MORTALITY OF GOVERNMENT LIFE ANNUITANTS.

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COPY of Report of the Actuary of the National Debt Office, dated the 12th day of October, 1910, on the Mortality of Government Life Annuitants.

NATIONAL DEBT OFFICE,
12 *October* 1910.

SIR,

I have the honour to submit the following report respecting an investigation which has been made into the Mortality Experience of Government Life Annuitants.

The last investigation was made by Mr. A. J. Finlaison, and his report thereon, under date of the 10 February 1883, was published in House of Commons Paper No. 8 of 1884.

The observation on that occasion extended over the long period of 67 years, from 1808 to 1875, and comprised the experience of the nominees of all annuities which had been granted under the Acts 48 Geo. 3. c. 142. and 10 Geo. 4. c. 24.

If variations in rates of mortality were only of a temporary character the inclusion of the experience of a further series of years with that of the period 1808 to 1875 would be desirable in order to obtain as many facts as possible from which the requisite average deductions could be drawn. There may, however, be causes at work in a later period of years, which were either absent or of less force in an earlier one, tending to render these variations permanent.

The conditions of life in this country are now somewhat different from what they were in the earlier years of the nineteenth century.

A knowledge of hygiene is more general, considerable improvements in sanitary matters have been effected by legislation, and great advances have been made in medical and surgical skill; all causes which favour an increase of vitality. On the other hand there may be causes tending to a contrary effect, for instance, a greater density of population and the stress of modern life; the latter cause probably affecting male more than female lives. The duration of life in a special class of lives like annuitants may not be influenced by all the causes mentioned, but that some of them will have an effect upon it seems to be more than probable. The object sought in investigating the mortality experience of a past period is to obtain data from which the probable rates of mortality in future may be deduced. With the view, therefore, of obtaining the experience of lives living under conditions more closely resembling those of the present time it has been decided to commence the present investigation with the experience of the year 1875, and to omit altogether that of previous years. If any later year were taken as the commencement of the observation it would hardly allow of the collection of sufficient data.

The lives comprised in the present experience are accordingly—

(1) The nominees of annuities granted before 1875 who were living in that year on the anniversary of the date of purchase of the annuity.

(2) The nominees of annuities granted under the Acts 10 Geo. 4. c. 24 and 51 & 52 Vict. c. 15, between the 1 January 1875 and 31 December 1903.

Where two or more annuities have been granted on the same life only the experience afforded by the one first granted has been included.

The following table gives a summary of the data :—

Government Annuity Experience, 1875-190	Summary of Data		
	Males	Females	Total
Number of lives under observation ...	5,504	13,863	19,367
Number of Years of Risk	57,652	163,378	221,030
Number of Deaths	4,168	9,333	13,501
Number of Lives existing at the close of the observation	1,336	4,530	5,866

There does not appear to be any material alteration as regards the ages at which annuities were purchased in the periods before and after 1875 respectively. In both periods about 5 per-cent of the total number of lives were under 40 years of age at the time of purchase and 65 per-cent between the ages of 50 and 70. The

following statement shows the number and ages of the lives on which annuities were purchased during the period 1875 to 1903 :—

Age	No. of Males	No. of Females	Total Number	Per-cent
15–19	5	8	13	...
20–29	35	103	138	1
30–39	134	323	457	4
40–49	351	909	1,260	11
50–59	867	2,473	3,340	29
60–69	1,123	3,122	4,245	36
70–79	601	1,325	1,926	17
80 and above	101	160	261	2
	3,217	8,423	11,640	100

The number of lives included in the experience on which annuities were purchased before 1875 is 7,727.

In order to deduce rates of mortality from the experience of annuitants during the period 1808–1875 Mr. Finlaison arranged the data so that the mortality could be traced through each year of age. For the first year the lives were assumed to come under observation, on the average, four months after their preceding birthday, and the annual rate of mortality based on the experience of the following eight months was taken as the rate for the whole year. After the first year the mortality was traced through exact years of age, that is, from birthday to birthday.

A different method has been followed in the present investigation, viz., that adopted by the Joint Committee of the Institute of Actuaries and the Faculty of Actuaries in the investigation into the mortality experience of annuitants in British Life Offices during the period 1863–1893. According to that method the lives are assumed to come under observation at the age attained upon the birthday nearest to the date of purchase, and the mortality is traced through each year of the duration of the annuity. The accuracy of this method is conditional upon no error of importance being involved in the assumption that the exact age at date of purchase is, on the average, equal to the nearest integral age. With the object of measuring the extent of the error when the method is applied to the case of the Government life annuitants an examination has been made of all cases included in the experience where the actual dates of birth are stated. It was found that the assumption mentioned had the effect of understating the tabular ages at purchase by about 20 days in the case of male lives and about 15 days in the case of female lives, and, in view of these slight differences, the method was considered to be sufficiently exact for the purposes of the present experience.

In all cases where only the date of baptism was available and not the actual date of birth, it was assumed that birth preceded

baptism by a period of 30 days. The average deviation between the tabular and exact ages at purchase, when the birthday was obtained on this assumption, was 18 days for male lives and 16 days for female lives, or practically identical with that shown in the cases where the exact date of birth was known.

The ages at date of purchase having been obtained in the manner described, the elementary facts relating to males and females respectively were tabulated under each age according to the duration of the annuity.

The period each life was under observation began either from the anniversary in 1875 of the date of purchase of the annuity or from the actual date of purchase in the years 1875 to 1903, and continued until the anniversary in 1904 of the date of purchase, or until previous death. The numbers of the lives coming under observation at their contract anniversaries in 1875 are stated in the column "Entered" of the tables opposite the appropriate number of "Years elapsed since purchase." The latter are the number of years between the actual date of purchase and its anniversary in 1875. The lives on which annuities were granted during the years 1875 to 1903 all come under observation from the date of purchase, and the numbers are stated on the first line opposite year "0" as there was no previous duration of the annuities in these cases.

The column "Existing" refers to the lives who were living in 1904 on the anniversary of the date of purchase, and who passed out of observation after their annuities had continued for the number of years given in the column "Years elapsed since purchase," the number between the date of purchase and its anniversary in 1904.

The duration in the case of the "died" was obtained by taking the number of integral years between the date of purchase and the date of death, and represents the number of contract years through which the life had passed previous to the contract year in which death occurred. In some instances the actual date of death was not known, and death could only be presumed owing to the cessation of the usual application for payment of the annuity. The assumption made in these cases was that the nominee of any annuity of which the instalment had remained unclaimed for three or more years could be taken as having died in the middle of the period between the date when the last paid instalment became due and the date of the first unclaimed instalment becoming due.

A series of independent tables for each age at purchase showing the variation in the rate of mortality with the duration since date of purchase are known as Select or Analyzed Tables. A considerable number of annuities are purchased at the National Debt Office under wills, &c., where the nominees do not exercise any option in the matter, but in most cases the nominees themselves are the purchasers. It is unlikely that the latter would choose a mode of investment which involves the risk of losing, in the event of premature death, a considerable part of the money invested, unless they considered themselves to be in a good state of health at the date of purchase. The effect of this self-selection, which corresponds in some measure

to that of the medical examination in the case of assurances against death, is shown by the rates of mortality during the years immediately following the grant of an annuity being lower than for lives of the same age whose annuities have been longer in force. It is usually found, however, that the duration, apart from the age, ceases to have much effect upon the mortality after a few years have elapsed from the date of purchase, and thereafter the rate of mortality may be considered to be dependent upon the age only. Tables constructed upon this basis and formed by grouping together all lives of the same attained age irrespective of the time elapsed since the annuities were purchased are known as "Aggregate Tables." Such tables may be either a complete aggregate of the Select Tables comprising all ages at purchase and all durations, or they may consist of an amalgamation of the Select Tables after excluding the experience relating to a certain number of years immediately following the date of purchase. Tables of the latter class are intended to represent the normal rate of mortality after the effect of the selection which is exercised at purchase is more or less exhausted. There are no means at present available of accurately measuring the full effect of this selection, and no definite limit can therefore be assigned to the period following purchase during which the rate of mortality is affected by its influence. The extent, however, to which the different tables are affected by the varying degrees of selection present in the data upon which they are based may be illustrated by a comparison of the expectations of life. In the case of males the values of $e_{[x]}$ are given for age groups 50–54 to 70–74 only, owing to the paucity of the select data relating to ages outside these limits. The table also serves to show the considerable differences which exist between the expectations of life of males and females for all the ages at which annuities are usually purchased.

Table showing the Mean Values of the Expectations of Life for various Groups of Ages according to different Classifications of the Data.

Group of Ages	Males				Females				Group of Ages
	Mean of Five Values of Expectations of Life				Mean of Five Values of Expectations of Life				
	Select Tables	Full Aggregate Table	Aggregate Table excluding First 5 Years	Aggregate Table excluding First 10 Years	Select Tables	Full Aggregate Table	Aggregate Table excluding First 5 Years	Aggregate Table excluding First 10 Years	
	$e_{[x]}$	e_x	$e_x^{(5)}$	$e_x^{(10)}$	$e_{[x]}$	e_x	$e_x^{(5)}$	$e_x^{(10)}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
40–44	...	24·97	24·42	24·00	29·01	29·47	28·54	27·52	40–44
45–49	...	21·78	21·31	21·16	25·17	25·82	24·99	24·42	45–49
50–54	18·89	19·13	18·69	18·52	22·21	22·23	21·60	21·19	50–54
55–59	16·15	16·01	15·66	15·23	18·13	18·51	18·04	17·67	55–59
60–64	12·80	12·89	12·61	12·34	15·42	15·00	14·61	14·37	60–64
65–69	10·26	10·07	9·83	9·71	12·35	11·78	11·58	11·39	65–69
70–74	8·58	7·59	7·41	7·27	9·36	8·84	8·73	8·67	70–74
75–79	..	5·51	5·43	5·39	6·98	6·43	6·38	6·34	75–79

NOTE.—The expectations of life given in the above Table and elsewhere in this Report are the "curtate" expectations (e_x). If comparison is made with the corresponding values of $\mathring{e}_x$ (the "complete" expectations), as given in Mr. A. J. Finlaison's Report, ·5 should be deducted from the latter values at all ages.

It will be of interest to compare some of the results brought out by the present investigation with those shown by (1) the Government annuity tables at present in force, based upon the experience of annuitants during the period 1808–1875, and (2) the British Offices annuity tables, based upon the combined experience in respect of life annuities of 43 British Assurance companies during the period 1863–1893.

MALES.

Table showing, in Decennial Age Groups, the actual Number of Deaths recorded in each of the First Five Years following Purchase and the corresponding Number that would have been expected to occur if the Rate of Mortality had been similar to that shown by (1) *the Government Annuity Experience* (1808–1875), (2) *the British Offices Annuity Experience* (1863–1893).

Group of Ages at Purchase	ACTUAL DEATHS (Government Annuity Experience, 1875-1904)						EXPECTED DEATHS (Government Annuity Experience, 1808-1875)						EXPECTED DEATHS (British Offices Annuity Experience, 1863-1893)						Group of Ages at Purchase
	Number of complete Years elapsed since Purchase					Total First 5 Years	Number of complete Years elapsed since Purchase					Total First 5 Years	Number of complete Years elapsed since Purchase					Total First 5 Years	
	0	1	2	3	4		0	1	2	3	4		0	1	2	3	4		
Under 30	...	...	..	...	1	1	...	...	1	1	1	3	...	..	4	1	...	5	Under 30
30–39	1	...	2	3	1	7	1	1	1	2	1	6	1	3	2	...	...	6	30–39
40–49	5	6	5	8	10	34	3	6	8	6	7	30	2	2	5	6	5	20	40–49
50–59	15	9	19	21	20	84	15	19	23	22	22	101	10	17	17	19	21	87	50–59
60–69	25	47	54	58	36	220	35	35	38	42	59	209	31	39	44	44	52	210	60–69
70–79	23	47	31	55	54	210	27	43	47	49	53	219	33	43	40	47	58	221	70–79
80 and upwards.	9	14	11	13	15	62	11	11	15	13	14	64	11	12	15	9	11	58	80 and upwards.
	78	123	122	158	137	618	92	115	133	135	157	632	88	116	127	126	150	607	

Females.

Table showing, in Decennial Age Groups, the actual Number of Deaths recorded in each of the First Five Years following Purchase and the corresponding Number that would have been expected to occur if the Rate of Mortality had been similar to that shown by (1) *the Government Annuity Experience* (1808–1875), (2) *the British Offices Annuity Experience* (1863–1893).

Group of Ages at Purchase	Actual Deaths (Government Annuity Experience, 1875-1904)						Expected Deaths (Government Annuity Experience, 1808-1875)						Expected Deaths (British Offices Annuity Experience, 1863-1893)						Group of Ages at Purchase
	Number of complete Years elapsed since Purchase					Total First 5 Years	Number of complete Years elapsed since Purchase					Total First 5 Years	Number of complete Years elapsed since Purchase					Total First 5 Years	
	0	1	2	3	4		0	1	2	3	4		0	1	2	3	4		
Under 30	1	...	...	...	...	1	1	1	2	...	2	6	...	...	1	..	1	2	Under 30
30–39	...	3	3	2	2	10	2	1	4	3	2	12	2	4	4	2	3	15	30–39
40–49	7	3	7	12	11	40	7	7	10	10	8	42	8	8	6	12	6	40	40–49
50–59	30	26	31	26	41	154	18	31	40	45	44	178	22	27	33	38	45	165	50–59
60–69	44	63	89	93	93	382	51	67	78	99	122	417	54	73	73	102	104	406	60–69
70–79	51	74	73	78	97	373	55	87	74	88	98	402	53	68	73	81	91	366	70–79
80 and upwards.	14	12	13	13	15	67	18	16	13	15	15	77	14	25	15	19	21	94	80 and upwards.
	147	181	216	224	259	1,027	152	210	221	260	291	1,134	153	205	205	254	271	1,088	

It will be seen, in the case of male annuitants, that the total number of deaths recorded in the five years was 618 as compared with 632 expected deaths according to the previous Government experience and 607 according to the British Offices experience. On the whole, therefore, there is little indication of any important change in the rate of mortality, although considerable divergencies exist in respect of individual age groups. If the ages below 60 are excluded the total deaths are practically identical in all three cases. Owing to the paucity of the data, the deaths occurring in successive years of duration exhibit various irregularities, but the light mortality in the year following purchase, year "0", is a noticeable feature of the new experience at the older ages.

In the case of females it will be observed that the results lead to more definite conclusions, the most significant feature being the comparatively light mortality exhibited by the lives included in the new experience. There are clear indications of a distinct improvement in the mortality of Government female annuitants during the period 1875–1904 as compared with the previous period, the total number of deaths recorded being less than the expected deaths for each year of duration—the aggregate difference for the five years being approximately 10 per-cent.

The Government annuity tables at present in force are based upon the assumption that the effect of selection may be neglected after four years have elapsed from date of purchase. A table based upon similar principles has been deduced from the present experience, and a comparison is given below between the actual deaths recorded at durations of four years and upwards and the corresponding number expected according to the experience of the period 1808–1875.

[*See* Table on p. 75.]

The table tends to confirm the conclusions based upon the relative rates of mortality in the first five years following purchase. In both cases the results indicate that the improved vitality of annuitants during the period under consideration, although to a certain extent common to nominees of either sex, was much more marked in the case of female lives.

The aggregate tables based upon the new experience, after excluding the facts relating to the early years of duration, are more in accord with the corresponding British Offices tables. This will be apparent from the comparative statement [given on page 76] representing the mortality according to the aggregate experience, after the exclusion in each case of the first five years following purchase.

Comparison of the Actual Number of Deaths recorded amongst Nominees whose Annuities had been in force Four Years and upwards at Date of Death, with the Number that would have been expected to occur at corresponding Ages if the Rate of Mortality had been similar to that shown by the Government Annuity Experience, 1808–1875.

Group of Ages at Death	Males				Females			
	Actual Deaths. (Government Annuity Experience, 1875-1904)	Expected Deaths. (Government Annuity Experience, 1808-1875)	Excess (+), or Defect (−), in Expected as compared with Actual Deaths		Actual Deaths. (Government Annuity Experience, 1875-1904)	Expected Deaths. (Government Annuity Experience, 1808-1875)	Excess (+), or Defect (−), in Expected as compared with Actual Deaths	
			Col. (3) minus Col. (2)	Per-cent			Col. (7) minus Col. (6)	Per-cent
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Under 30	2	3	+ 1		...	3	+ 3	
30–39	3	8	+ 5	+ 21·62	9	14	+ 5	+ 8·82
40–49	32	34	+ 2		59	57	− 2	
50–59	120	133	+ 13	+ 10·83	233	278	+ 45	+ 19·31
60–69	514	533	+ 19	+ 3·70	1,112	1,219	+ 107	+ 9·62
70–79	1,519	1,529	+ 10	+ ·66	3,268	3,528	+ 260	+ 7·96
80–89	1,283	1,319	+ 36	+ 2·81	3,276	3,464	+ 188	+ 5·74
90 and upwards	215	211	− 4	− 1·86	608	674	+ 66	+ 10 86
	3,688	3,770	+ 82	+ 2·22	8,565	9,237	+ 672	+ 7·85

Comparison of the Actual Number of Deaths recorded amongst Nominees whose Annuities had been in force Five Years and upwards at Date of Death, with the Number that would have been expected to occur at corresponding Ages if the Rate of Mortality had been similar to that shown by the British Offices Annuity Experience.

Group of Ages at Death	MALES				FEMALES			
	Actual Deaths. (Government Annuity Experience, 1875-1904)	Expected Deaths. (British Offices Annuity Experience, 1863-1893)	Excess (+), or Defect (−), in Expected as compared with Actual Deaths		Actual Deaths. (Government Annuity Experience, 1875-1904.)	Expected Deaths. (British Offices Annuity Experience, 1863-1893)	Excess in Expected as compared with Actual Deaths	
			Col. (3) minus Col. (2)	Per-cent			Col. (7) minus Col. (6)	Per-cent
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Under 30	1	...	− 1		..	1	1	
30–39	2	6	+ 4	−29·03	9	14	5	25·00
40–49	28	16	−12		51	60	9	
50–59	106	136	+30	+28·30	210	216	6	2·86
60–69	482	488	+ 6	+ 1·24	1,037	1,058	21	2·03
70–79	1,465	1,433	−32	− 2·18	3,158	3,163	5	·16
80–89	1,252	1,253	+ 1	+ ·08	3,234	3,271	37	1·14
90 and upwards.	215	206	− 9	− 4·19	607	622	15	2·47
	3,551	3,538	− 13	− ·37	8,306	8,405	99	1·19

In deducing the expected deaths in the preceding tables the unadjusted rates of mortality were employed.

The extent to which the lives of annuitants entering at different ages were prolonged as a consequence of the lower rates of mortality to which they were apparently subject during the period 1875–1904 may be measured by comparing the select expectations of life according to the three different experiences already mentioned. This comparison is shown in the following table [p. 78], where the values appearing opposite each group of ages represent the mean expectations of life for the five consecutive ages at purchase comprised in the group. The values are deduced from the unadjusted rates of mortality, with the exception of those representing the Government Annuity Experience, 1808–1875, which are taken from the graduated tables given in Mr. Finlaison's report.

In the case of males it will be seen from column (5) that, although the expectations of life according to the new experience are throughout in excess of those based upon the experience of the period 1808–1875, there is a close agreement between the values for the important age groups 60–64 and 65–69. The corresponding differences for females are given in column (10), and it would appear from these figures that a considerable increase has taken place in the duration of life of female annuitants at all ages. The figures in column (11) show that the expectations of life for females, according to the new experience, are also at most ages in excess of those based upon the British Offices experience, but in this case the differences are smaller and vary in sign, superior vitality being exhibited by the British Offices annuitants entering at ages 55–59 and 70–74. The explanation of the comparatively low expectation of life for age group 55–59 according to the new experience is to be found in the low values of the expectations for ages at purchase 55 and 56. For some reason the female annuitants entering at those ages were inferior lives, and the heavy mortality amongst them had the effect of reducing the average expectation for the group to a lower level than would have ruled if these nominees had maintained the normal standard of vitality.

The rates of mortality deduced from the crude facts of the experience require to be subjected to a process of adjustment, in order to remove irregularities due to limitation of the data, and to obtain a smooth series of values which will serve as a basis for the construction of monetary tables. The graduated table must be in a convenient form for practical use, and produce results which do not materially differ from those given by the ungraduated table. When it may be assumed that the factor of selection is of no practical importance after a few years have elapsed from the date of purchase, such a table may be constructed on the basis of a combination of rates of mortality varying with the age and duration during a limited number of years, and subsequent rates varying only with the age. The present Government annuity tables are based on the assumption that the effect of selection is practically exhausted after the expiration of four years, and consequently after that time

Comparison of Expectations of Life at Date of Purchase, based upon the Government Annuity Experience, 1875–1904, *with the corresponding Values according to the previous Government Experience and the British Offices Experience.*

Group of Ages at Purchase	MALES					FEMALES					Group of Ages at Purchase
	Mean of Five Values of Expectations of Life			Differences.		Mean of Five Values of Expectations of Life			Differences		
	Government Annuity Experience		British Offices Annuity Experience, 1863-1893	Col. (3) minus Col. (2)	Col. (3) minus Col. (4)	Government Annuity Experience		British Offices Annuity Experience 1863-1893	Col. (8) minus Col. (7)	Col. (8) minus Col. (9)	
	1808-1875	1875-1904				1808-1875	1875-904				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
						27·97	29·01	28·55	+1·04	+·46	40–44
						24·51	25·17	24·71	+·66	+·46	45–49
50–54	18·60	18·89	18·83	+·29	+·06	21·07	22·21	21·78	+1·14	+·43	50–54
55–59	15·66	16·15	16·09	+·49	+·06	17·72	18·13	18·66	+·41	−·53	55–59
60–64	12·77	12·80	13·16	+·03	−·36	14·50	15·42	15·08	+·92	+·34	60–64
65–69	10·22	10·26	10·32	+·04	−·06	11·48	12·35	12·13	+·87	+·22	65–69
70–74	7·98	8·58	8·22	+·60	+·36	8·85	9·36	9·49	+·51	−·13	70–74
						6·74	6·98	6·74	+·24	+·24	75–79
						*5·64	5·73	5·00	+·09	+·73	80–84

* This is the value for age 80, which is the limit of the table.

the select rates of mortality are merged into an aggregate table. In the case of the British Offices annuity experience the effect of selection was found to extend over a longer period, and the results of the present investigation lead to a similar conclusion. The new tables based upon the present experience have been constructed in such a manner that when rates of mortality varying with the age and duration during the first five years are merged into those of an aggregate table varying only with the age, the expectations of life deduced therefrom will approximate closely to the true values at the date of purchase and at the expiration of five years thereafter.

In the graduation of mortality tables considerable use has been made in recent years of methods involving the assumption of a law of mortality, more especially the law known as "Makeham's first Modification of Gompertz's Law." According to the law of human mortality, as originally propounded by Gompertz, the function known as the "force of mortality" increases with the age in geometrical progression. This theory was further developed by Makeham, who introduced an important modification resting on the hypothesis that the force of mortality is composed of two parts, one of which is constant and the other a geometrical series. The law thus modified has been found to represent with great accuracy the rates of mortality of adult lives, as shown by many important tables. Expressed algebraically Makeham's formula for the law of mortality is $\mu_x = A + Bc^x$, where μ_x represents the force of mortality at age x and A, B, and c are constants, the values of which are to be derived from the facts. Tables graduated by this formula present a perfectly smooth series of values, and, moreover, embody an important principle known as the "Law of Uniform Seniority", which enables the value of an annuity upon two or more joint lives of different ages to be deduced from that of an annuity upon the same number of joint lives of equal ages. Makeham's formula, therefore, possesses many advantages as an instrument of graduation if it can be applied without any material distortion of the facts. A graduation by this formula of the aggregate tables based upon the present experience was found to yield satisfactory results, and the method has accordingly been adopted.

The degree to which the features of the original experience are reproduced in the graduated tables constructed to represent the experience after the exclusion of the first five years following purchase may be judged from a comparison of the ungraduated and graduated expectations of life as shown in the two following tables.

MALES.

Comparison of Ungraduated and Graduated Expectations of Life.

Group of Ages	Mean of Five Values of Expectations of Life			Excess (+), or Defect (−), in the Graduated Expectations of Life as compared with the Values of	
	Ungraduated Select Tables, 5 Years after Purchase	Ungraduated Aggregate Table excluding First 5 Years	Graduated Table		
	$e_{[x-5]+5}$	$e_x^{(5)}$		$e_{[x-5]+5}$	$e_x^{(5)}$
(1)	(2)	(3)	(4)	(5)	(6)
55–59	15·44	15·66	15·63	+·19	−·03
60–64	12·77	12·61	12·68	−·09	+·07
65–69	9·82	9·83	10·01	+·19	+·18
70–74	7·58	7·41	7·67	+·09	+·26
75–79	5·81	5·43	5·68	−·13	+·25
80–84	...	3·97	4·06	. .	+·09
85–89	...	2·86	2·78	. .	−·08

FEMALES.

Comparison of Ungraduated and Graduated Expectations of Life.

Group of Ages	Mean of Five Values of Expectations of Life			Excess (+), or Defect (−), in the Graduated Expectations of Life as compared with the Values of	
	Ungraduated Select Tables, 5 Years after Purchase	Ungraduated Aggregate Table excluding First 5 Years	Graduated Table		
	$e_{[x-5]+5}$	$e_x^{(5)}$		$e_{[x-5]+5}$	$e_x^{(5)}$
(1)	(2)	(3)	(4)	(5)	(6)
45–49	25·36	24·99	25·08	−·28	+·09
50–54	21·20	21·60	21·57	+·37	−·03
55–59	18·23	18·04	18·14	−·09	+·10
60–64	14·42	14·61	14 86	+·44	+·25
65–69	11·84	11·58	11·80	−·04	+·22
70–74	9·00	8·73	9·04	+·04	+·31
75–79	6·60	6·38	6·66	+·06	+·28
80–84	4·79	4·48	4·68	−·11	+·20
85–89	. .	3 12	3·12	...	...

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The expectations of life for males commence with age group 55–59 owing to the paucity of the original data at the younger ages. It will be seen from the differences in columns (5) and (6) that for ages 70–79 in the case of males and ages 65–84 in the case of females, the graduated expectations of life are considerably in excess of those based upon the aggregate experience as shown in column (3), but agree approximately with the select expectations given in column (2). These ages comprise the sections of the tables where it was found necessary to modify the values based upon the aggregate experience as shown in column (3), which did not adequately represent the true expectations of life five years after purchase. The abnormally low vitality of the female entrants at ages 55–59, which is reflected in the reduced values of the select expectations of life both at date of purchase and five years afterwards, has already been referred to, but this feature has not been retained in the graduation, and it will be observed that the graduated value for age group 60–64, corresponding to ages 55–59 at date of purchase, is considerably higher than either of the ungraduated values for this group. The mean of the graduated expectations of life for each group of ages of both males and females lies either between the corresponding values of $e_{[x-5]+5}$, and $e_x^{(5)}$, or is in excess of both.

Makeham's formula, as originally expressed, is not suitable for the graduation of rates of mortality which vary both with the age and with the duration, but Mr. G. F. Hardy, recently President of the Institute of Actuaries, has shown that the formula may be adapted to this purpose, and demonstrated its practical application in the graduation of the British Offices tables. A similar course has been followed on the present occasion and advantage has been taken of the methods indicated by Mr. Hardy in his memorandum on the subject. It will not be necessary to reproduce here the various mathematical expressions employed to establish the requisite relations between the select and ultimate sections of the tables in order to preserve the law of uniform seniority.

The graduated rates of mortality for each of the first five years after purchase and for five years and upwards after purchase, together with the values of the functions p_x, e_x, and a_x, at 3 per-cent interest, as deduced therefrom, are given in the tables appended to this report.

The following table gives a comparison between the values of annuities at 3 per-cent interest, as deduced from the graduated experience, and the corresponding values based upon the original data. For ages under 65 in the case of males, and under 60 in the case of females, where the select expectations of life five years after purchase were found on the average to be in close agreement with those based on the corresponding aggregate experience, the values appearing in columns (2) and (6) have been obtained by using the select rates of mortality for the first five years following purchase in combination with the five years and upwards aggregate table. For ages above 65 in the case of males, and above 60 in the case of females, the ungraduated values are based entirely upon the select tables.

Comparison of Ungraduated and Graduated Annuity Values at 3 per-cent Interest.

Group of Ages	MALES				FEMALES				Group of Ages
	Mean of Five Values of $a_{[x]}$		Deviations Col. (3) minus Col. (2)		Mean of Five Values of $a_{[x]}$		Deviations Col. (7) minus Col. (6)		
	Ungraduated	Graduated	+	−	Ungraduated	Graduated	+	−	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	0)
					16·69	16·71	·02	...	45–49
50–54	13·54	13·57	·03	...	15·13	15·11	..	·02	50–54
55–59	11·83	11·86	·03	...	13·22	13·38	·16	..	55–59
60–64	9·96	10·12	·16		11·60	11·55	...	·05	60–64
65–69	8·29	8·43	·14	...	9·70	9·69	...	·01	65–69
70–74	7·20	6·83	...	·37	7 67	7·88	·21	...	70–74
75–79	5·07	5·38	·31	...	5·94	6·19	·25	.	75–79
					5·01	4·70	..	·31	80–84
Average	9·31	9·36	·05	...	10·62	10·65	·03	.	

The larger deviations at the older ages are mainly due to the irregularities which arise from the scarcity of the select data upon which the ungraduated values at those ages are based.

In the following table a comparison is shown of the values of annuities according to the new tables with the corresponding values of the annuity tables published in 1903, which were based upon the British Offices graduated experience.

Comparison of Annuity Values at 3 per-cent Interest according to the new Graduated Tables and according to the British Offices Annuity Tables.

Age	MALES			FEMALES		
	Value of an Annuity of 1		Col. (2) minus Col (3)	Value of an Annuity of 1		Col. (5) minus Col. (6)
	New Government Table	British Offices Annuity Table		New Government Table	British Offices Annuity Table	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
40	17·34	17·60	−·26	18·69	18·26	+·43
45	15·85	16·06	−·21	17·31	16·93	+·38
50	14·24	14·40	−·16	15·78	15·51	+·27
55	12·55	12·66	−·11	14·09	13·96	+·13
60	10·82	10 88	−·06	12·29	12·23	+·06
65	9·10	9·12	−·02	10·43	10·33	+·10
70	7·45	7·44	+·01	8·59	8·41	+·18
75	5·93	5·90	+·03	6·84	6·61	+·23
80	4·59	4·54	+·05	5·27	5·05	+·22

It will be seen from the differences in column (4) that the annuity values on male lives are in close agreement for ages above 55. At the younger ages the British Offices male life values were made to correspond approximately with those deduced from the experience of assured lives, and they are slightly higher at these ages than the values given by the new tables.

In the case of female lives the new values are throughout in excess of those based upon the British Offices graduated experience, the largest differences occurring at the beginning and near the end of the table.

The results of the present investigation point to the conclusion that the mortality tables on which the cost of Government life annuities is now based underestimate the vitality of annuitants, and, consequently, that these annuities are granted on too favourable terms to the persons who purchase them.

The following tables show the effect which the adoption of the new tables would have upon the cost of Government life annuities when the rate of interest is taken at 3 per-cent per annum.

Cost of an Annuity of £100 payable in Quarterly Instalments with an additional Payment of one-fourth part of the Annuity after Death; the Interest of Money being 3 per-cent per annum.

Male Lives.

Age	New Table of 1910	Present Table	Variation	
			In Amount	Per-cent
	£	£	£	
40	1,783	1,687	+ 96	+5·7
45	1,635	1,566	+ 69	+4·4
50	1,476	1,433	+ 43	+3·0
55	1,308	1,284	+ 24	+1·9
60	1,136	1,114	+ 22	+2·0
65	965	946	+ 19	+2·0
70	801	786	+ 15	+1·9
75	651	639	+ 12	+1·9
80	518	514	+ 4	+ ·8

Female Lives.

Age	New Table of 1910	Present Table	Variation	
			In Amount	Per-cent
	£	£	£	
40	1,917	1,867	+ 50	+2·7
45	1,781	1,732	+ 49	+2·8
50	1,628	1,578	+ 50	+3·2
55	1,460	1,413	+ 47	+3·3
60	1,282	1,232	+ 50	+4·1
65	1,098	1,046	+ 52	+5·0
70	915	856	+ 59	+6·9
75	741	694	+ 47	+6·8
80	585	552	+ 33	+6·0

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These tables show that the cost of annuities should be increased at all ages for both male and female lives. In the case of males the most noticeable feature is the large increase in the cost at the younger ages. Comparatively few annuities are purchased at ages under 50, and the present experience, in common with other annuity experiences, suffers from a scarcity of data at the younger ages. The new values at these ages, however, would be still somewhat lower than the corresponding values according to the British Offices annuity tables. In the case of females the new values, when compared with those at present in force, show a considerable increase at all ages, the additional cost being approximately equal to half a year's purchase of the annuities throughout the greater part of the table. The average increase in the cost would be 2·2 per-cent for male lives and 4·5 per-cent for female lives at the ages at which annuities are usually purchased, viz., 50 to 75.

I have to acknowledge the valuable assistance rendered by Mr. W. R. Jarman of this Department during the course of the investigation, especially as regards the graduation of the tables.

I am, Sir,

Your obedient servant,

J. BLAKEY,

Actuary to the National Debt Commissioners.

Male Life Annuitants.

The Value of an Annuity of 1, *at* 3 *per-cent Interest, for the Year in which Life Annuities are purchased, for each of the Four subsequent Years, and for Five Years and upwards from Purchase.*

Age at Purchase $[x]$	Years elapsed since Purchase						Age attained $x+5$
	0	1	2	3	4	5 and upwards	
	$a_{[x]}$	$a_{[x]+1}$	$a_{[x]+2}$	$a_{[x]+3}$	$a_{[x]+4}$	a_{x+5}	
40	17·342	16·953	16·604	16·276	15·957	15·641	45
41	17·054	16·659	16·303	15·970	15·645	15·324	46
42	16·761	16·360	15·998	15·659	15·328	15·003	47
43	16·463	16·055	15·687	15·342	15·007	14·677	48
44	16·160	15·746	15·372	15·021	14·681	14·347	49
45	15·852	15·431	15·051	14·696	14·351	14·011	50
46	15·539	15·112	14·726	14·365	14·016	13·673	51
47	15·222	14·789	14·398	14·032	13·677	13·331	52
48	14·900	14·461	14·064	13·693	13·336	12·986	53
49	14·574	14·129	13·727	13·352	12·990	12·638	54
50	14·245	13·794	13·387	13·007	12·642	12·287	55
51	13·911	13·455	13·043	12·660	12·292	11·935	56
52	13·575	13·113	12·697	12·310	11·940	11·580	57
53	13·236	12·769	12·349	11·958	11·586	11·225	58
54	12·895	12·423	11·998	11·605	11·230	10·869	59
55	12·552	12·075	11·646	11·251	10·874	10·513	60
56	12·206	11·725	11·293	10·895	10·518	10·156	61
57	11·860	11·374	10·939	10·539	10·162	9·801	62
58	11·513	11·022	10·585	10·184	9·807	9·447	63
59	11·165	10·671	10·231	9·830	9·453	9·095	64
60	10·817	10·320	9·878	9·476	9·101	8·744	65
61	10·470	9·969	9·527	9·125	8·750	8·397	66
62	10·123	9·620	9·177	8·776	8·403	8·053	67
63	9·779	9·274	8·830	8·429	8·059	7·712	68
64	9·436	8·929	8·485	8·086	7·719	7·376	69
65	9·097	8·588	8·144	7·747	7·383	7·045	70
66	8·759	8·250	7·806	7·412	7·052	6·719	71
67	8·426	7·915	7·473	7·082	6·726	6·399	72
68	8·096	7·585	7·145	6·757	6·406	6·085	73
69	7·770	7·260	6·822	6·438	6·093	5·777	74
70	7·450	6·940	6·505	6·125	5·786	5·477	75
71	7·135	6·626	6·194	5·819	5·485	5·184	76
72	6·825	6·318	5·889	5·519	5·193	4·899	77
73	6·522	6·016	5·592	5·228	4·908	4·622	78
74	6·225	5·722	5·302	4·944	4·631	4·354	79
75	5·934	5·435	5·019	4·668	4·363	4·094	80
76	5·652	5·155	4·745	4·400	4·103	3·842	81
77	5·376	4·883	4·479	4·141	3·852	3·600	82
78	5·107	4·619	4·221	3·890	3·610	3·367	83
79	4·847	4·363	3·972	3·649	3·377	3·143	84
80	4·594	4·116	3·732	3·417	3·153	2·928	85
...	...	...	...	...	...	2·723	86
...	...	...	...	...	...	2·526	87
...	...	...	...	...	...	2·339	88
...	...	...	...	...	...	2·161	89
...	...	...	...	...	...	1·993	90
...	...	...	...	...	...	1·833	91
...	...	...	...	...	...	1·681	92
...	...	...	...	...	...	1·538	93
...	...	...	...	...	...	1·404	94
...	...	...	...	...	...	1·277	95
...	...	...	...	...	...	1·158	96
...	...	...	...	...	...	1·046	97
...	...	...	...	...	...	·941	98
...	...	...	...	...	...	·842	99
...	...	...	...	...	...	·746	100

FEMALE LIFE ANNUITANTS.

The Value of an Annuity of 1, *at* 3 *per-cent Interest, for the Year in which Life Annuities are purchased, for each of the Four subsequent Years, and for Five Years and upwards from Purchase.*

Age at Purchase $[x]$	YEARS ELAPSED SINCE PURCHASE						Age Attained $x+5$
	0	1	2	3	4	5 and upward	
	$a_{[x]}$	$a_{[x]+1}$	$a_{[x]+2}$	$a_{[x]+3}$	$a_{[x]+4}$	a_{x+5}	
40	18·689	18·311	17·951	17·616	17·304	17·008	45
41	18·428	18·042	17·676	17 333	17·015	16·712	46
42	18·159	17·767	17·394	17·045	16·720	16·410	47
43	17·885	17·486	17·106	16·750	16·418	16·102	48
44	17·604	17·198	16·810	16·449	16·110	15·787	49
45	17·315	16·902	16·509	16·140	15·795	15·465	50
46	17·021	16·600	16·200	15·824	15·473	15·137	51
47	16·719	16·292	15·884	15·503	15·145	14·803	52
48	16·411	15·977	15·563	15·174	14·810	14·463	53
49	16·097	15·656	15·234	14·840	14 471	14·117	54
50	15·776	15 327	14·900	14·500	14·125	13·766	55
51	15·451	14·994	14·560	14·154	13·774	13·410	56
52	15·118	14·654	14·215	13·803	13·418	13·048	57
53	14·781	14·310	13·864	13·448	13·057	12·684	58
54	14·438	13·960	13·509	13·087	12·692	12·315	59
55	14·090	13·606	13·148	12·722	12·323	11·942	60
56	13·738	13·247	12·784	12·353	11·950	11·567	61
57	13·381	12·884	12·416	11·981	11·575	11·189	62
58	13·021	12·517	12·044	11 606	11·197	10·809	63
59	12·657	12·147	11·670	11·229	10·818	10·429	64
60	12·291	11·775	11·294	10·850	10·438	10·048	65
61	11·922	11·401	10·916	10·470	10·057	9·666	66
62	11·551	11·025	10·537	10·089	9·675	9·285	67
63	11·179	10·648	10·157	9·708	9·295	8·906	68
64	10·807	10·271	9·777	9·328	8·916	8·528	69
65	10·434	9·894	9·399	8·950	8·538	8·154	70
66	10·062	9·518	9·022	8·573	8·164	7·783	71
67	9·691	9·144	8·646	8·199	7·793	7·416	72
68	9·322	8·772	8·274	7·829	7·426	7·054	73
69	8·955	8·403	7·905	7·463	7·064	6·697	74
70	8·592	8 038	7·541	7·102	6·708	6·346	75
71	8·232	7·677	7·182	6·746	6·358	6·002	76
72	7·877	7·320	6·828	6·397	6·014	5·666	77
73	7·526	6·970	6·480	6·054	5·678	5·338	78
74	7·182	6·626	6·139	5·719	5·351	5·019	79
75	6·844	6·288	5·806	5·392	5·031	4·708	80
76	6·512	5·958	5·480	5·074	4·721	4·408	81
77	6·188	5·636	5·163	4·764	4·421	4·117	82
78	5·872	5·323	4·856	4·464	4·130	3·836	83
79	5·564	5·018	4·557	4·175	3·850	3·567	84
80	5·265	4 723	4·269	3·895	3·581	3·308	85
...	...	...	...	...	...	3·060	86
...	...	..	...	...	...	2·824	87
...	..	...	..	..	...	2·599	88
...	.	...	...	...	...	2·385	89
...	.	...	. .	...	...	2·182	90
..	. .	.	...		.	1·991	91
...	...	.	...	...	...	1·811	92
...	..	...	..	..	...	1·641	93
...	...	..	...	...		1·482	94
...	...	...	...	.	...	1·334	95
..	...	...	...	...	. .	1·195	96
...	...	...	..	..	.	1·066	97
..	...	...	...	...	...	·946	98
...	...	...	...	...	...	·834	99
..	...	...	...	...	...	·728	100

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