

OBITUARY



WALTER WILSON STOTHERS
(1946–2009)

Walter Wilson Stothers was born in Glasgow on 8 November 1946. A third (youngest) son, he had the identical name to his father. From childhood, however, he had always been known by his middle name ‘Wilson’, so that his father, a Glasgow GP, would never be referred to as ‘Old Walter’. His mother, as Jean Young Kyle, had herself graduated in Mathematics in 1927, a rare achievement for a woman at that time. After attending the local primary school 1952–1956, Wilson completed his primary education in the preparatory classes in Allan Glen’s School, then a distinguished Glasgow boys school with a scientific emphasis, progressing to the secondary school in 1958 and ending by becoming Dux in 1964. He also played in the school rugby first XV. From 1964–1968 he was a student in the Science Faculty of Glasgow University. His original intention was to take Honours in Chemistry and, indeed, he won the Chemistry prize in his first year. But he excelled in all subjects, winning the Faraday medal in the Intermediate Honours (second year) class in Natural Philosophy (Physics). After that he concentrated on Mathematics and became the top student, gaining a First Class Honours degree, as well as the Cunninghame Medal and a Jack Scholarship to Peterhouse College, Cambridge (1968). Before commencing postgraduate studies he married Andrea Watson in September 1968. At Cambridge from 1968–1971 he studied for a Ph.D. in number theory under the supervision of Peter Swinnerton-Dyer

and graduated in 1972 with the thesis *Some Discrete Triangle Groups*. By then he was becoming aware of the strange realm inhabited by mathematicians that he seemed to be entering. So when his Cambridge room-mate Bob Odoni, at a research meeting they were attending as postgrads, asked, 'Wilson, do you realise that we are the only normal people here', Wilson felt compelled to respond, 'What makes you think that we are normal?'

Wilson returned to Glasgow University in 1971 to take up a lectureship in Mathematics and, eventually in 1989, was promoted to Senior Lecturer. He was fully involved in teaching at all levels. At different times he was involved in major rewrites of level 1 notes, one outcome being co-authorship of a text *Fundamentals of University Mathematics* that, despite changing times, is still used in various universities in Scotland and beyond. It will shortly be reprinted in a third edition. At higher levels he taught a distinctive portfolio of courses on topics such as number theory, discrete mathematics, geometry, coding theory, topology, complex analysis, Lebesgue integration, Galois theory and computability and logic. His teaching was popular: for example, his third year class on discrete mathematics, though not compulsory, was invariably attended by almost all honours students including those taking the Applied Mathematics degree.

From 1980 Wilson, partly through the connection with the computer business founded by his oldest brother, became practically involved in all aspects of computing, software, hardware, programming and algorithms. This was in the days of punched cards and well before the advent of dedicated IT support. He was largely responsible for the introduction of computers to the Mathematics department at Glasgow and provided hardware and software support until 1991.

For many academics, it is the research aspect that is most significant. In this regard Wilson described himself as fundamentally a number theorist. Nevertheless, his main interest throughout lay in the associated area of discrete groups, which is a blend of group theory, complex analysis, Galois theory and non-euclidean geometry. Many of the methods he applied were combinatorial with the use of diagrams. The main seam was the study of subgroups of the modular group and other Fuchsian groups and algebraic generalisations of these. Although there were relatively inactive periods, he returned to the topic in later years using tools, such as computer algebra software, which enable the investigation of problems that had previously seemed impossible. Wilson was also interested in 'elementary' number theory (in the technical sense of the term) and some of his joint papers reflect this. He also collaborated with colleagues on various themes. He supervised one successful Ph.D. student, Philip Stephenson, whose 1992 thesis was also on subgroups of triangle groups. For many years Wilson was one of the technical editors of the *Glasgow Mathematical Journal*. He also wrote 30 reviews for *Mathematical Reviews*.

It does seem that Wilson found life as an academic at Glasgow somewhat stifling, colourless and unrewarding. Fortunately, there were other outlets for his mathematical skills and energies. The Open University (OU) had admitted its first students in 1971. From 1972 Wilson began to be involved with the OU as Tutor and Counsellor in Scotland and, sometimes, Ireland, on all the pure mathematics that the OU produced, from foundation level to honours level. From 1980 he taught at summer schools in Stirling, Durham, Reading and Nottingham. Many flocked to his lectures, delivered in a quiet manner and laced with perceptive and laconic humour. A feature was his colourful shirts, chosen to match the cover of the day's text. He certainly enjoyed the fact that OU students, especially those within pure mathematics, are well motivated. He continued with the OU until becoming ill in 2007. At annual student-organised revision

weekends he (voluntarily) attended as tutor, he gained a reputation and popularity as a predictor of the content of exam papers. He also served as a lecturer, consultant and external examiner on courses in Combinatorics and the OU's first-ever course on Mathematics for Computing.

Wilson's teaching involvement in geometry led him to the development of his (famous!) geometry web pages (to be found at www.maths.gla.ac.uk/~wws/cabripages/cabri0.html). Begun as an experiment in teaching projective conics using Cabri, these absorbed much time and effort. They were expanded to include other geometries illustrating the Klein view of geometry, such as Cinderella, and provide interactive diagrams and a 'dynamic geometry' facility. These have been widely appreciated by students but also held in high regard by professionals (for which there is testimonial evidence). For his part, in later years Wilson developed a research interest in the area of classical Euclidean geometry, perhaps unfashionable to some, but still attracting a band of devotees. He wrote some papers on projective cubics and left behind a number of unpublished items on such matters: one is to be published *in memoriam*.

In his mathematical writing, Wilson liked to express himself elegantly and economically but without flourish or affectation, as can be detected even in some of the titles. But this was a reflection of his personality which was self-effacing and modest. At the same time, he was always patient and displayed a willing helpfulness in everything. He displayed a quiet but apt wit of the kind that most of us who are mathematicians appreciate and admire. My favourite memory was to witness a eureka moment, perhaps in 1983, when we were in a busy coffee-room puzzling over the nature of an algebraic field element thrown up by a research problem. 'It's in $K!$ ' he yelled uncharacteristically, totally oblivious to all around.

Wilson and Andrea have four children Christopher (born 1977), Veronica (1979) and twins Simon and Paul (1981). Within the family he was regarded as fun-loving and a master of word play. More generally, he was cultured and well read, a lover of colour and an expert photographer with his own darkroom. He had gone part-time at Glasgow in 2006 and, naturally, it was sad for all when Wilson became ill in 2007. In April while he underwent heart valve pre-operative tests, a shadow on the lung was found and cancer was diagnosed. He had the heart valve operation in June and a lung operation in August. In April 2008, his illness was found to be terminal. He finally retired altogether in August 2008. His last months were filled with pain, borne with dignity and fortitude. As a final gift to his children, knowing that there was only a short time remaining, he scanned hundreds of photographs and compiled a unique photo album for each. He died peacefully at home on July 16, 2009. Tributes of appreciation have been paid by Open University colleagues and students and even from members of *Hyacinthos*, a Yahoo group of geometers.

Despite his unassuming nature, Wilson Stothers leaves an enduring if unexpected mathematical legacy. Around 1977/1978 he resumed research on discrete groups and distilled, through the medium of complex function theory, a beautiful theorem on complex polynomials. It states that if f, g, h are relatively prime polynomials, not all constant, then the number of *distinct* zeros of the product fgh is at least $\max(\deg(f), \deg(g), \deg(h)) + 1$. (Additionally, his argument also gives conditions for equality to hold.) This bound yields an elegant proof of one of Davenport [2] on the zeros of $f^3(t) - g^2(t)$. Although a statement of the theorem appears prominently on the opening page of his 1981 paper *Polynomial Identities and Hauptmoduln* (a title that simultaneously manages to be understated and obscure), Wilson's brief

introduction simply announces his work as a new proof of Davenport's inequality, with a generalisation. This is reflected in the entries in the main reviewing journals which failed to state Stothers' new theorem. A few years later, R. C. Mason, a student of Alan Baker, independently discovered the same result, again by relatively deep methods, the work being published (1983) initially in the proceedings of a 1982 colloquium [3]. At the time Wilson was discovering various elementary proofs of his result generalised to all (algebraically closed) fields: thus f, g, h are assumed not to be all p th powers if the characteristic p is positive. An extant manuscript, perhaps from 1984, remains unpublished. Again there are comments on the case when equality holds. In any case, Wilson presented the theorem at various seminars throughout UK. At one in Cambridge, Baker pointed out Mason's independent work. In 1985 David Masser formulated the *abc* conjecture (a refinement of one of Oesterlé) in number theory, unproved to this day. If true, it would have profound implications for many fundamental problems in number theory. It was observed that the polynomial analogue of the *abc* conjecture was already a theorem, specifically the one under discussion. The latter began to be referred to as 'Mason's theorem' and various elementary proofs appeared, the most notable being that of a high school student Snyder [4], given prominence in Serge Lang's influential graduate and undergraduate texts on Algebra. More recently, Edward Formanek (Pennsylvania State University), in some way uncovered Stothers' original version, presumably in connection with his own research (he gave a conference talk *Some Polynomial Differential Equations and a Theorem of W.W. Stothers* in 2007), and later there was some correspondence about this. With this impetus there has now been a proper attribution of the 'Stothers–Mason' theorem, at least in the most knowledgeable quarters. Thus the latest editions on Lang's books do refer to the Mason–Stothers theorem. There is now a considerable body of literature on the theorem and its applications, such as to the AKS primality testing algorithm as described in [1], where following a familiar pattern in mathematics, the proof of the Stothers–Mason inequality is reduced to a polished ten lines. I believe that Wilson would have liked that.

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