V_T^2\,(x + \mu \sin\psi)^2\,cRdx$$

Integrating along the blade to obtain the total excess thrust coefficient we have, ignoring root and tip losses,

$$\frac{\Delta T}{\frac{1}{2}\,\rho\,V_T^2\,c\,R} = a\,\Delta\theta\,\left(\tfrac{1}{3} + \frac{\mu^2}{2} + \mu\sin\psi - \tfrac{1}{2}\,\mu^2\cos 2\,\psi\right)$$

Thus this form of unbalance yields a first harmonic vibration

$$\Delta C_T = a\,\Delta\theta\,\mu\sin\psi$$

and a second harmonic vibration

$$\Delta C_T = -\,a\,\Delta\theta\,\tfrac{1}{2}\,\mu^2\cos 2\,\psi$$

Note that both are proportional to the rigging error which causes them, and that the second harmonic vibration is of the order of one tenth of the first harmonic amplitude

If the twist of a blade is different from that of its fellows much the same result is obtained Suppose the total linear twist of one blade is $\Delta\theta_T$ less than the standard, then its incidence at any element will be $\Delta\theta_T x$ greater Following the same reasoning as for a rigging error, we have first and second harmonic vibrations resulting, of magnitude

$$\Delta C_T = \tfrac{2}{3}\,a\,\Delta\theta_T\,\mu\sin\psi$$

$$\Delta C_T = -\,a\,\Delta\theta_T\,\tfrac{1}{4}\,\mu^2\cos 2\,\psi$$

Thus errors in blade twist lead to the same vibrations as errors in root incidence

The importance of the third form of unbalance—the second moment

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of mass—can be demonstrated by considering its effect through the mechanism of coning angle It is easily shown (for example Ref 12) that coning angle is inversely proportional to the second mass moment Suppose therefore that this moment is greater on one blade, so that the coning angle is correspondingly reduced by a value Δa_o Then from Ref 12 the velocity component of U_P normal to the blade no feathering plane, due to coning will be reduced by an amount

$$\Delta U_p = \Delta a_o V \cos \psi$$

It follows that the blade angle of attack will be increased by the increment

$$\Delta \alpha = \frac{\Delta U_p}{U_T} = \frac{\mu \Delta a_o \cos \psi}{x + \mu \sin \psi}$$

Then following the reasoning used for rigging errors we shall find that we have again a first and a second harmonic vibration

$$\Delta C_T = \tfrac{1}{2} a \Delta \alpha_o \mu \cos \psi$$

$$\Delta C_T = \tfrac{1}{2} a \Delta a_o \mu^2 \sin 2 \psi$$

Once again the result confirms to the pattern set by the first In all cases we have assumed that only one blade is out of balance, but it is obvious that the trends revealed for these cases are generally applicable In all cases

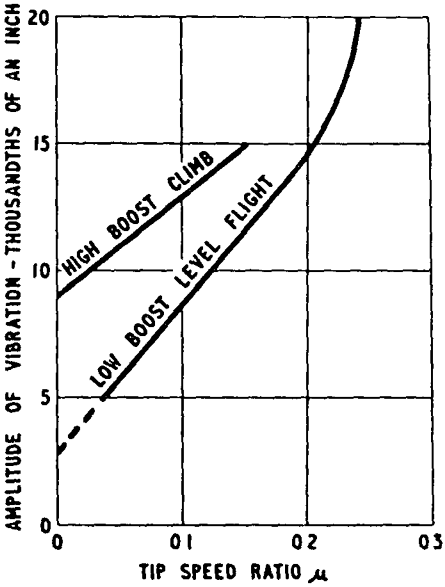


Fig 7

TYPICAL VARIATION OF ONCE ROTOR VERTICAL VIBRATION WITH TIP SPEED RATIO μ

VIBROGRAPH MEASUREMENTS ON STRUCTURAL MEMBER

the important first harmonic vibration is a linear function of the tip speed ratio μ, (for moderate speeds) and this result is of great assistance when one approaches the problem of suppressing the first harmonic vibration of a particular machine In Fig 7 are plotted two typical vibration amplitude variations with μ It is evident that the linear relationship breaks down at high speed, and this is attributed to the increase in coning angle associated with the increased rotor thrust at high speed This characteristic is fairly universal in plots of this nature

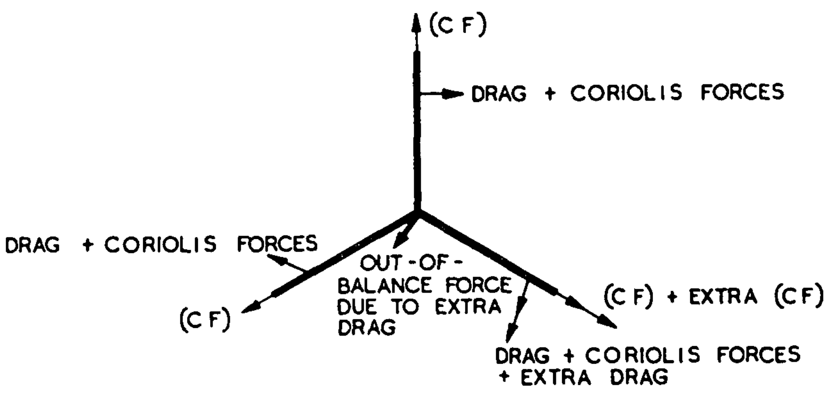
The second obvious conclusion is that in the case of the high boost climb, the greater part of the first harmonic vibration is independent of forward speed, and is therefore not caused by any of the mechanisms so far described On these particular tests the constant component could be due to fuselage "wallowing" under the influence of a large first harmonic in-plane vibration, since the measurements were made on one side of the fuselage There is, however, some evidence for believing that true first harmonic vertical vibrations can occur in hovering, but it is difficult to see what mechanism could account for this

In-Plane Vibration Due to Unbalance (In Hovering)

By far the most important in-plane vibration due to unbalance is that which appears as a first harmonic vibration in the fuselage Remembering the rules for in-plane forces, this vibration is obviously due to a steady out-of-balance force in the rotating system, and since the simplest case is that of a rotor without drag hinges, we will consider this first

Freely flapping rotor without drag hinges

Bearing in mind the table of descriptive properties compiled for a blade in the section Vertical Vibration due to Unbalance, Fig 8 depicts the two



CAUSES OF SIMPLE IN-PLANE UNBALANCE (WITHOUT DRAG HINGES)

Fig 8

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ways in which a steady out-of-balcnce force can be caused in the rotating system The most obvious is when the first moment of mass of one (or more) blades differs from the datum value This results in a steady rotating force which is felt as a first harmonic shake in the fuselage The problem of eliminating this vibration is experienced in all rotating systems, and is simply a matter of accurately balancing blades to a datum level during production Balancing to a datum level must be emphasised even today there are some helicopters whose blades are balanced in sets at the factory, so that the changing of one blade necessitates changing the entire set and its return to the factory This is obviously very unsatisfactory from an operational point of view, and it can usually be avoided by tightening up tolerances during manufacture

The second cause of an unbalanced force vector at the hub is a difference in blade drags This can be caused by a difference in

(1) Radial dimensions
(2) Section dimensions
(3) Twist }
(4) Root incidence }
(5) Surface finish
(8) Second moment of mass
(9) Tab size and setting

Some of these effects are obvious Differences in profile drag are caused by (1), (2), (3), (4), and less obviously by (8) (due to unequal coning), but normally the most important cause is surface finish variations (5) Induced drag variations are caused by (1), (3), (4) and (9) This means that isolating the cause of first harmonic in-plane vibration on a given helicopter can be a complex business, since we have to find which blade is causing the trouble, and for what reason

We shall take as a starting point the usually justifiable assumption that the blades have been correctly balanced, and that they have the same weight, first and second moments of mass, to within limits which were specified by the designer at an early stage in the design

We have first to determine which blade is out of balance This is done by means of a hand-held vibrograph which is fitted with an electrically operated reference base It is necessary to use a simple trigger mechanism somewhere on the rotating system of the helicopter which can be used to mark the vibrograph record so that the amplitude can be related to the azimuth position of the rotor It is evident from Fig 8 that if the drag of one blade is greater than datum, the out-of-balance vector at the hub will be at right angles to that blade, and pointing in the direction shown If the drag is less than datum, on the other hand, the vector will be in the opposite direction This result can be extended to the case when all blades differ from the one selected as datum, so that by determining the phase of the vector it is possible to determine the magnitude and sign of each blade's difference from the datum In practice this is best done by establishing a standard vibrograph and a standard fitting for it on the aircraft The machine is run up to speed, and a record taken in the tracking condition Then a specially constructed glass slide is laid over the recorded trace, which enables the drag increment of each blade to be read off

This " drag increment " must appear on the slide as a practical quantity which describes the curative measures the Ground Engineer must take to achieve a balanced rotor This entails some means of differentiating between profile and induced drag increments Now without going too deeply into the argument it is obvious that blade profile drag will be largely independent of pitch angle, and that any variations in profile drag due to excrescences and surface finish differences will be almost entirely independent of pitch angle It follows that if a vibrograph record is taken of the rotor running at zero pitch it will reveal differences which are mainly due to profile drag variations Ideally these variations should be balanced out by altering only profile drags, such as by adjusting a variable excrescence on the blade This need only be a short stud which could be screwed in or out of the under surface of the blade with a screwdriver, the number of turns needed being read off the glass slide The alternative procedure of altering the blade incidence until an induced drag increment is achieved which exactly balances the profile drag increment is very bad practice, although often used Obvious objections are that balance is only achieved in hovering, and that vertical vibration is induced as described in Section 6 when the aircraft is in translational flight

Having balanced the profile drags, a second vibrograph record can be taken with the rotor generating full thrust Any unbalance revealed will now be largely due to differences in induced drag, and this can be corrected by adjusting root incidences Accordingly the glass slide will also have to give readings in degrees of root pitch angle

Using the vibrograph and glass slide technique we have so far specified two rotor runs instead of the many currently used It is suggested that the rotor balancing should be rounded off with a third and last run, in which the collective pitch lever is moved up through the entire range, the record of this run being handed to Inspection as a normal routine check

A cruder method of determining the phase of the out-of-balance force is to hold a china marking pencil lightly against the rotor shaft, so that a mark occurs when the shaft is moved against the pencil The " shaft critical " rotor speed will be known, and assuming that the undercarriage damping is small, the mark on the shaft will be in phase with the out-of-balance force if the " shaft critical " speed is above that at which the test is carried out If it is below the mark will be 180° out of phase

When highly damped oleo legs are used on the undercarriage, and when the rotor speed is within 50% of the " shaft critical " speed, then the ratio of undercarriage damping to critical damping must be estimated, and the phase angle ϕ obtained from the simple relationship

$$\tan \phi = \frac{2 \, \dfrac{d}{d_o} \left(\dfrac{\omega}{\Omega_w} \right)}{1 - \left(\dfrac{\omega}{\Omega_w} \right)^2}$$

Where $\left(\dfrac{d}{d_o} \right)$ is the ratio of damping to critical damping, ω is the rotational speed of the rotor and Ω_w the speed at which " shaft whirl " occurs

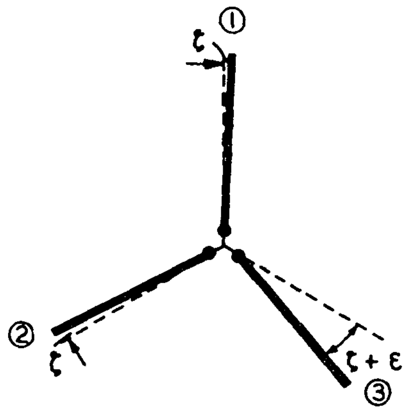
Fully articulated rotor

The causes of first harmonic in-plane vibration of a rotor with drag hinges are exactly the same as above, but with the big difference that the hinges magnify the blade forces many times Thus a fully articulated rotor is much more difficult to balance than one without drag hinges

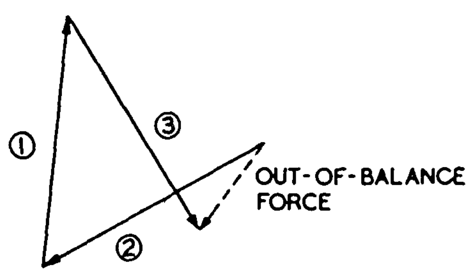
Because of hinge amplification, we need consider only blade centrifugal forces in Fig 9 to understand the conception of drag tracking which is so important with fully articulated rotors It is evident that if one blade has a greater lag angle than its fellows, then the polygon of forces will not be closed The gap will represent the resulting out-of-balance force

This result was generalised and expressed analytically in Ref 1 If on a three bladed rotor two lag angles differ from the third by the small angles ϵ_1 and ϵ_2 then the resulting out-of-balance force, expressed as a fraction of the C F in one blade root is,

$$\frac{\triangle F}{(CF)} = \sqrt{\epsilon_1^2 + \epsilon_2^2 - \epsilon_1\epsilon_2}$$



FULLY ARTICULATED ROTOR OUT OF DRAG TRACK



APPROPRIATE POLYGON OF CENTRIFUGAL FORCES

Fig 9

This equation is plotted in Fig 10 Any incremental lag angle ϵ will be due to an incremental force on the blade exerting an incremental torque $\triangle Q$ about the drag hinge

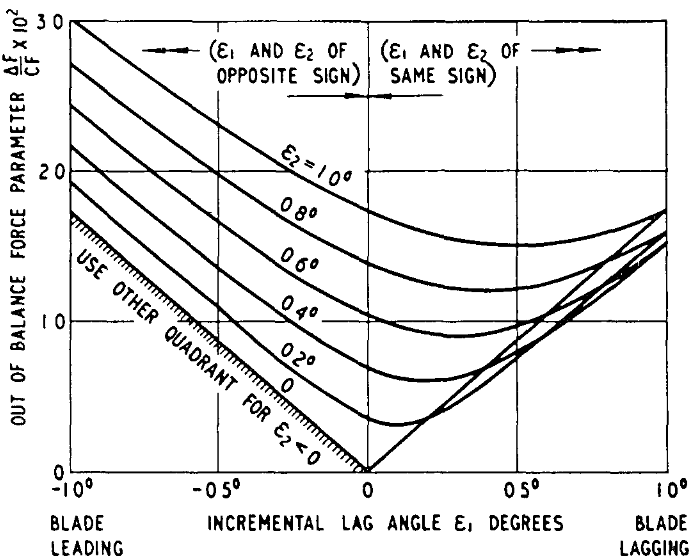
$$\epsilon = \frac{\triangle Q}{l\ (CF)}$$

$$\text{therefore}\ \triangle F_{HUB} = \frac{r_F}{l} \sqrt{\triangle F_1^2 + \triangle F_2^2 - \triangle F_1\ \triangle F_2}$$

As before l is the drag hinge offset, and r_F the radius at which the blade force is applied It is again evident that the "magnification factor" is r_F/l But for this term the equation is exactly the same for a hinge-less rotor

A fully articulated rotor is therefore "drag tracked" in exactly the same manner as we should balance one without hinges, except that much finer adjustments have to be made Also care must be taken, if inter-blade snubbers are fitted, to see that these are not unequally adjusted, thus tending to distort the blades from their correct azimuth spacing (On a constant speed rotor profile torque errors can be balanced by snubbers, but this is only satisfactory if the aircraft is limited to translational flight at low tip speed ratios)

The increased difficulty experienced in drag tracking a fully articulated rotor has an obvious corollary , it is more liable to go out of track in service, and it is suggested that this is yet another black mark against it This conclusion is borne out by the remarks of Mr J H Willans in his lecture



VARIATION OF OUT OF BALANCE FORCE ΔF WITH 'DRAG TRACK'

Fig 10

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last March, when he compared the Bell rotor very favourably with fully articulated designs

General Observations on " Automatic Rotor Balancing "

The development of this method requires a certain design effort by the Company producing the helicopter, because the skill of the design team is being substituted for the skill of the Ground Engineer In other words " tracking " is relegated to the position of " just another servicing job " instead of highly skilled art in which years of experience and some psychic ability still sometimes fail to achieve results

I once asked an eminent Chief Designer why the Boscombe Down report on his aircraft gave such a high vibration level " Oh " he replied, " it was a very bad rotor on that machine We were never able to get it smooth "

This reply indicated how far we have to go before we reach a product comparable in efficiency and uniformity with aircraft propellors Fundamentally the term " crank rotor " can mean nothing more than that the balancing techniques employed are inadequate The glass slide and vibrograph technique will go a long way to meet this deficiency but at best it constitutes an interim solution Ideally all new blades should be tracked against a master at the factory, and under no circumstances should balancing be subsequently necessary in service

In case the importance of tracking is not fully accepted, some remarks Mr J H Willans made in his lecture on March 11th should be recalled He said that " the problem of main rotor tracking is one which simply must be solved if the helicopter is ever going to be a commercial vehicle The smallest American design with a two blade rotor (and no drag hinges) is the only one which is almost trouble free At the other end of the scale is a British design where the defect assumes alarming proportions Blade tracking consumes an enormous quantity of man-hours, and even that is not the end of the trouble Quite apart from the time consumed and the number of men required to do this job, almost perfect weather conditions and daylight are required Tracking itself must be abolished "

This implies metal and/or plastic blade construction to fairly close limits—propeller limits in fact—and the elimination of tabs Most firms seem to have embarked on the design of metal rotor blades in any case mainly from consideration of performance, and in the cases where tight limits can be adhered to they should effect a substantial reduction in the total balancing time in the life of a helicopter

Many of the present metal blade designs use aluminium alloy and it is difficult for anyone connected with blade vibration to pass over this or to remain polite on the subject To guarantee a life of 1,000 hours with a conventional structure the fluctuating stress must not exceed 2% ultimate (Ref 14) or 5% with a fully bonded structure Liquid honing and polishing will raise these limits, but not to the figures of 30% ultimate which are sometimes caused by the fluctuating force imputs Thus it is only a question of time before an aluminium alloy rotor blade breaks, and although this might be acceptable if no alternative existed, it is absurd when steel is available to give infinite fatigue life Correctly designed, a steel blade is also easier to construct than a light alloy one, and a compound taper steel and fibreglass blade can, and has been constructed for a lower cost than would have been possible with improved wood construction The only real objection to steel blades seems to be that someone somewhere once observed that they would be difficult to make and heavy

SECTION 2 CONTROL VIBRATION

General Considerations

Periodic pitching moments about the pitch change axis of a blade produce cyclic stick forces in the same manner as periodic in-plane forces at the drag hinge produce longitudinal and lateral oscillations of the hub For a rotor with b blades, stick vibrations of frequencies b/rev , 2b/rev , 3b/rev , may occur due to blade pitching moments which oscillate at frequencies (b + 1)/rev , (b — 1)/rev , (2b + 1)/rev , (2b — 1)/rev , etc This rule includes steady forces, because steady stick forces are produced by first harmonic blade torque variations, and may be either stable (centralising) or unstable (divergent) depending on the input

These statements assume that the control system is completely reversible, but can be applied with some reservation to control systems such as that of the Bristol Sycamore where inertia dampers are fitted to reduce force feed-back Where irreversible power operated controls are fitted no feed-back is possible of course, but it will be a long time before the majority of helicopters are so equipped Even so equipped, provision must be made for emergency manual reversion, so that a limited amount of stick balancing is still essential

As an example of the basic laws, let us consider a three bladed rotor controlled by a fully reversible system The relation between blade pitching moments and stick forces will then be

Blade pitching moment	*Cyclic Stick forces produced*
1/rev	Steady force
2/rev	3/rev shake
3/rev	none in cyclic stick (3/rev shake in collective pitch lever)
4/rev	3/rev shake
5/rev	6/rev shake
6/rev	none in cyclic stick (6/rev shake in collective pitch lever)

The amplitude of the moment in the stick due to each blade is always one half the blade moment, when due allowances for gear ratio have been made

Stick balancing is primarily concerned with the cyclic stick, since in a correctly designed collective pitch lever, vibration is usually lost in its friction damper In hovering or on the ground, the conditions when stick balancing is carried out, the important factors are a difference between the constant components of blade torques, which appears as a 1/rev stick shake, and the first harmonic blade fluctuation, which appears as a steady stick force We shall therefore roughly assess the causes of blade steady and first harmonic torque fluctuation in order to understand the problems involved in balancing them Simple equations will be presented for the effects described, but it is unfortunately impossible to include proofs, which are often quite lengthy

Torque Due to Positions of Blade Axes

Considering the blade element in Fig 11 we see that there are four forces exerting moments about the hinge axis, of which only blade elemental weight can be regarded as negligible If we consider only first harmonic rotor flapping, which is justifiable for hovering in the absence of blade distortion, it can be shown that the resulting blade torque is a constant, except when the stick is actually being moved The total torque at any radius is

$$Q_{r_1} = z \int_{r_1}^{R} (dT - a_0\omega^2 \Delta m \, r) \, dr + a_0\omega^2 \int_{r_1}^{R} \Delta m \, r \, \Delta y \, dr$$

The second integral implies that C F forces can be taken as acting at the principal axis of the blade, and one of the most important checks made

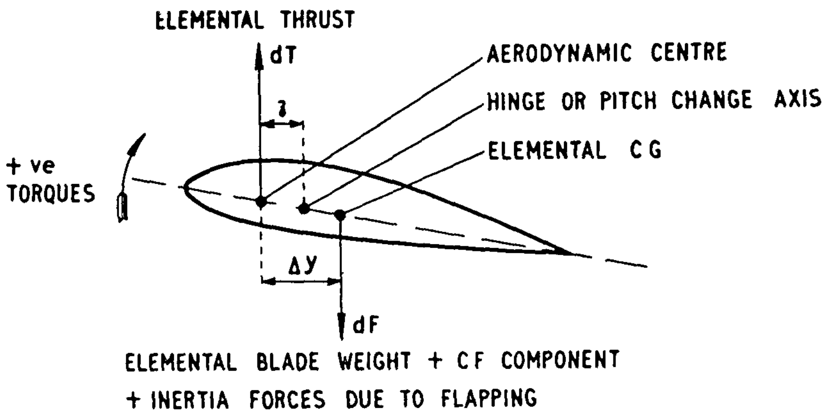

Fig 11

at the factory is ensuring that the principal axis (the line for which $\int \Delta m r \Delta y = 0$) lies within certain specified limits of the aerodynamic centre and hinge axis This is done by " swinging " the blade as a compound pendulum The blade C G is always on the principal axis, and when the pendular attachment is also on the axis, the blade will swing without any torsional oscillation about its axis, but only a precession due to air damping

It can be proved that even if the rotor blade is bent in the drag plane, the straightening out process when C F is applied will not affect the principal axis position as determined statically

If we assume that the principal axis is Δy_T behind the aerodynamic centre at the tip, and Δy_R at the root, and if we further assume that aerodynamic and hinge axes are parallel, then the non-dimensional root torque is

$$\frac{Q}{\frac{1}{8}\rho V_T{}^2 R^2 a C_o} = C_T \, z - \left[\frac{t_4}{t_3} C_T + \left(\frac{t_2 t_4}{t_3} - t_3\right) \sqrt{C_T \frac{\sigma_R}{16e}} + \right.$$

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$$+ \left(\frac{t_4^2}{t_3} - t_5\right) \theta_T\Big] \left[(\Delta y_R - z)\frac{\chi}{\gamma} + \Delta y_T - \Delta y_R\right]$$

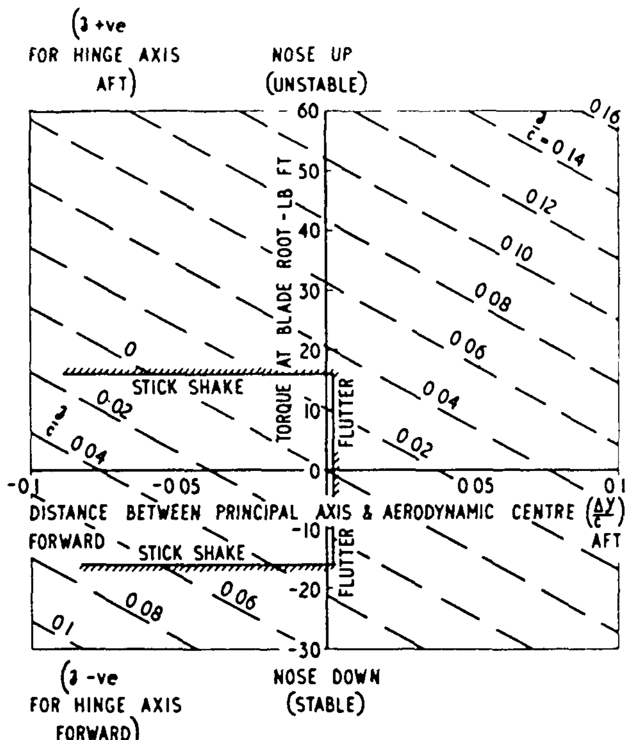
To obtain a clearer picture, assume that the principal axis is also parallel to the other axes, and that we are concerned with a constant chord untwisted rotor If the hinge and principal axes coincide, the torque coefficient is

$$C_Q = C_T \Delta y$$

as we should expect If the hinge and aerodynamic axes coincide, on the other hand

$$C_Q = a_o \chi \Delta y$$

which is also what we should expect from general considerations



EXAMPLE BLADE

VARIATION OF TORQUE AT BLADE ROOT WITH PRINCIPAL AND HINGE AXIS POSITION

l DISTANCE OF HINGE AXIS BEHIND AERODYNAMIC CENTRES

NOMINAL THRUST AND TIP SPEED

Fig 12

It is evident that in the general case there are too many factors to permit of easy generalisation of the effect of relative positions Each blade must be

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treated as a separate problem, and wherever possible the problem itself should be eliminated by arranging for the three axes to coincide In establishing balance limits at the design stage it is important to remember that apart from flutter and system instability considerations, it is not absolute torque on the blade which is important to the cyclic stick, but the maximum torque difference which can occur between blades

The establishment of balancing limits for a typical blade is illustrated in Fig 12

Torque Due to Aerodynamic Pitching Moment

Since aerodynamic pitching is a constant torque in hovering there is a temptation to regard it as unimportant In forward flight however, the pitching moment is increased on the advancing blade, due to the increased air velocity, and reduced on the retreating side This leads to first and second harmonic torque fluctuations about the mean hovering value, which appear as steady and 3/rev cyclic stick forces in a three bladed rotor Today most people seem to agree with Hafner (Ref 6) that symmetrical blade sections are to be preferred for general use

Torque Due to Propeller Moment

A rotating blade will always tend to throw off pitch until the mean effective chord of all its elements is in the plane of rotation The phenomenon is considered in detail in Ref 13 and we need only note the result that propeller moment can be expressed as

$$Q = - I_R R \omega^2 (A\theta_R - B\theta_T)$$

where A and B are given by Ref 13 Thus propeller moment yields a steady and first harmonic torque variation in the blade

Inertia Torque and Torsion Bearings

If a cyclic pitch variation is applied to a rotor, a harmonically varying root torque is necessary to cause the blades to oscillate If I_P is the polar moment of inertia of the blade, this torque is

$$Q = - I_P \frac{d^2\theta}{dt^2} = - \omega^2 I_P (A_1 \cos \psi + B_1 \sin \psi)$$

Inertia torque therefore appears in the cyclic stick as a steady destabilising force

Conventional torque bearings increase this destabilising force, by causing a " square wave " torque which opposes the direction of blade movement A typical value for this torque is

$$Q = 2\,7 \times 10^{-4} \text{ (CF)} \qquad \text{lb ft}$$

On a two bladed rotor torsion bearing torque will cause a second harmonic vibration in the stick On a three blader it results in a substantially steady destabilising force, provided the torque from each bearing is the same

Torque-Bar Torque

The elegant torque bar first used by Hafner (Ref 6) is an attractive method of eliminating the two destabilising torques described in the previous section If C_B is the stiffness of the bar (which is made up of a constant stiffness due to its physical nature, and a variable stiffness due to the blade C F, the latter being proportional to ω^2) and if $(\theta_R)_{B=O}$ is the pitch angle at which the bar is untwisted, then the torque is

$$Q = - C_B [\theta_R - (\theta_R)_{B=O}]$$
$$= - C_B [A_o - (\theta_R)_{B=O}] + C_B [A_1 \cos \psi + B_1 \sin \psi]$$

It is evident that so far as cyclic stick loads are concerned, the zero position of the bar is of no importance

Comparing the oscillating component with that of the inertia torque equation in the section Inertia Torque and Torsion Bearings, it is seen that the two brackets are of opposite sign If C_B is chosen so that

$$C_B = \omega^2 I_P$$

there would then be no resultant torque in the controls This property is the basis of the torque bar as applied to helicopters, although this simple relationship is modified by the introduction of the propeller moment oscillating terms

For the remainder of this lecture it is assumed that a torque bar is in fact being used Thus the results will have to be slightly modified to apply to rotors using torsion bearings

Total Torque of Balanced Blade

If we assume that the three blade axes of the section Torque due to Positions of Blade Axes, are coincident, then the total torque on a blade is

$$Q_T = - I_R R A \omega^2 (A_o - A_1 \cos \psi - B_1 \sin \psi) + I_R R B \omega^2 \theta_T$$
$$- \omega^2 I_P (A_1 \cos \psi + B_1 \sin \psi) -$$
$$- C_B [A_o - (\theta_R)_{B=O}] + C_B (A_1 \cos \psi + B_1 \sin \psi)$$

= Propeller moment +
+ Inertia torque +
+ Torque bar torque

If torsion bearings are used the last line will be different

It is evident from this equation that the fluctuating terms will be zero if the torque bar stiffness is

$$C_B = \omega^2 (I_P - I_R R A)$$

If the bar is insufficiently stiff there will be a resultant first harmonic which will appear as a steady destabilising force on the stick of a three bladed rotor As in the case of a blade with torsion bearings, trimmers or centralising springs will then have to be fitted to make the system stable

A more common fault is for the bar to be too stiff, due to neglect of the C F stiffness term, in which case the stick will experience a stabilising force (three or more blades) or twice rotor vibration (two blades) In both cases the forces will vary linearly with stick displacement

The steady torque component in one blade is

$$\triangle Q_T = - A\ (I_R R \omega^2 A + C_B) + I_R R \omega^2 B \theta_T + C_B\ (\theta_R)_{B=O}$$

The maximum value of $\triangle Q_T$ can be minimised by the correct choice of $(\theta_R)_{B=O}$ but the linear variation with collective pitch angle A_o must be externally balanced by some external spring and linkage or by making the A_o coefficient equal to zero,

$$I_R RA = \frac{C_B}{\omega^2}$$

This second solution is rather ugly since it entails mounting a mass balance on an arm projecting substantially normal to the chordal plane of the blade, but is a welcome simplification on very light helicopters, and is lighter than external springs

Additional Causes of Vibration in a Balanced Control System

Spider Instability The Hafner spider is in one form prone to a special instability in which collective pitch stick loads appear as a destabilising force in the cyclic control with a magnitude which is a linear function of applied cyclic It is for this reason that the collective system loads should be balanced out above the " dangle berry " on large helicopters, but in any case this instability makes the use of centralising bias mandatory on the cyclic pitch stick It should, however, be noted that this form of instability is an advantage if for some reason a torque bar of low enough stiffness cannot be made for the " classic tuning " of the section Total Torque of Balanced Blade

Control system resonance A helicopter control system is made up of inertias and stiffnesses, and is subjected to fluctuating force inputs Thus resonance can arise, the nature of which will be intimately bound up with the physical properties of the actual system One manifestation of such resonance is stick circling, the violence of which increases with time This group of problems can be tackled with the conventional and well established techniques

Discussion of Overall Control Balancing

Collective Pitch Stick Balancing of controls can be divided under the two headings of Design and Correction Generally speaking the collective pitch stick forces are balanced in the design stage, and any corrections which are found to be necessary, due to manufacturing errors, etc , can be rectified by adjusting the balance springs In many helicopters a simple adjustment point is provided for this purpose

On some helicopters trailing edge tabs situated near the root of the blade are used for balancing out collective pitch stick loads This is not only inelegant, but often causes more trouble than it cures The only point in its favour is that both the causes of unbalance and the cure vary with the square of the rotational speed

Most collective pitch sticks are in any case fitted with friction nuts, which cover a multitude of vibrations In general this is a good idea, since it allows the pilot to leave the stick alone when the occasion demands, but if friction is used to conceal bad balancing the control is tiring for the pilot, and may also tend to " creep " under load

The procedure for balancing is very simple in principle , the friction is removed from the system, and the rotor run up to speed Then at any collective pitch setting the stick should remain in position when released from the hand If it does not, the necessary corrections can usually be made by tightening or slackening off the balance spring For complete balancing to be always possible it is essential to have two independent balance springs

An elegant method of balance for ultra-light helicopters is the mass balance described in the section Total Torque of Balanced Blade The most effective form of adjustment here lies in varying the length of the arm on which the mass balance is carried

Cyclic Pitch Stick

As described in the section Total Torque of Balanced Blade the most effective design action to balance the cyclic stick lies in the use of correctly tuned torque bar It is normal for stick forces to be caused by additional effects however, which are fundamentally due to manufacturing errors

We can express the root torque of one blade as

$$Q_T = M_o + M_1 \ \theta_R + M_2 \ \frac{d^2\theta}{dt^2}$$

If the torque bar is not exactly tuned we have

$$Q_T = M_o + (M_2 \ \omega^2 - M_1) \ \theta_R$$

Now if the torque of one blade differs from that of its fellows, either or all of $M_o M_1$ and M_2 must be affected If M_o only is affected then a simple trailing edge tab is the ideal way of correcting it, but M_1 and M_2 require a correction whose magnitude varies with θ_R Probably the simplest way of doing this is to fix small balance weights on to the pitch change arm of the blade

We have therefore two rules for balancing out stick shake

(1) If the shake is *the same at all collective pitch settings* then it should be corrected by tabs or deforming the trailing edge

(2) If the shake *increases or decreases with collective pitch* then adjustment should be made by balance weights on the pitch change arm

Weights on the pitch change arm can also be used to find the phase, and therefore the cure of stick shake This is done by first measuring the amplitude of the shake, usually by clipping a small horizontal board to the stick, covering it with a sheet of paper, and holding a pencil to the paper whilst the stick is shaking This is repeated with a known weight attached to one pitch change arm

With a two bladed rotor this information is generally sufficient If the weight increases the stick shake, then the blade to which it was added was too nose heavy If shake is diminished, it was tail heavy The curative action to be taken depends on the two rules given above

The same principle can be applied to a three bladed rotor, and a method of phasing the shake was described by Turner in a lecture to the Association some years ago

It is suggested that tabs alone will never satisfactorily balance the cyclic stick, since they can only generate moments which are independent of blade pitch Provision should be made for one tab sufficiently near the blade root to avoid twisting the blade when it is in use, and a mass balance attachment fitting on the blade root arm Both should be constructed in such a manner that they can only be adjusted with special equipment In particular the tab should never shift in service, when once set, and on metal blades it is better to deform the trailing edge than to use tabs at all

Finally, I should like to thank the Bristol Aeroplane Company, Ltd, for permission to present this paper The opinions expressed are my own and do not necessarily represent those of the Engine Division I should also like to thank Mr P Brotherhood, of the R A E, for helpful criticism, and the many discussions we have had on vibration over the last two years

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LIST OF SYMBOLS

Units are Perry System slugs — mass
lb — force
ft — length
sec — time

a		slope of lift curve
a_o	=	coning angle
A_o	=	collective pitch angle at theoretical root
a_1	=	rotor flap back from no-feathering orbit
A_1	=	cos ψ coefficient of feathering relative to control orbit
A,B	=	constants given in Ref 13
b	=	number of blades
B_1	=	sin ψ coefficient of feathering relative to control orbit
b_1	=	lateral tilt of rotor to no-feathering orbit
C_F	=	force coefficient = $F/\frac{1}{8}\rho V_T{}^2$ a R Co
C_Q	=	torque coefficient = $Q/\frac{1}{8}\rho V_T{}^2$ a R^2 Co
C	=	chord at any radius
C_o	=	theoretical root chord at hub C_L
CF	=	centrifugal force in blade root
C_b	=	torque bar stiffness
C_T	=	thrust coefficient
	=	$T/\frac{1}{8}\rho V_T{}^2$ a b R Co or $T/{}^1\rho V_T{}^2$C R
C_L	=	lift coefficient
COR	=	suffix denoting coriolis effects
e	=	effective disc area
F	=	a force or a factor
g	=	acceleration due to gravity
I_D	=	moment of inertia of blade drag hinge
I_F	=	second moment of blade mass about the flapping pin
i	=	suffix denoting induced effects

I_R	=	moment of inertia difference at the theoretical blade root defined in Ref 13
I_P	=	Polar moment of inertia of blade about pitch change axis
l	=	drag hinge off-set
L	=	lift force
M_m	=	first moment of mass about hub $C/_L$
M_{mD}	=	first mass moment about drag hinge
m	=	blade mass
δm	=	elemental mass
n	=	any integer or harmonic number
o	=	suffix denoting profile effects
Q	=	a torque
r	=	radial distance along the blade
r_F	=	radius of an applied force
R	=	rotor radius
t_n	=	a taper constant defined in Ref 4
t	=	time
T	=	thrust of rotor
U	=	velocity relative to blade element
U_T U_P	=	horizontal and vertical components of U relative to no-feathering plane
V	=	forward speed of helicopter
V_T	=	rotor tip speed
W	=	all up weight of helicopter
x	=	r/R
y	=	distance between aerodynamic centre and elemental C G
z	=	amplitude of vibration
z	=	distance between aerodynamic centre and pitch change axis
α	=	blade elemental angle of attack
β	=	blade angle of flap relative to the no-feathering orbit
β_S	=	blade angle of flap relative to the shaft orbit
γ	=	an inertia number
	=	$I_{F/\frac{1}{8}\rho}R^4 a\ Co$
ϵ	=	lag angle difference
ζ	=	lag angle
θ	=	blade pitch angle at any element
θ_R	=	blade pitch angle at theoretical root
$(\theta_R)_{B=O}$	=	θ_R at which torque bar is untwisted
θ_T	=	blade twist, root to tip (positive for 'washout')
μ	=	tip speed ratio $V/_{V_T}$
ρ	=	mass density of air
σ_R	=	root solidity of rotor $= b\ Co/_{\pi R}$

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ψ = azimuth angle of blade

ϕ = inflow angle

χ = first moment of mass number = $Mm/\frac{1}{8}\rho R^3 a\, Co$

ω = rotor angular velocity

Ω = a circular frequency

References

No	*Author*	*Title*
1	P R Payne	" Helicopter Rotor Vibration in the Tip-path Plane " *Aircraft Engineering* June, 1955
2	W Stewart	" Higher Harmonics of Flapping on the Helicopter Rotor " *R A E Report Aero* 2459
3	R Hafner	" Simulation of Helicopter Operating Conditions in Ground Tests " April, 1953, issue of the *Journal of the Helicopter Association*
4	P R Payne	" The Flight Envelope of a Helicopter " *Journal of the Royal Aeronautical Society* November, 1954
5	R M Howarth & C H Jones	" Ground Resonance of the Helicopter " *Journal of the Helicopter Association* Vol 7, No 4 April, 1954
6	R Hafner	" Rotor Systems and Control Problems in the Helicopter " Paper read before the Anglo-American Aeronautical Conference September 5th, 1947
7	J A Cameron	" Recent Helicopter Flight Testing Experience " *Journal of the Helicopter Association* Vol 7, No 1 July, 1953
9	R Hafner	" The Domain of the Helicopter " *Journal of the Royal Aeronautical Society* Vol 58, No 526 Oct, 1954
10	P R Payne	" Hub Moments and Forces of a High Offset Rotor " *Aircraft Engineering* January, 1955
11	R P Coleman and C W Stempin	" A Preliminary Theoretical Study of Aerodynamic Instability of a Two-Blade Helicopter Rotor " N A C A RM L6H23, 1946
12	P R Payne	" A General Theory of Helicopter Rotor Dynamics " August, 1954, issue of *Aircraft Engineering*
13	P R Payne	" A Rapid Method of Estimating Propeller Moment " April, 1954, issue of *Journal of Royal Aeronautical Society* Addendum in December, 1954, issue
14	Ken Williams	" Fatigue Life of Wing Components for Civil Aircraft " *Journal of the Royal Aeronautical Society* November, 1952
15	P R Payne	" The stiff-hinged rotor " *Aircraft Engineering* Oct, 1955

Discussion

The **Chairman** referred to the Author's suggestion that some of the vibration could be absorbed by a pendulum He had seen a photograph in a Dutch periodical recently, of a new Italian helicopter which appeared to have pendulous weights attached to the hub This had mystified him at the time, but possibly they were devices of the nature referred to by the Author

The use of metal blades seemed to be regarded as the panacea for all ills, but the Chairman recalled remarks made to him by the engineering officer of a Canadian Helicopter Squadron which he visited last year The Squadron had small tandem