Analogue Computer Development with Reference to Helicopter Applications*

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DR G S HISLOP *(Chairman of the Executive Council) occupying the Chair*

INTRODUCTION BY THE CHAIRMAN

The CHAIRMAN, in introducing the Author, said that Mr VENNING had spent from 1941 to 1946 in the Royal Signals on mobile communications and from 1946 to 1949 at the City and Guilds Engineering College From 1949 to 1952 he had been in the Royal Naval Scientific Service, working on underwater acoustics From 1952 to 1955 Mr Venning had been at the College of Technology, University of Manchester, as a Lecturer in Electrical Engineering In 1955 he joined the University of Southampton in the Department of Electronics where, in addition to lecturing in electronics he was undertaking a research contract for the Ministry of Supply basically on the subject of his present paper

MR B H VENNING

Summary

This paper arises from work which has been carried out at the University of Southampton on the use of an electronic analogue computer in the study of the Ground Resonance problem in helicopter design Before detailing the work on this problem, the background of analogue computing is discussed both in method and in application, together with the limitations likely to be of interest to the user of this means of computation Illustrations of the use of the present computer in dealing with the linear equations arising from Coleman's treatment of Ground Resonance are given, and also the various ways in which the simple treatment needs to be extended As a start in this direction, the problem of " bouncing " of a helicopter from wheel to wheel has been simulated using switched parameters of damping and stiffness, and a start is now being made on simulating the stepped friction damper used for hub damping

* This work has been carried out under a research grant from the Ministry of Supply, to whom thanks are also due for establishing contacts between the University and the Helicopter Industry Acknowledgments must also be made to Dr H Fuchs, now of Blackburn and General Aircraft Ltd , who constructed the original computer , to Mr T Ciastula of Saunders-Roe Ltd , to Messrs R M Howarth and C H Jones of Bristol Aircraft Ltd , Mr J M Harrison of Westland Aircraft Ltd and Professor Hemp and Mr D Howe of the College of Aeronautics, Cranfield, all of whom have assisted in supplying data relating to helicopter design

(1) INTRODUCTION

The growth in complexity of many problems of engineering design over the last two decades has increased the volume of computation required to a stage where it is often beyond the capabilities of manual computors The processes of evolution in the scientific world seem to mean that in all branches of engineering we work nearer and nearer the limits of satisfactory operation, and more aspects of design have to be settled in the planning stage and less left to chance and the factor of safety (or ignorance) As the calculations become more involved, so the faith in the correctness of the answer becomes less, leading to constant cross-checking and re-calculation The need, therefore, is for a machine that solves the problem as a whole, giving the solution in a form that can be appreciated by the designer concerned, with an accuracy that is consistent with that of the data used

The ideas of calculating machines are not new, and the principles on which the modern digital calculators work were expounded early in the 19th century, when hardly any of the present-day concepts of electronics, by which they have been realised, were known Digital computers perform all their work by counting, addition, subtraction, discrimination of magnitude and a capacity for memory being the backbone of all their operations, and the earliest calculating machines attempted to use purely mechanical devices These failed, being too complex and too slow, and electronic counting had to be evolved before successful machines were produced

The alternative of translating the awkward variables of a problem to some other variable, more amenable to the desired mathematical operations, is not new either This is the principle behind analogue computing, the most familiar example being the engineer's slide-rule with the numbers of a problem represented by lengths on a logarithmic scale, so that addition of two lengths is equivalent to multiplication of the two numbers The first analogue machines to be applied to complex problems were the mechanical differential analysers, using mechanical integrators Electronic analogue computers were the outcome of developments during the war, when electronic circuits were produced that could add and integrate voltages with sufficient accuracy and stability to replace the earlier mechanical integrators Electrical voltage became the analogue of the quantities in the problem to be solved, and the mathematical equations were simulated using a number of electronic units to perform the required mathematical operations This is the basis of the " simulator " type of analogue computer, or differential analyser as it was originally called when concerned wholly with the solution of differential equations

Two other types of computers also exist that fall under the category of analogue devices In one, the " electrolytic tank " type, the computing element is a continuous conducting medium in which the distribution of the electric potential within the volume satisfies the same conditions as the problem variable This forms an analogue capable of solving a partial differential equation, but the general utility of the method is not great The other form is the " network analyser " type, utilising a network of passive elements, usually resistors, to set up a static analogue of the problem The network effectively forms a matrix of a number of electrical admittances which can represent the coefficients of certain types of equations, and the method finds application in complex structure problems, such as wing loadings

Neither the electrolytic-tank nor the network analyser have become general purpose computers, owing to the limitations in the size and scope of problems that they can handle, and so the term electronic analogue computer will nearly always be found to refer to the simulator type

(2) The Solution of a Simple Vibration Problem

If the problem is one that can be stated in terms of a set of simultaneous linear differential equations, the computing elements are arranged to solve each of the equations, with interconnections between the units to represent the whole matrix Consider a relatively simple problem with two degrees of freedom, of the type which is a little tedious to calculate by hand, but is ideally suited for solution on an analogue computer[1] The equations of the system (Fig 1a) are —

$$Mx_1 + dx_1 + (K + k)\,x_1 \qquad - dx_2 - kx_2 = P_o \sin \omega t$$

$$- dx_1 - \quad kx_1 \qquad + mx_2 + dx_2 + kx_2 = 0$$

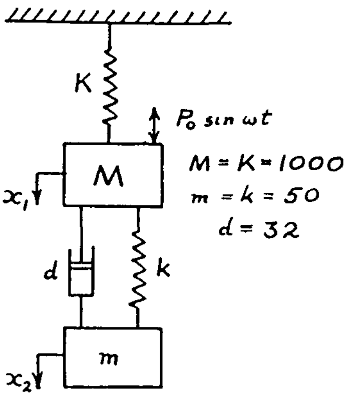

FIG 1(a) COUPLED MASS – SPRING SYSTEM

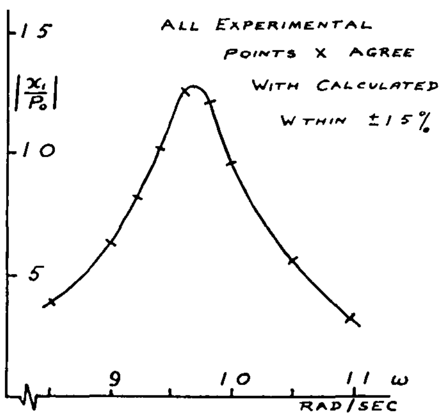

FIG 1(b) STEADY-STATE RESPONSE OF x_1

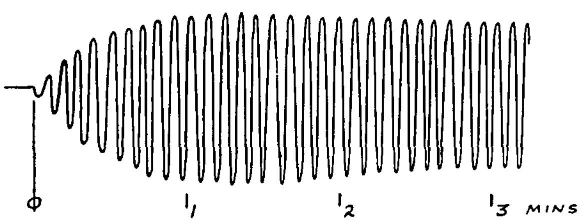

FIG 1(c) TRANSIENT RESPONSE OF x_1

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which for the values chosen become —

$$1000\,\ddot{x}_1 + 32\,\dot{x}_1 + 1050\,x_1 \qquad - 32\,\dot{x}_2 - 50\,x_2 = P_0 \sin \omega t$$

$$- 32\,\dot{x}_1 - \quad 50\,x_1 + 50\,\ddot{x}_2 + 32\,\dot{x}_2 + 50\,x_2 = 0$$

(The notation $\ddot{x}$ and $\dot{x}$ is used throughout to denote $\frac{d^2x}{dt^2}$ and $\frac{dx}{dt}$)

The calculated frequency response of this system is shown in Fig 1b, with amplitude of response of x_1 plotted against the angular forcing frequency ω The steady state response has also been computed on an analogue computer at a number of values of ω, and these experimental points will be seen to agree within about $\pm 1\ 5\%$ with the calculated curve (The error includes that of the pen recorder used to observe the motion x_1, which is of this order on its own) This type of problem, involving a number of simultaneous differential equations, is constantly recurring in all branches of engineering, and the response of any variable in it to any exciting waveform can easily be found on the computer Fig 1c shows the transient response of this system, as observed at one particular value of ω

Each of the equations above is of the form $a\ddot{x} + b\dot{x} + cx + R = 0$ where R is the sum of contributions from other degrees of freedom, together with the driving function, if any This equation can be solved by taking the variable $\ddot{x}$, integrating it once with respect to time to obtain $\dot{x}$, integrating it again to obtain x, taking fractions $a\ddot{x}$, $b\dot{x}$ and cx, adding R and forming a closed loop so that the sum of $a\ddot{x} + b\dot{x} + cx + R$ must be zero Simultaneously, each other degree is being solved, with interconnections between them to ensure that the correct fractions are available within R The essential requirements for an analogue computer, using voltage as the analogue quantity, are therefore components capable of integrating a varying voltage with respect to time, and of simultaneous addition of a number of voltages, together with provision for making reversal of signs

(3) Linear Computing Elements

The variables of the original problem are represented on the computer by voltages which vary in amplitude with time in the same manner as the original variables of the problem, except for any scaling by known factors to satisfy certain operating limits It is these voltages which are therefore integrated, added and reduced by constant coefficients to produce the solution, which appears as another voltage The basic element that features in integrating and adding circuits is the feedback amplifier shown in Fig 2a

The triangular symbol containing A represents a high-gain d c amplifier between V_g and V_{out}, i e an amplifier that has an output voltage V_{out} many thousand times the voltage at its input terminal V_g, at frequencies from zero to some upper limit f_2 (Fig 2b) There is also a phase reversal of 180° between V_g and V_{out} The output voltage is now fed back to the input point V_g through an impedance Z_2, and the input to the unit is applied through an impedance Z_1 It can be shown that this arrangement now gives a relation between input voltage V_{in} (to the unit as a whole) and output voltage V_{out} of $V_{out}/V_{in} = -Z_2/Z_1$, being independent of the gain of A as long as this is high The 180° phase reversal is essential, due to the requirement that V_{out} must always be in antiphase to V_g for stability around the feedback loop (Electronic designers also have their problems of stability, although the consequences of instability are much less spectacular than those in the aeronautical world)

Sign changing

If Z_1 is made a pure resistance R_1, and Z_2 a second resistance R_f, then

$$V_{out} = -\frac{R_f}{R_1} V_{in} = -k\, V_{in}$$ where k is a constant multiplier

If $R_1 = R_f$ then $V_{out} = -V_{in}$, giving only a change of sign

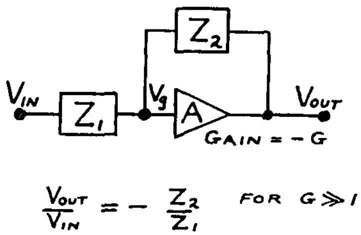

FIG 2(a) GENERAL FEEDBACK AMPLIFIER

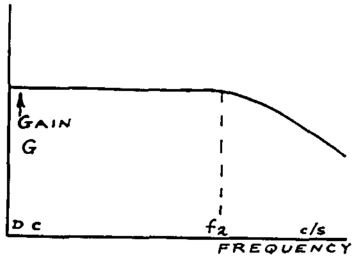

FIG 2(b) GAIN – FREQUENCY CHARACTERISTICS OF AMPLIFIER A

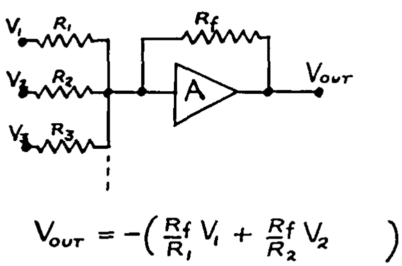

FIG 2(c) ADDITION

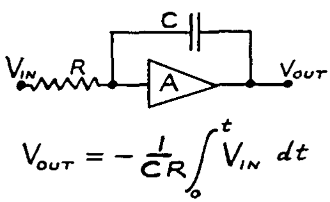

FIG 2(d) INTEGRATION

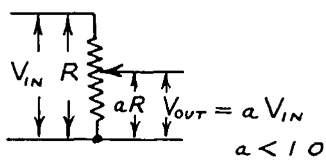

FIG 2(e) MULTIPLICATION BY A CONSTANT COEFFICIENT

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Addition

A number of input resistors can be used in parallel, connecting a similar number of input voltages (Fig 2c) The relationship now becomes

$$V_{out} = -\left[\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 + \qquad \right]$$

$$= -[k_1V_1 + k_2V_2 + k_3V_3 + \qquad] \text{ or } -[V_1 + V_2 + V_3 + \qquad]$$

$$\text{if } k_1 = k_2 = k_3 = 1\ 0$$

This has therefore performed the operation of addition of a number of variables, with or without multiplication by constant factors

Integration

If Z_2 is a pure capacitance C and Z_1 a resistance R (Fig 2d), then the relationship between V_{out} and V_{in} becomes

$$V_{out} = -\frac{1}{CR}\int_o^t V_{in}\ dt \text{ or if } CR = 1\ 0,\ V_{out} = -\int_o^t V_{in}\ dt$$

With a number of input voltages V_1, V_2 etc applied through R_1, R_2 etc

$$V_{out} = -\left[\frac{1}{CR_1}\int_o^t V_1\ dt + \frac{1}{CR_2}\int_o^t V_2\ dt + \qquad \right]$$

i e a summing integrator

Multiplication by a Constant Coefficient

It is often desired to multiply by a constant coefficient which can be made to be less than unity This is easily carried out by a simple resistive potentiometer (Fig 2e) If the total resistance is R_1 and the portion between the lower end and the moving contact is aR, with " a " less than 1 0, then $V_{out} = a\ V_{in}$

All the above computing operations depend only on the values of two or more passive impedances, subject to the original qualification that the gain of the amplifier A is large This amplifier is therefore made as a standard unit in the computer with a performance, in terms of gain-frequency and output voltage drift, which will allow it to be used in any of the computing roles without introducing appreciable error The input and feedback impedances are then added as required to make units which perform addition, integration or sign-changing, and these impedances again are chosen in magnitude and type to maintain an accurate calibration The errors and limitations actually introduced are discussed later

(4) The Solution of the Response of a Single Degree-of-Freedom System

With the elements described above, a system such as a mass-spring combination governed by the second-order differential equation ax + bx + cx = 0 can easily be solved The equation can be re-written as

$$x = -\left[\frac{b}{a} x + \frac{c}{a} x\right]$$

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Two integrating amplifiers are used (Fig 3), each containing the elements shown in Fig 2d, together with an adding amplifier and a sign-reversing amplifier to correct the reversal of sign occurring at each integration Co-efficient potentiometers P_1 and P_2 introduce the coefficients $\frac{b}{a}$ and $\frac{c}{a}$ and the solution is effectively performed by the adding amplifier which equates $\frac{b}{a}x + \frac{c}{a}x$ to $-x$ If all the voltages are initially at zero, it corresponds to a system with zero initial conditions A disturbance voltage is now injected into the system at the input of the adder, and then removed, corresponding to a small displacement of the mass in a mass-spring system The voltages now move in accordance with the equation, and an observation of the behaviour of x will give a response analogous to that of the amplitude of motion of the original system Acceleration or velocity could be observed at the same time by choosing x or x

The response to any forcing function F(t) is simulated in the same way by applying the function in place of the disturbance, and methods can usually be devised to generate waveforms of almost any shape Any given initial conditions can be simulated by adding appropriate voltages to the circuit prior to the disturbance pulse

Typical responses of a system such as that described above are shown in Fig 3b The oscillation frequency recorded from the computer is within

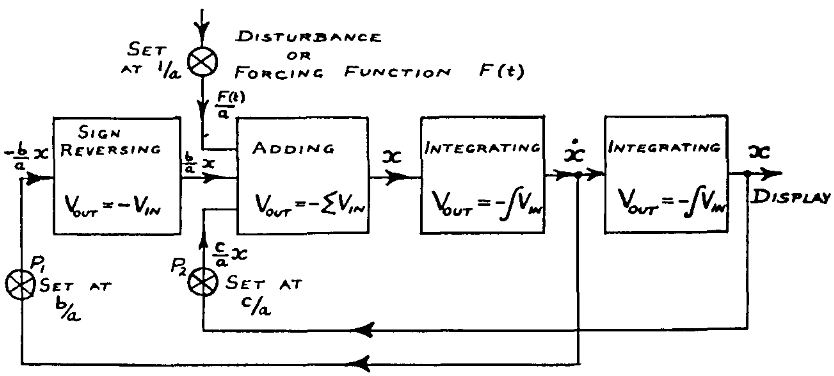

FIG 3(a) SOLUTION OF $ax + bx + cx = 0$ OR $F(t)$

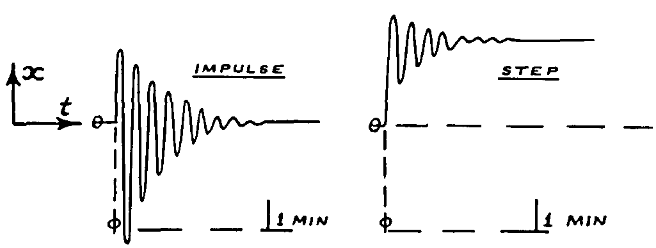

FIG 3(b) RESPONSE TO INPUTS

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0 5% of the theoretical value, and the observed damping is within 2% at a value of α (in $e^{-\alpha t} \sin \omega t$) of 0 01

Solution of vibration problems of several degrees of freedom

When a number of degrees of freedom are involved, the equations form a number of linear second-order equations with the possibility of coupling terms from each to each of the others, as in the example in paragraph 2 above In the solution of such problems, each degree of freedom requires the four units used above, together with the appropriate coefficient potentiometers for inertia, damping and stiffness terms, and for the coupling terms from the other equations which are fed to the input of the adding amplifier The equation $a_{11}x_1 + b_{11}x_1 + c_{11}x_1 + S = F(t)$ is now solved for each degree simultaneously, where S includes all the contributions from the other degrees of freedom, which are oscillating at the same time and giving and receiving contributions to and from each other The stability of such a system is usually studied by applying a momentary disturbance to one degree, and observing the subsequent oscillation

(5) Speed of Solution

The chief elements needed for the solution of linear differential equations have now been dealt with, and will follow a similar pattern on all types of analogue computers, though differing in complexity If the problem is set up on the computer with its time scale unchanged, then the solution will be performed in the same time as the original problem, i e if it refers to the free oscillations of a system with a natural frequency of 5 c/s, then five cycles of the observed response will be observed in a second and assuming damped conditions, the response will decay to zero over a matter of seconds In this case, a pen recorder would have to be used to obtain a satisfactory display A similar problem with a natural frequency of 50 c/s would give an output too fast for even a high-speed pen recorder to follow, and a cathode-ray oscilloscope would have to be used if the computer-time was kept the same as the real problem-time

With the oscilloscope type of display, it is essential to complete the solution within 1 second at the most, and preferably in less time than that, and so such displays are used with what are called Repetitive Computers These repeat the solution continuously, providing a disturbing pulse at regular intervals, followed by a period for display of the solution on a suitable time scale, than an action that effectively brings all the computer voltages to rest, or to chosen initial conditions, and finally a repeat of the disturbance for the next solution Computers have been produced (not commercially) that change the equation parameters in given steps for each successive solution, coupled with a three-dimensional display to present a series of solutions following a given sequence The advantage of an oscilloscope type of display is the speed at which the information is presented, so that the effects of changes in parameters can quickly be seen The chief disadvantage lies in the lack of any permanent record of the response unless photography of some sort is utilised, so that measurement, as opposed to pure observation, becomes more difficult

The alternative method is to use the relatively slower pen recorder in its simple moving-coil type for responses of the order of 1 c/s, or as a servodriven recorder for higher frequencies to 15 c/s These offer the advantage of a permanent record, and probably a more accurate time-base for measurements of oscillation frequency, but the speed of response of the recorder may not match that of the problem, so that it often becomes necessary to scale the problem in time before setting it upon the computer This is not necessarily a disadvantage as the frequency characteristics of the amplifiers used in the computing units (Fig 2b) may prevent the solution of the problem in real time on account of the errors introduced (paragraph 6) Any problem may be scaled in time —

The natural angular frequency of the basic problem equation ax + bx + cx = 0 is $\left(\frac{c}{a}\right)^{\frac{1}{2}}$ If a new time variable t_1 is introduced (" computer " time) where $t_1 = \beta t$ " t " being " problem " time and β a constant, then $dt_1 = \beta\, dt$ and $d^2t_1 = \beta^2 dt^2$ Hence a computer equation β^2 ax + β bx + cx = 0 can be formed in a time scale $t_1 = \beta t$ To make computer time equal to 10 times problem time, all the first-order differential coefficients are multiplied by 10, and all the second-order coefficients by 100, and a problem with a natural frequency of 5 c/s would then give a response on the computer that gave only 0 5 cycles in each second

(6) Accuracy and Limitations

Many factors contribute to the overall accuracy of a computation by analogue methods, so that it becomes very difficult to predict the limits to be expected in the final answer under all conditions In a simple two or three degree of freedom problem, 1% or better can be achieved fairly easily, but as the number of units involved increases, then this figure will of course deteriorate, roughly by the square root of the number of units involved if the errors are assumed to be of random occurrence

The sources of error are —

(a) Due to the operational impedances These are the input and feedbank impedances which obviously have a first-order effect on integration and addition irrespective of any additional errors introduced by the characteristics of the amplifier The values of the individual resistances and condensers can be selected to 0 1% fairly easily, but resistance values, even of the high-stability carbon resistors now used, are affected by temperature and ageing and cannot be expected to maintain the original values to better than 0 5% over all conditions However, if all change together with age, and also with temperature, the overall effect is less than that of the individuals Resistors should therefore be grouped together, away from valve heat if possible, so that all suffer the same temperature change Wire-wound resistors of manganin alloy would be a great improvement, but cost is prohibitive in the numbers used

(b) Due to coefficient setting The coefficients of the equations are set up on potentiometers or resistive dividers which should divide by the desired fraction to within 0 1% This requires an extremely linear potentiometer with high resolution A " loading " error always affects the actual linearity obtained as the input resistance of the next stage is always in parallel with the lower portion of potentiometer resistance, giving an error in the linearity which can easily be 2% with typical circuit values To obtain a satisfactory accuracy, a coefficient setting bridge is used A plug-in lead temporarily disconnects the coefficient potentiometer from the computer circuit and connects it to a bridge circuit in which the loading error is also simulated The desired coefficient fraction is set up on the bridge standard, and the coefficient potentiometer brought to a balance against this All the potentiometers can thus be correctly set independently of any errors in their individual linearities or in their dial readings (which can move out of alignment with constant use) Satisfactory resolution is usually obtained by using 10-turn helical potentiometers or three decade resistance dividers, giving a setting accuracy better than 0 1% at full scale, but obviously less than this at the lower end Coefficients differing greatly in magnitude cannot therefore be set to the same order of accuracy, and the smaller coefficients must inevitably suffer

(c) Due to recording the output As indicated in paragraph 5, the variable of interest is either displayed on an oscilloscope or on a pen recorder, with amplitude along the Y-axis and a suitable time-scale along the X-axis If only the frequency of the oscillation is of interest, a fast time sweep can be used, particularly on a pen recorder, and the time between zero crossings of the axis can be measured to a few parts in a thousand, certainly better than 1% Where amplitude is of interest, the accuracy is much poorer, both on a photographed trace from an oscilloscope and on a pen recorder, neither of which could be guaranteed to much better than $\pm 3\%$ Measurements of damping would also normally include errors of this order, but a stability plot of damping v frequency to determine the stability boundary could avoid the error at zero damping

(d) Due to amplifier characteristics The remainder of errors can be said to be due to deficiencies in the performance of the amplifiers Each amplifier should obviously give zero output for zero input, and most types of amplifiers have a " balance " control to ensure this Once adjusted, subsequent changes in the output level are due to " drift " in the amplifier, caused by thermal changes in the valves and resistances, variation in power supplies and to minute currents at the input of the first valve of the amplifier Reduction of these spurious voltages to acceptable limits, whilst still maintaining the transmission at zero frequency, accounts for most of the complications and expense of the computing amplifiers and their associated power supplies One class of amplifiers known as " drift corrected " incorporate " chopper " relays to convert the d c signals to a form of a c and so allow much of the drift to be removed , another type used in a commerical repetitive computer effectively samples the drift at the end of every solution and injects an equal and opposite voltage to compensate for it during the next , the simplest amplifiers rely on a number of balanced stages to reduce the effects of drift to a minimum Such drift voltages occurring within the computing loop of problems like the second-order equation in paragraph 4 will obviously affect amplitude levels, particularly near zero, but will not necessarily have any measurable effect on the frequency, decay of oscillations or stability

The accuracy of addition in a summing amplifier depends on the number of input resistors in use when related to the gain of the amplifier, e g for 12 input resistors it is necessary that the gain exceeds 13,000 to maintain an accuracy of 0 1% on the summation Similarly, the accuracy of integration is dependent upon the gain of the associated amplifier, except that here the error increases with time, so that it is necessary to consider how long the integrator is required to be in action during the computation If an integrator using an amplifier of gain 50,000 and a C $\times$ R product of 1 sec was used to integrate a constant input voltage, the observed integral would depart from the true value by 0 1% after 50 seconds which limits the integration time in slow computers These errors in addition and integration are also affected by the frequency response of the amplifier (the frequency f_2 in Fig 2b), when related to the highest frequencies appearing in the solution, and unless the amplifier will maintain its maximum value of gain up to frequencies of the order of 1,000 times that of the solution frequency, errors again occur which increase with time

From the point of view of the user, little can be done to control the errors introduced by the amplifier characteristics, except to recognise that they can become appreciable (a) with the higher frequency solutions, (b) with lower frequency solutions continued for a minute or more to observe a number of cycles, (c) when a large number of inputs are added together, (d) if the amplifier zero drifts without any correction

(7) Extension to More Complicated Equations

So far, it has been shown that the solution of a number of simultaneous linear differential equations can be obtained using only integrating and adding amplifiers, coefficient potentiometers and the necessary interconnections A set of ten, or twenty or more equations could be solved with the addition of more of these units and masses of potentiometers and wire The scope of analogue computing is greatly increased by introducing other units such as multipliers to cater for coefficients that are not constant, and non-linear elements where output does not follow a linear relationship to input These latter can be designed to produce almost any arbitrary exciting function, or to simulate such mechanical effects as " back-lash " in gearing, or a co-efficient varying as a power law, or a sudden change in a damping as an absorber becomes compressed and reaches a stop Such devices can often include effects which are observed to occur in practice, but which cannot easily be incorporated in straightforward mathematical analysis

(i) *Units with non-linear input-output characteristics*

The simulation of non-linear effects brings into use the rectification characteristic of the simplest of all valves, the diode, which presents a substantially linear resistance characteristic in one direction when conducting, and an infinite resistance in the reverse direction A single diode used with the input resistance of a feedback amplifier gives an input-output characteristic with a single break occurring at zero (Fig 4a) The point at which this break occurs can be controlled by applying a bias voltage to the diode, and if a number of these are used at the input, biassed to different voltages, the number in circuit varies with the amplitude of the input, giving a slope which can be made to follow any desired law (Fig 4b) As the number of diodes is increased, the tangents approximate to a curve, and a square law can be represented to an accuracy of better than 1% using about 20 diodes

(ii) *Multipliers*

One type of multiplier has been dealt with already, namely, the potentio-

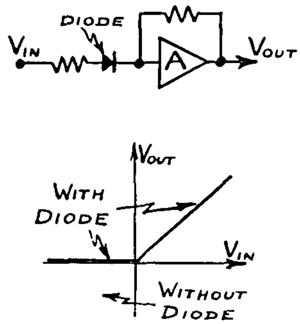


FIG 4(a) DIODE USED WITH FEEDBACK AMPLIFIER

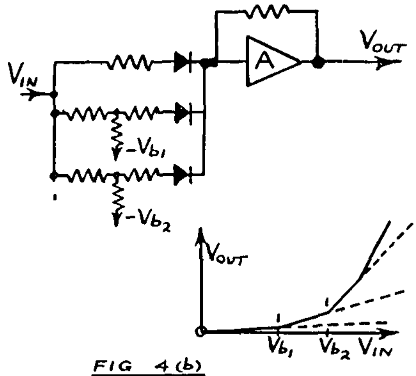


FIG 4(b) BIASSED DIODE UNIT

meter which introduces a constant coefficient, remaining at this value until charged by hand Multipliers which operate continuously on two functions are considerably more complicated An electro-mechanical servo-multiplier can be produced if a small servo-control is made to set the position of the moving contact on a potentiometer in accordance with the input variable This can be made to give an accuracy of about 1%, but is limited to operate at the slower speeds Much thought and ingenuity has gone into producing other types, but the accuracy achieved is never much better than 1%, and often much worse

Two principles met with are —

(*a*) Multiplication by Quarter Squares —This makes use of the relationship $(x_1 + x_2)^2 - (x_1 - x_2)^2 = 4\, x_1 x_2$ The two input variables are added, and also subtracted, the sum and difference being passed through non-linear units having a square-law characteristic The difference of the outputs from these is proportional to the product of the original quantities

(*b*) Multiplication by Logarithms —This is the familiar action of using logarithms in its electronic analogue, with circuits which give an output proportional to the log of the input, and to others that give an antilog effect These are used to obtain the logs of the input variables, which are added and then passed to the antilog circuit

(iii) *Switched coefficients*

Another class of non-linearities can be introduced by arranging for a sudden change in one or more coefficients of the linear equation to occur as one of the variables passes through a critical value This is easily arranged by feeding the variable to an electronic switch which triggers at a pre-determined voltage level The appropriate change in the coefficient can be made by a simple relay if the computing speed is slow enough, or by diode switches if not One example of this technique is the solution of a simple system in which the damping force arises from Coulomb friction, being of constant magnitude independent of velocity, but always opposing it in sign The system equation for free oscillation is $a\ddot{x} \pm b + cx = 0$, the sign of " b " depending upon that of x and therefore changing as x passes through

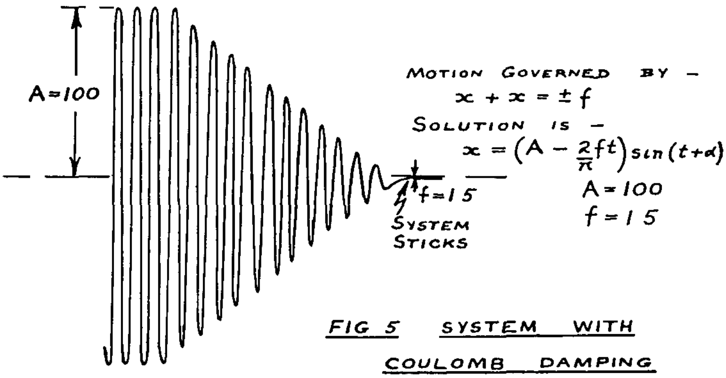


FIG 5 SYSTEM WITH COULOMB DAMPING

zero The variable x is connected to an electronic switch which changes the polarity of the voltage representing the constant damping force b as x passes through zero A typical solution obtained on a computer is shown in Fig 5, and the linear decay obtained agrees with the theoretical solution to within 3%

(8) Application to Helicopter Problems

A large part of the development of analogue computers in this country has been stimulated by the application to aeronautical problems, one example of which is the " flutter " problem which resulted in the Flutter Simulator at R A E , Farnborough[4] The effort there has since resulted in the extension to three-dimensional problems with TRIDAC, a most comprehensive machine with 2,000 computer units and 600 KVA power requirement[5] Some firms have produced their own computers for problems like the aircraft flutter, and commercial general-purpose computers are now available, including some marketed by firms with a distinctly aeronautical background However, there seems to have been little application to the field of helicopter design, and as a result of suggestions put forward in about 1953, a research contract was placed by the Ministry of Supply with the University of Southampton which has enabled a computer to be constructed for the express purpose of studying one of the problems of helicopter design, namely, that of Ground Resonance This problem lends itself to simulation on an analogue computer because the work of Coleman in America resulted in an analysis of the problem which leaves a number of simultaneous differential equations to be solved to ascertain the stability of a system of rotor blades coupled to an undercarriage system on the ground[6]

Ground Resonance Problem

The background of this problem has been presented to this Association before, and one paper[7] by Howarth and Jones in December 1953 reviewed the work of Coleman and extended the application of it to the Bristol Type 173 helicopter Briefly, the result of a judicious choice of a system of coordinates results in the problem for rotor systems of three or more blades resolving to two blade equations and a number of chassis equations, with mutual coupling terms between them, but all with constant coefficients at any particular rotor speed A typical set of equations follow for a system with a single lateral degree of hub freedom, using a notation similar to that of Howarth and Jones in their paper (except for the co-ordinate symbols)

$$X + \lambda_\beta X + [\Lambda_2 - \omega^2 (1 - \Lambda_1)] X + 2 \omega Y + \lambda_\beta \omega Y + \tfrac{1}{2} x = 0$$

$$- 2 \omega X - \lambda_\beta \omega X + Y + \lambda_\beta Y + [\Lambda_2 - \omega^2 (1 - \Lambda_1)] Y = 0$$

$$\mu X + x + \lambda x + \omega_x{}^2 x = 0$$

X and Y are the variables representing the rotor co-ordinates,

x = lateral co-ordinate of the hub,

λ_β = blade damping coefficient,

$\Lambda_2 = \frac{K_\beta}{I_h}$ where K_β = stiffness (if any) at the drag hinge,

I_h = moment of inertia about the hinge,

$\Lambda_1 = \frac{a}{b}$ where a = drag hinge offset,

b = distance from drag hinge to centre of percussion of the blade,

μ = mass ratio $= \frac{n\, m_b}{m}$ where n = number of blades

m_b = mass of each blade

m = total mass at hub

λ = hub damping coefficient,

ω_x = natural frequency of hub in lateral direction

This problem can be solved by conventional analytic methods to give a plot such as Figs 6a and 6b, where the real and imaginary frequency equations are plotted, with an indication of instability in the region shown, between the limits of rotor speed equal to 1 15 and 1 26 rads/sec In an investigation into the effects of blade and hub damping on this instability, a plot such as the above would have to be produced for each value of

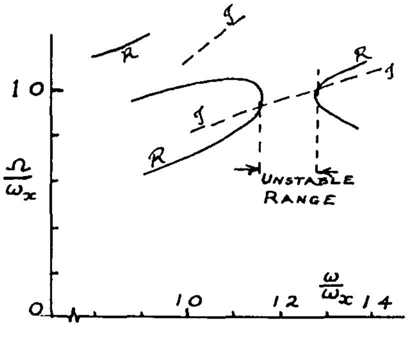


FIG 6(a) COLEMAN PLOT

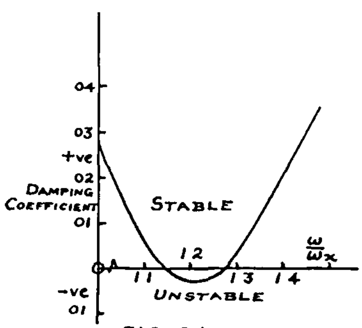


FIG 6(b) CORRESPONDING PLOT FROM COMPUTER

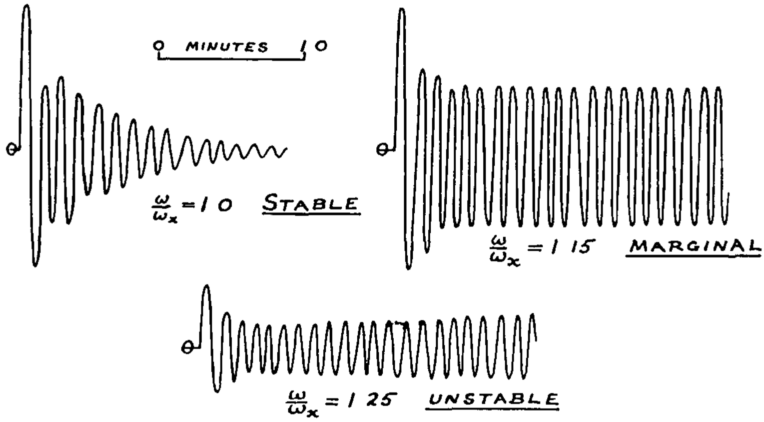


FIG 6(c) LATERAL HUB DISPLACEMENT

damping chosen Each plot is tedious to produce—just how long it takes seems to be a trade secret, but it certainly seems to be a matter of many man (or woman) hours

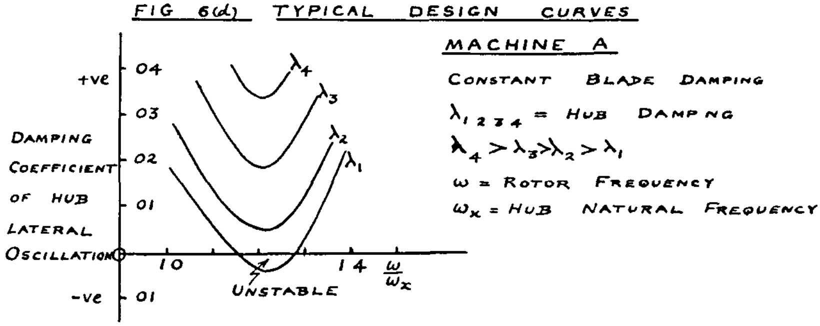
The set of equations is ideally suited for solution on a straight-forward analogue computer, requiring four amplifiers for each degree of freedom considered (two integrating, one adding and one sign reversing) At present at Southampton there is a four-degree of freedom computer available, and this problem has been set up many times using data relating to several different helicopters Usually the free response to a short impulsive disturbance is taken, and typical recorder traces of amplitude of lateral hub displacement are shown in Fig 6c These are taken with the time variable rescaled to give an angular velocity on the computer of about 1 rad/sec By taking a number of such traces at different values of rotor speed, and with different hub and chassis damping parameters, design curves such as Fig 6d can be constructed, giving indication of the unstable ranges, and of the damping necessary to remove the instability

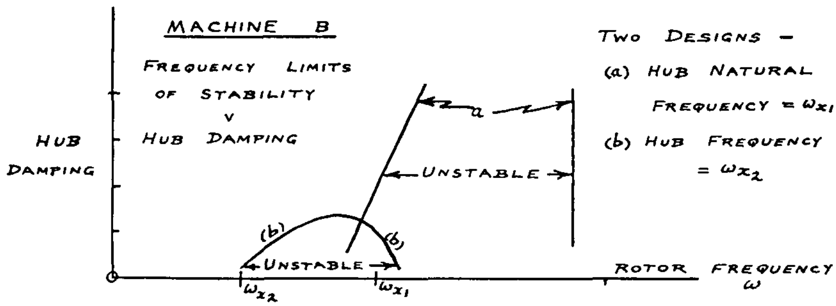
In each blade equation, two of the coefficients are proportional to rotor speed, and one to (rotor speed)^2, and to reduce the time spent in re-setting coefficients at each change of rotor speed, a control has now been fitted which changes these coefficients by amounts corresponding to changes of $\pm 20\%$, $\pm 10\%$, $\pm 5\%$, $\pm 2\%$ and $\pm 1\%$ about a set rotor speed, whilst still maintaining a setting accuracy of 0 1% This greatly speeds the production of stability v rotor speed plots, and the frequency boundaries for a given set of damping conditions can now be obtained with an accuracy of about 1% in a matter of minutes, together with a permanent record of the response at each frequency

In the theoretical treatment used by Coleman, it is assumed that the hub moves with fore and aft, and lateral freedoms only (no vertical movement), and that rigid blades rotate about each drag hinge (no flapping motion or bending of the blades) All aerodynamic forces are assumed zero, motion in pitch and roll is ignored and the stiffnesses from hub to ground are assumed linear A more realistic representation of the behaviour on the ground, both at rest and when moving, will obviously require some at least of the additional factors indicated above to be incorporated Additional degrees of freedom in roll and pitch and in vertical movement can be added, but these may well affect the assumption of linear stiffness That such a representation is necessary is indicated by the fact that ground resonance has been observed in some machines during take-off and landing when they are apparently perfectly stable when at rest on the ground There is also the complication that some machines use multiple-plate friction dampers at the blade hinges, not the viscous dampers assumed in the analytic treatment An equivalent viscous damping coefficient can be determined for the friction damper by equating energy dissipation, but the action of it must introduce non-linear terms into the equation which are not taken account of by the viscous coefficient on its own

Some of the above modifications are being incorporated into solutions of ground resonance problems on the analogue computer, which is particularly suitable for this because, as has been indicated already, discontinuous parameters can be introduced and non-linear functions simulated As a start to the problem of the stability of a helicopter which is bouncing from wheel to wheel after landing, the case of an abrupt change in the lateral stiffness has been considered, with a simultaneous change in lateral damping[8] The lateral freedom was chosen simply because figures of stiffness and damping were available, and a study of the system at rest had already been made This is an attempt to simulate the condition of one wheel leaving the ground,

FIG 6(d) TYPICAL DESIGN CURVES





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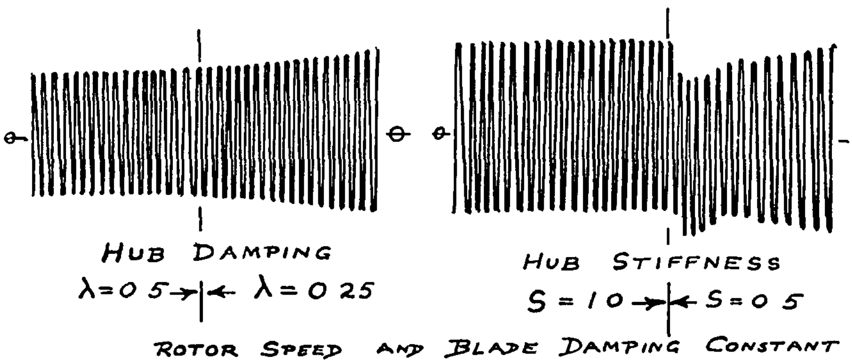
so that only one tyre remains in contact and the lateral stiffness, derived from this tyre alone, is therefore halved At the same time, with one undercarriage leg effectively out of action, the total lateral damping is also halved The amplitude at which this change takes place has been varied, and a typical response of the system under these conditions is shown in Fig 7 At the value of rotor speed chosen, the system is stable with normal stiffness and damping, but with the changes occurring as shown instability arises as the portion of the cycle over which the full stiffness and damping act decreases This particular example has undoubtedly been over-simplified to be of much practical use, but the principle of coefficient switching could easily be extended Instead of a single degree of lateral freedom, there should be freedom in vertical movement and in roll and pitch, so that the variations in stiffness and damping could be controlled by the correct displacements The magnitude of changes occurring is also open to discussion, together with the amplitudes at which they change

FIG 7 SWITCHING OF PARAMETERS



The other non-linearity which is being considered is that of the frictional forces If a true viscous damper is used, then frictional forces are proportional to velocity and the equations remain linear With a friction damper, an equivalent viscous damping coefficient can be produced by equating energy losses per cycle of oscillation, with a value inversely proportional to the vibration frequency at the damper Problems of this nature have been put on the computer and a trial and error method used to arrive at the correct damping coefficient The use of this coefficient implies that the equations remain linear, which is not true if the damping is high The example of Coulomb damping (paragraph 7 and Fig 5) is one result of using a direct analogue of friction damping on the computer, and would be equivalent to a single plate damper, with constant frictional force independent of amplitude but in phase with velocity The next step is to make the force proportional to amplitude whilst remaining in phase with velocity, thus simulating the stepped friction damper with a large number of small steps, in fact the characteristic used to obtain the equivalent viscous coefficient This work

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is in hand at present, together with the consideration of how such an effect can be included in the derivation of Coleman's equations

(9) The General Use of Analogue Computers

The chief features of the use of analogue computers have now been described, together with a few applications which may be regarded as typical Some of the more important points involved in considering the use of computers may be summarised

(*a*) The electronic analogue computer is a simulator, in that it retains all the essential features of the original problem, with the input quantity, and the various controlling parameters available for alteration, and all the variables immediately accessible for observation, being changed only by scaling in amplitude and possibly in time

(*b*) Once the problem is set up by interconnecting units and adjusting coefficient potentiometers, the speed of solution may be arranged to be so rapid that repetitive presentation on an oscilloscope is used, or it may occupy many seconds to allow presentation on a pen-recorder

(*c*) It can be employed to solve problems (such as simultaneous linear differential equations) which are capable of solution by conventional mathematical processes, but it can also be used with elements specially designed to simulate non-linearities which occur in practical systems, but which often have to be ignored to obtain a mathematical solution by simple means

(*d*) There are certain limits to the problems that can be set up on an electronic analogue computer, *e g* all integration is with respect to time and integration with any other variable is difficult, although pure mechanical analogues could be used to overcome this This is not normally a great disadvantage for engineering problems Computation in real time allows the inclusion of actual mechanical linkages such as servo controls in studying auto-pilot responses, or a human link between the presentation of some data and the operation of a control

(*e*) Once the problem is set up, the effect of variation of different parameters is quickly observed In such work, accuracy is not of prime importance as most solutions are inevitably rejected in arriving at one satisfactory design If the design parameters are known with such certainty that accuracy better than 2-5% is significant then the final design can be recalculated by another means The economy has been achieved in the material that has been rejected

(*f*) If the purpose for which the computer will be employed is clear, then it can be designed economically to fulfil the requirements with a minimum of computer elements There is a strong case for such a machine when similar calculations are continually being performed by hand and final accuracy is not of prime importance

(*g*) When a general purpose computer is required, then it will inevitably be fairly complex, with a capacity that is not always utilised Commercial

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computers offer many systems to allow the amplifier units to serve different functions, and to provide flexible interconnections via plug leads, or by plugs alone

(h) An electronic analogue computer is only another form of slide-rule, and if accuracy beyond " slide-rule accuracy " is required, then the problem must be placed on a digital machine or be computed by hand The actual accuracy obtained depends upon many factors and will become poorer as the complexity of the problem increases and more units are introduced It may be as good as 0 1% for a simple problem under favourable conditions, but can usually be considered more in the order of 1-5% There are practical limits to the accuracy which can be attained at all the intermediate stages of input data, computation and reading of output

(i) Digital computers have an economic minimum in size, due to the timing circuits, pulse generators, input and output data units that are required no matter what the capacity of the computer is Consequently, they tend to be built as complex general-purpose computers and the small special-purpose machine is virtually unknown outside accounting offices Their cost, and the consequent need for continuous employment means that they will be found in computing centres rather than in individual laboratories and design offices In the accurate handling of large amounts of statistical data, they are of course in a supreme position, but for " trial and error " problems which are so often the basis of most of a designer's life, where so many of the figures are not known with certainty, the analogue computer is often more satisfactory Here, the output is a visual plot against a time scale, instead of a mass of figures which have to be translated into a recognisable form, and the effect upon the output of a change in any of the design parameters can normally be seen immediately at the turn of a dial

(j) It is not always necessary to be able to formulate the mathematical equations of a problem before setting it up on an analogue computer This can be achieved simply by considering the transfer characteristics of each stage, and replacing the actual stages by analogue elements with the same transfer characteristics

Conclusions

An electronic analogue computer capable of dealing with four linear second-order differential equations has been applied to the solution of the equations arising from the analysis of the ground resonance problem of single-rotor helicopters by the treatment originally due to Coleman The frequency stability boundaries determined from the computer agree with those obtained by the Coleman plot, and the speed at which solutions can be obtained is such that design curves can be obtained catering for a wide range of damping and stiffness in the chassis and blades In order to eliminate some of the simplifications used in the analytic treatment, provision is being made for the computer to deal with various non-linear parameters such as changes in stiffness, and in simulating damping forces other than those due to viscous damping

A review of some of the chief features of analogue computing is added to assist in the understanding of its application to the problem of design

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Discussion

The **Chairman** said that they were very grateful to Mr Venning for his paper He would like, first of all, to invite the users, or potential users of analogue computers to put forward their views

Mr C H Jones (*Dynamics Engineer, Bristol Aircraft*) said that he would like to thank Mr Venning for his lecture which had helped his own understanding of analogue computors quite considerably

First of all he would like to ask a question about the coefficient correction which Mr Venning said that he had included The flutter equation had terms proportional to velocity squared and terms proportional to velocity The Coleman equation had an extra term, which was proportional to velocity and, using the Southampton University computer it was necessary to keep on changing the coefficients appropriately to change the rotor speed He wondered whether it was now possible to change the rotor speed readily

Secondly, he was very much in favour of analogue computers in the sense that if one desired to know the answer to a stability problem one gave the analogue an impulse and waited to see what it did when left to itself That gave him more confidence than having to plough through the whole field in order to find a brick in the middle Using other methods one had to cover the whole range of rotor velocity and frequency of the oscillation

He would like to know what other people's views were on the subject of stability criteria With the analogue computer it was possible to measure the logarithmic decrement at any particular speed and assess the stability by how rapidly the oscillation died out Other methods of solution, such as the classical Coleman method, gave no idea of how severe the oscillation was likely to be Instead, one found the extent of any unstable range

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