EXCESS OF LOSS REINSURANCE WITH REINSTATEMENTS REVISITED

BY

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ABSTRACT

The classical evaluation of pure premiums for excess of loss reinsurance with reinstatements requires the knowldege of the claim size distribution of the insurance risk. In the situation of incomplete information, where only a few characteristics of the aggregate claims to an excess of loss layer can be estimated, the method of stop-loss ordered bounds yields a simple analytical distribution-free approximation to pure premiums of excess of loss reinsurance with reinstatements. It is shown that the obtained approximation is enough accurate for practical purposes and improves the analytical approximations obtained using either a gamma, translated gamma, translated inverse Gaussian or a mixture of the last two distributions.

KEYWORDS

Excess of loss, reinstatement, aggregate deductible, aggregate limit, compound Poisson, Pareto, stop-loss order

1. INTRODUCTION

Despite their importance in practice, excess of loss reinsurance with reinstatements has been rather neglected in the actuarial literature. The main papers on the subject have been Simon(1972) and Sundt(1991). More recent discussions include Walhin(2001/2002), Walhin and Paris(2000/2001a/b), Mata(2000), Hess and Schmidt(2004).

The classical evaluation of pure premiums for excess of loss reinsurance with reinstatements, which is based on the collective model of risk theory, requires the knowledge of the claim size distribution and is exemplified in Sundt(1991). However, in practice, there is often incomplete information and only a few characteristics of the aggregate claims to an excess of loss layer can be estimated. This information includes the expected number of claims as well as the mean, variance and range of the claim size. In this situation, the method developed in Hürlimann(1996) (see also the applications in Hürlimann(2001/2003)) yields simple analytical distribution-free approximations to pure premiums of excess of loss

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reinsurance with reinstatements. As numerical examples suggest, the calculation requires usually only few operations and the obtained approximation is often quite accurate for practical purposes. Under certain conditions it improves the analytical approximations obtained from the recent proposals in Chaubey et al.(1998). The paper is organized as follows.

Sections 2 and 3 follow closely Sundt(1991). Section 2 recalls the main types of excess of loss reinsurance including aggregate deductibles, aggregate limits and reinstatements. The main formula by Sundt(1991) for the calculation of pure premiums for excess-of-loss reinsurance with equal reinstatements is presented in Section 3. The core of our analytical distribution-free approximation method is summarized in Section 4, which is based on Hürlimann(1996). There, we compare the numerical results by Sundt(1991) with those obtained using the distribution-free approach. The final Section 5 compares results obtained using the analytical approach by Sundt(1991) with those obtained from analytical approximations using a gamma, translated gamma, translated inverse Gaussian and a mixture of the last two distributions, as well as with those obtained using the distribution-free approach and the rate on line method introduced by Walhin(2001).

2. Types of excess of loss reinsurance

The non-proportional reinsurance covers discussed in the present paper have been described in detail in Sundt(1991). Let us recall the main definitions and notations.

Given an insurance portfolio over a one-year period, let N denote the *number of claims* occurring during the year and Y_i the i-th *claim size*, $i = 1,\ldots,N$.

An excess of loss reinsurance or for short an *XL reinsurance* for the *layer m in excess of* ℓ, written m *xs* ℓ, covers the part of each claim that exceeds the *deductible* ℓ but with a limit m on the payment of each claim, that is the reinsurer covers for each claim

$$Z_i = \min\{(Y_i - \ell)_+, m\}, \quad i = 1,\ldots,N, \tag{2.1}$$

where $(x)_+ = x$ if $x > 0$ and $(x)_+ = 0$ if $x \leq 0$. Let X denote the *aggregate claims to the layer*, that is the random sum

$$X = \sum_{i=1}^{N} Z_i. \tag{2.2}$$

An *XL reinsurance* for the *layer m xs* ℓ *with aggregate deductible* L, written m *xs* ℓ *xs* L, covers only the part of the aggregate claims to the layer that exceeds L, that is

$$X_L = (X - L)_+. \tag{2.3}$$

In case there is also an *aggregate limit* M to the layer, the reinsurer covers the aggregate claims to the layer that exceeds L but with a limited payment of M, that is

$$X_{L,M} = \min\{(X-L)_+, M\}, \tag{2.4}$$

and the cover is called an *XL reinsurance* for the *layer m xs ℓ with aggregate layer M xs L*. If the aggregate limit M is a whole multiple of the limit m, that is if $M = (K+1)m$, one speaks of an *XL reinsurance* for the *layer m xs ℓ in the aggregate with K reinstatements*. The reinsurance coverage of this XL reinsurance is given by

$$X_L^K = \min\{(X-L)_+, (K+1)m\}. \tag{2.5}$$

In this situation, the reinsurance has to be *reinstated* if the aggregate payment exceeds a whole multiple of the limit, that is $K \geq 1$. If $K = 0$ there are no reinstatements. In practice, there are *free* and *paid reinstatements*. A *reinstatement premium* is expressed as percentage of the *initial premium* P_L^K, say $c_k P_L^K$ for the k-th reinstatement with $c_k \geq 0$, and it is *paid pro rata* of the claims to the layer. If $c_k = 0$ the k-th reinstatement is free. The k-th reinstatement with premium $c_k P_L^K$ covers the amount

$$r_L^k = min\{(X - km - L)_+, m\}, \quad k = 1, \ldots, K. \tag{2.6}$$

In this terminology the *0*-th reinstatement is set at the initial premium P_L^K and covers the original layer, that is

$$r_L^0 = \min\{(X-L)_+, m\}. \tag{2.7}$$

Since the reinstatement is paid pro rata, the *random premium* for the k-th reinstatement is $\frac{r_L^{k-1}}{m} c_k P_L^K$. The *random total premium* income required for this XL reinsurance equals

$$_{tot}P_L^K = P_L^K \cdot \left(1 + \frac{1}{m}\sum_{k=1}^{K} c_k r_L^{k-1}\right). \tag{2.8}$$

The aggregate claims paid by the reinsurer for this XL reinsurance clearly satisfies the identity

$$\sum_{k=0}^{K} r_L^k = X_L^K, \tag{2.9}$$

with X_L^K defined above in (2.5).

3. Pure premiums for XL reinsurance with reinstatements

To calculate premiums for XL reinsurance with reinstatements, one needs to know the distribution function $F_X(x) = \Pr(X \leq x)$ of the aggregate claims X to the layer *m xs ℓ* defined in (2.2), as well as the associated *stop-loss transform* defined by $\pi_X(x) = E\left[(X - x)_+\right] = \int_x^\infty \bar{F}_X(x)dx$, where $\bar{F}_X(x) = 1 - F_X(x)$ denotes the survival probability function. Clearly, the pure premium should satisfy the expected value equation $E\left[{}_{tot}P_L^K\right] = E\left[X_L^K\right]$, that is by (2.8) the equation

$$P_L^K \cdot \left(1 + \frac{1}{m}\sum_{k=1}^{K} c_k E\left[r_L^{k-1}\right]\right) - E\left[X_L^K\right], \tag{3.1}$$

which expresses the fact that the expected premium income should be equal to the expected claim payments.

For simplicity consider only the practical case of equal reinstatements $c_k = c, k = 1,\ldots,K$ (e.g. $c = 100\%$, 40% or $c = 0$ if the reinstatements are free). Then (3.1) reads

$$P_L^K \cdot \left(1 + \tfrac{c}{m} E\left[X_L^{K-1}\right]\right) = E\left[X_L^K\right] \tag{3.2}$$

But $X_L^k = \min\{(X - L)_+, (k+1)m\} = (X - L)_+ - (X - L - (k+1)m), k = 1,\ldots, K$, and using the definition of the stop-loss transform, one obtains the main premium formula (e.g. Sundt(1991), formula following (3.4)):

$$P_L^K = \frac{\pi_X(L) - \pi_X(L + (K+1)m)}{1 + c \cdot \dfrac{\pi_X(L) - \pi_X(L + Km)}{m}} \,. \tag{3.3}$$

In particular, if $c = 0$ and $K \to \infty$ (infinitely many free reinstatements) and $L = 0$ (no aggregate deductible) one has $P_0^\infty = E[X]$, which is the pure premium of the usual XL reinsurance with layer m *xs* ℓ.

4. Distribution-free approximations

The formula (3.3) shows that the pure premium of the XL reinsurance with fixed reinstatements depends on the knowledge of the stop-loss transform associated to the aggregate claim

$$X = \sum_{i=1}^{N} Z_i = \sum_{i=1}^{N} \min\{(Y_i - \ell)_+, m\} \tag{4.1}$$

defined in (2.1) and (2.2).

In practice, the evaluation of the stop-loss transform of X relies often on numerical methods, especially Panjer like recursive algorithms or Fast Fourier Transform methods. The calculations in Sundt(1991) are based on the well-known Panjer recursion, which requires the distribution of the claim size Y_i in discrete form. The present distribution-free approach uses only a few characteristics of X, namely the *expected number of claims* $\lambda = E[N]$, the *mean* $\mu = E[Z_i]$ and the *variance* $\sigma^2 = Var[Z_i]$ of the i-th excess of loss claim size Z_i. Since the latter random variable is bounded, its *range*

$[0, m]$ is also known. In this situation, the method developed in Hürlimann(1996) yields simple analytical lower and upper bounds to the stop-loss transform of X, which directly provide distribution-free approximations to pure premiums of the XL reinsurance with reinstatements. It is shown that the use of the upper bound only does not always produce conservative pure premiums. Therefore, a calculation using the lower bound as well as the average of these approximations might be an attractive practical alternative.

Recall the essential idea of the distribution-free approach. One assumes that the XL claim size $Z = Z_i$ belongs to the set $D = D([0, m]; \mu, \sigma)$ of all random variables with fixed mean $\mu = E[Z]$, variance $\sigma^2 = Var[Z]$ and support contained in the interval $[0, m]$. Consider the quantities

$\upsilon = \left(\frac{\sigma}{\mu}\right)^2$: the relative variance of the claim size Z

$\upsilon_0 = \frac{m - \mu}{\mu}$: the maximal relative variance over the set D

$\upsilon_r = \frac{\upsilon}{\upsilon_0}$: the relative variance expressed in terms of the maximal relative variance

The main result of Section 2 in Hürlimann(1996) provides the construction of explicit stop-loss ordered random variables Z_*, Z^* with the property $Z_* \;{}_{sl}\, Z \;{}_{sl}\, Z^*$ for all $Z \in D$, that is $\pi_{Z_*}(x) \le \pi_Z(x) \le \pi_{Z^*}(x)$ for all $x \ge 0$, and all $Z \in D$, where $\pi_Z(x) = E[(Z - x)_+]$ denotes the stop-loss transform of the random variable Z. The distributions $F_*(z)$ and $F^*(z)$ of the normalized random variables $(Z_* - \mu)/\mu$ and $(Z^* - \mu)/\mu$ are summarized in Table 4.1.

TABLE 4.1

STOP-LOSS ORDERED EXTREMAL DISTRIBUTIONS

z	$F_*(z)$
$-1 \le z \le -\upsilon_r$	0
$-\upsilon_r \le z < -\upsilon$	$\upsilon_0/(1+\upsilon_0)$
$\upsilon \le z < -\upsilon_0$	1
z	$F^*(z)$
$-1 \le z \le z(\alpha^*)$	$\upsilon/(1+\upsilon)$
$z(\alpha^*) \le z \quad z(\beta^*)$	$\frac{1}{2}\left(1 + z/\left(\upsilon + z^2\right)^{\frac{1}{2}}\right)$
$z(\beta^*) \le z < \upsilon_0$	$\upsilon_0/\left(\upsilon_r + \upsilon_0\right)$
$z = \upsilon_0$	1
$z(\alpha^*) = (\alpha^* - \mu)/\mu = \frac{1}{2}(\upsilon - 1)$	
$z(\beta^*) = (\beta^* - \mu)/\mu = \frac{1}{2}(\upsilon_0 - \upsilon_r)$	

The idea of the distribution-free approach consists to replace the claim size Z by the stop-loss ordered bounds Z_*, Z^*, and more precisely a stop-loss ordered discrete version Z_d^* of Z^*. Then the aggregate claims to the layer m *xs* ℓ is also stop-loss ordered such that

$$X_* = \sum_{i=1}^{N} Z_{*i} \leq_{sl} X = \sum_{i=1}^{N} Z_i \leq_{sl} X^* = \sum_{i=1}^{N} Z_i^* \leq X_d^* = \sum_{i=1}^{N} Z_{d,i}^* . \tag{4.2}$$

Clearly, replacing $\pi_X(x)$ in (3.3) by $\pi_{X_*}(x)$ and $\pi_{X_d^*}(x)$ will provide lower and upper bounds to XL premiums with reinstatements. The present paper offers an analysis of the approximation obtained replacing $\pi_X(x)$ in (3.3) by $\pi_{X_d^*}(x)$ and $\pi_{X_*}(x)$.

From a *statistical viewpoint* this method has the advantage to be distribution-free, that is only dependent on the few risk characteristics λ, μ, σ and m. The numerical illustrations in Section 5 suggest that the distribution-free approximations $\pi_{X^*}(x)$ and $\pi_{X_d^*}(x)$ are very good approximations, especially for a small number of expected claims and higher layers. However, it is not always an upper bound to the pure premium. It is worth to investigate this matter a bit further. Denote the distribution-free approximation to the pure premium by

$$P_L^{*K} = \frac{\pi_{X^*}(L) - \pi_{X^*}(L + (K+1)m)}{1 + c \cdot \dfrac{\pi_{X^*}(L) - \pi_{X^*}(L + Km)}{m}} . \tag{4.3}$$

In the particular case of free reinstatements $c = 0$ and no aggregate deductible $L = 0$, the approximate premium is always anticonservative provided the mean XL claim size remains fixed.

Proposition 4.1. For stop-loss ordered claim sizes with equal means, the approximate premium is always lower than the exact premium if the reinstatements are free and there is no aggregate deductible.

Proof. Noting that $\pi_X(0) = \pi_{X^*}(0) = E[X]$ and using the other assumptions, one sees that $P_0^K - P_0^{*K} = \pi_{X^*}((K+1)m) - \pi_X((K+1)m)$, which is always non-negative by (4.2). □

A look at the case of free reinstatements but positive aggregate deductible shows that the approximate premium is conservative provided the following inequality holds:

$$\pi_{X^*}(L) - \pi_X(L) \geq \pi_{X^*}(L + (K+1)m) - \pi_X(L + (K+1)m). \tag{4.4}$$

This is always fulfilled when the right-hand side becomes smaller and smaller. This is likely to be the case for a higher number of reinstatements, higher capacities and deductibles. As the numerical examples in Section 5 suggest, this is less likely fulfilled for small capacities and deductibles and higher number of expected claims. In any case, the approximation cannot be better than the smallest error of approximation $\pi_{X^*}(L) - \pi_X(L)$ for large $(K+1)m$, which will be maximum for the aggregate deductible satisfying the equation $F_{X^*}(L) = F_X(L)$.

From a *computational viewpoint* the distribution-free approach is easy to implement. We assume that the aggregate claims $\sum_{i=1}^{N} Y_i$ of the original insurance portfolio is compound Poisson distributed such that N is Poisson distributed with mean parameter $\lambda = E[N]$. Calculations for other claim number distributions will be similar but are not explicitly stated. As already stated, instead of Z^* we will use a discrete stop-loss ordered approximation Z_d^* of it such that $Z^* \;_{sl} Z_d^*$. It is obtained by the method of mass dispersal, which is known to preserve the stop-loss order. From Hürlimann(1996), Section 3.2, one knows that Z_d^* is a 4-atomic random variable with support $\{x_0, x_1, x_2, x_3\}$, and probabilities $\{p_0, p_1, p_2, p_3\}$ determined as follows:

$$x_0 = 0, \quad x_1 = \frac{1}{2}\mu(1+\upsilon), \quad x_2 = \mu\left(1 + \frac{1}{2}(\upsilon_0 - \upsilon_r)\right), \quad x_3 = \mu(1+\upsilon_0), \tag{4.5}$$

$$p_0 = \frac{\upsilon}{1+\upsilon}, \quad p_1 = \frac{\upsilon_0 - \upsilon}{(1+\upsilon_0)(1+\upsilon)}, \quad p_2 = \frac{\upsilon_0 - \upsilon}{(1+\upsilon_0)(\upsilon_r + \upsilon_0)}, \quad p_3 = \frac{\upsilon_r}{\upsilon_r + \upsilon_0}. \tag{4.6}$$

The stop-loss transform of X_d^* can be evaluated using the series representation

$$\pi_{X_d^*}(x) = \lambda\mu - x + e^{-\lambda(1-p_0)} \cdot \sum_{n_1, n_2, n_3 = 0}^{\infty} \frac{(\lambda p_1)^{n_1}(\lambda p_2)^{n_2}(\lambda p_3)^{n_3}}{n_1!\, n_2!\, n_3!} \left(x - \sum_{i=1}^{3} n_i x_i\right)_+. \tag{4.7}$$

One notes that the infinite series are always finite sums because summation occurs only for $\sum_{i=1}^{3} n_i x_i < x$. In fact, often only very few terms must be evaluated. To evaluate the stop-loss transform obtained from the lower stop-loss bound Z_* to Z, one notes that Z_* is a biatomic random variable with support $\{x_1, x_2\} = \{\mu - \sigma^2/(m-\mu), (1+\upsilon)\mu\}$ and probabilities $\{p_1, p_2\} = \{1 - \mu/m, \mu/m\}$. The required stop-loss transform is calculated using the formula

$$\pi_{X_*}(x) = \lambda\mu - x + e^{-\lambda} \cdot \sum_{n_1, n_2}^{\infty} \frac{(\lambda p_1)^{n_1}(\lambda p_2)^{n_2}}{n_1!\, n_2!} \left(x - \sum_{i=1}^{2} n_i x_i\right)_+. \tag{4.8}$$

To illustrate the practical impact of the distribution-free approach, consider the example by Sundt(1991). Assume that the claim number N is Poisson distributed with parameter λ and the claim sizes Y_i are Pareto distributed with scale parameter OP (the so-called observation point) and index α, that is

$$P(N = n) = \frac{\lambda^n}{n!} e^{-\lambda}, \quad n = 0, 1, 2, \ldots, \tag{4.9}$$

$$P(Y \le y) = 1 - \left(\frac{y}{OP}\right)^{-\alpha}, \quad y \ge OP > 0, \alpha > 0. \tag{4.10}$$

To implement the distribution-free method, one needs the first two moments of the claim size $Z = (Y - \ell)_+ - (Y - m - \ell)_+$ from an XL reinsurance with layer m *xs* ℓ. One obtains through induction the recursive formula

$$E[Z^n] = OP^n \cdot \left(\frac{n}{n-a}\right)\left\{\left(\frac{m+\ell}{OP}\right)^{n-a} - \left(\frac{\ell}{OP}\right)^{n-a}\right\} - \sum_{k=1}^{n-1}\binom{n}{k}\ell^{n-k} \cdot E[Z^k], \quad n = 1,2,\ldots \tag{4.11}$$

The parameters required to evaluate the approximate stop-loss transforms (4.7) and (4.8) are besides λ and m, the mean $\mu = E[Z]$ and the variance $\sigma^2 = Var[Z]$ obtained from (4.11). The original analytical approach by Sundt(1991) requires an appropriate discretization of the claim size distribution, which is used to evaluate the stop-loss transform using Panjer's recursion. Based on the Pareto claim size (4.10), the distribution of the XL claim size reads

$$F(z) = \begin{cases} 1 - \left(\frac{\ell + z}{m}\right)^{-a}, & 0 \le z \le m, \\ 1, & z \ge m. \end{cases} \tag{4.12}$$

Applying the method of mass dispersal, one approximates $F(z)$ by a distribution on the $t+1$ points $ih, i = 0, \ldots, t$, with $h = m/t$. Let $\bar{F}(z) = 1 - F(z)$ denote the survival function and let f_i be the probability associated to the point $ih, i = 0, \ldots, t$. It is not difficult to obtain the following formulas

$$\begin{gathered} f_0 = \varepsilon_1 - x_1 + 1 - \left(\frac{\ell}{OP}\right)^{-a}, \\ f_i = x_i + \varepsilon_{i+1}, - x_{i+1}, \; i = 1, \ldots, t-1, \quad f_t = x_t, \end{gathered} \tag{4.13}$$

where one sets

$$\begin{gathered} \varepsilon_i = \left(\frac{\ell + (i-1)h}{OP}\right)^{-a} - \left(\frac{\ell + ih}{OP}\right)^{-a}, \quad i = 1, \ldots, t-1, \\ \varepsilon_t = \left(\frac{\ell + (t-1)h}{OP}\right)^{-a}, \quad x_i = \frac{\omega_i}{h} - (i-1) \cdot \varepsilon_i, \quad i = 1, \ldots, t, \\ \omega_i = \frac{OP}{a-1} \cdot \left\{\left(\frac{\ell + (i-1)h}{OP}\right)^{-a+1} - \left(\frac{\ell + ih}{OP}\right)^{-a+1}\right\} \\ + (i-1)h\left(\frac{\ell + (i-1)h}{OP}\right)^{-a} - ih\left(\frac{\ell + ih}{OP}\right)^{-a}, \quad i = 1, \ldots, t-1, \\ \omega_t = \mu - \sum_{i=1}^{t-1} \omega_i. \end{gathered} \tag{4.14}$$

Sundt(1991) calculates pure premiums for $m = \ell = 100$, $OP = 100$, $a = 1.2$, $\lambda = 0.5$. Table 4.2 compares the results of this analytical approach with those obtained using the distribution-free approach based on (4.7). With much less computational effort, in fact at most 11 terms are required in the formula (4.5), the approximate pure premiums are surprisingly accurate and in most cases on the safe side. Note that the case $K = 5$ is compared with the case of infinitely many reinstatements in Sundt(1991). A more detailed study including the approximation by (4.8) is pursued in the next Section.

TABLE 4.2

INITIAL PURE PREMIUMS SUNDT(1991) VS. DISTRIBUTION-FREE APPROACH (DF)

		$L = 0$		$L = 100$		$L = 200$	
K	c	Sundt	DF	Sundt	DF	Sundt	DF
0	0	27.85	27.73	4.088	4.176	0.3963	0.4238
1	0	31.94	31.90	4.485	4.600	0.4247	0.4559
	1	24.98	24.98	4.309	4.416	0.4230	0.4540
2	0	32.33	32.33	4.514	4.632	0.4264	0.4579
	1	24.51	24.51	4.319	4.428	0.4245	0.4558
5 (∞)	0	32.36	32.36	4.515	4.634	0.4263	0.4580
	1	24.45	24.45	4.320	4.429	0.4246	0.4559

5. ANALYTICAL APPROXIMATIONS

In view of the very accurate approximations obtained with the distribution-free approach in Table 4.1, it is natural to ask whether other analytical approximations of the aggregate claims to the layer m xs ℓ perform similarly. From Chaubey et al.(1998) one knows that the approximations based on the gamma, translated gamma, translated inverse Gaussian and a mixture of the last two distributions yield usually good approximations of aggregate claims distributions. Like the distribution-free approach, these approximations are based on the method of moments an do not use Panjer's recursive formula as in the original analytical approach by Sundt(1991). Even more, Chaubey et al.(1998) claim that the latter mixture seems to be a superb approximation to the aggregate claims distribution. The compound Poisson approximations use at most 5 parameters, namely the expected number of claims $\lambda = E[N]$ and the first four moments $E[Z^i], i = 1, \dots, 4$, of the claim size. Recall the formulas required to evaluate the analytical approximations to the aggregate claims X.

Gamma approximation

The distribution of X is approximated by a gamma distribution $\Gamma(\beta, \alpha)$ with parameters $\alpha = k_X^{-2}$, $\beta = \left(k_X^2 \mu_X\right)^{-1}$ where μ_X, k_X are the mean and coefficient of variation of X given by

$$\mu_X = \lambda E[Z], \quad k_X^2 = \frac{1}{\lambda} \cdot \frac{E[Z^2]}{E[Z]^2}. \tag{5.1}$$

This approximation matches the mean and variance of X.

Translated Gamma approximation

According to Dickson and Waters(1993), it is natural to approximate X by $X_{TG} = X_G + \gamma$, where $X_G \sim \Gamma(\beta, \alpha)$, with the parameters defined by

$$\alpha = 4\lambda \cdot \frac{E[Z^2]^3}{E[Z^3]^2}, \quad \beta = 2 \cdot \frac{E[Z^2]}{E[Z^3]}, \quad \gamma = \lambda \cdot \left(E[Z] - 2 \cdot \frac{E[Z^2]^2}{E[Z^3]} \right). \tag{5.2}$$

This approximation matches the mean, variance and skewness of X.

Translated inverse Gaussian approximation

Chaubey et al.(1998) propose to approximate X by $X_{TIG} = X_{IG} + \delta$, where $X_{IG} \sim IG(a,b)$ has an inverse Gaussian distribution, with the parameters defined by

$$a = 3\lambda \cdot \frac{E[Z^2]^2}{E[Z^3]}, \quad b = \frac{1}{3}\frac{E[Z^3]}{E[Z^2]}, \quad \delta = \lambda E[Z] - a. \tag{5.3}$$

This approximation matches the mean, variance and skewness of X.

Mixture of translated gamma and translated inverse Gaussian

Let $F_{TG}(x) = F_G(x - \gamma)$ and $F_{TIG}(x) = F_{IG}(x - \delta)$ be the distribution functions of the translated gamma and translated inverse Gaussian approximations. Following Chaubey(1989), the mixture defined by

$$F_{mix}(x) = w \cdot F_{TG}(x) + (1 - w) \cdot F_{TIG}(x) \tag{5.4}$$

matches the mean, variance, skewness and kurtosis of X provided

$$w = \frac{\kappa_X - \kappa_{TIG}}{\kappa_{TG} - \kappa_{TIG}}, \tag{5.5}$$

where the kurtosis parameters are defined by

$$\kappa_X = \frac{1}{\gamma} \cdot \frac{E[Z^4]}{E[Z^2]^2}, \quad \kappa_{TG} = \frac{6}{\alpha}, \quad \kappa_{TIG} = 15 \cdot \frac{b}{a}. \tag{5.6}$$

Another very simple approximation to pure premiums for XL reinsurance with reinstatements is the rate on line method introduced by Walhin(2001). Consider the rate on line of a layer m xs ℓ defined by $ROL = \lambda \cdot E[Z]/m$, which is the premium of an unlimited free reinstatements treaty divided by the capacity. The rate on line method assumes that there are only total losses hitting the layer completely with the frequency $\theta = ROL$. If one assumes that $Z = m$ with probability one, and N has the mean θ, then the rate on line approximate premium is given by the formula

$$P_L^K = m \cdot \frac{\pi_N\left(\frac{L}{m}\right) - \pi_N\left(\frac{L}{m} + (K+1)\right)}{1 + c \cdot \frac{\pi_N\left(\frac{L}{m}\right) - \pi_N\left(\frac{L}{m} + K\right)}{m}}. \tag{5.7}$$

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In case N is Poisson distributed with survival function $S(i;\theta) = P(N > i) = 1 - \sum_{j=0}^{i} e^{-\theta} \cdot \frac{\theta^j}{j!}$, and the aggregate deductible is an integer multiple of the cover m, one obtains the formulas

$$P_L^K = m \cdot S\left(\frac{L}{m};\theta\right), \quad \text{if } K = 0,$$
$$P_L^K = m \cdot \frac{\sum_{i=\frac{L}{m}}^{\frac{L}{m}+K} S(i;\theta)}{1 + c \cdot \sum_{i=\frac{L}{m}}^{\frac{L}{m}+K-1} S(i;\theta)}, \quad \text{if } K \geq 1. \tag{5.8}$$

The Tables 5.2 to 5.16 offer a sensitivity analysis of the different approximations by comparing the results obtained with the analytical approach by Sundt(1991) with those obtained using the above analytical approximations, the distribution-free approach and the rate on line method. It is important to note that the distribution-free approach always yields a lower and an upper bound obtained from (4.7) and (4.8). It is interesting to compare the average of these bounds with the results obtained from the other methods. The claim size has a Pareto distribution with observation point $OP = 100$ and index α, and the claim number is Poisson distributed with parameter λ. We restrict the analysis to the more common case of free reinstatements $c = 0$. The example by Sundt(1991) for $L = 0, 100, 200$ and additional higher layers is studied in the Tables 5.2, 5.7 and 5.12. The other Tables consider similar examples with varying expected number of claims $\lambda = 1, 2, 5, 10$ and fixed Pareto index $\alpha = 2.5$. Table 5.1 provides an overview of the three best approximations to the analytical premiums by Sundt(1991). In the situation of no aggregate deductible $L = 0$, the distribution-free approach based on (4.7) is always best for the smaller expected number of claims up to $\lambda = 2$, followed by the rate on line method and the Gamma approximation. In these situations, the mixture approximation by Chaubey et al.(1998) does not perform well. However, for the higher expected number of claims $\lambda = 5, 10$ and the lowest layer, the mixture approximation is best, followed by the translated gamma and the translated inverse Gaussian distributions. For $\lambda = 5, 10$, and the middle and higher layer, the distribution-free approach is again best, followed by the rate on line method and the Gamma approximation. By increasing the aggregate deductible, as for instance $L = 100$, similar remarks can be made with the following differences. The distribution-free approach and the rate on line method are only best for even smaller expected number of claims and higher layers, and the translated inverse Gaussian takes the role of the Gamma as third best approximation in these cases. The mixture approximation is best for the lowest layer already when $\lambda \geq 2$.

TABLE 5.1

OVERVIEW OF THE BEST THREE APPROXIMATIONS TO THE ANALYTICAL PREMIUMS BY SUNDT(1991)

Table	λ	a	L	Layer	Choice 1	Choice 2	Choice 3
5.2	0.5	1.2	0	100-200			
				200-300	DF	ROL	Gamma
				300-400			
5.3	1	2.5	0	100-200			
				200-300	DF	ROL	Gamma
				300-400			
5.4	2	2.5	0	100-200			
				200-300	DF	ROL	Gamma
				300-400			
5.5	5	2.5	0	100-200	Mixture	TG	TIG
				200-300	DF	ROL	Gamma
				300-400	DF	ROL	Gamma
5.6	10	2.5	0	100-200	Mixture	TG	TIG
				200-300	DF	ROL	Gamma
				300-400	DF	ROL	Gamma
5.7	0.5	1.2	100	100-200			
				200-300	DF	ROL	TIG
				300-400			
5.8	1	2.5	100	100-200	TG	TIG	Mixture
				200-300	DF	ROL	TIG
				300-400	DF	ROL	TIG
5.9	2	2.5	100	100-200	Mixture	DF	ROL
				200-300	DF	ROL	TIG
				300-400	DF	ROL	TIG
5.10	5	2.5	100	100-200	Mixture	TG	TIG
				200-300	TIG	TG	DF
				300-400	DF	ROL	TIG
5.11	10	2.5	100	100-200	Mixture	TG	TIG
				200-300	TIG	TG	DF
				300-400	DF	ROL	TIG

TABLE 5.2

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS,
$\lambda = 0.5$, $\alpha = 1.2$, $L = 0$, $c = 0$

Deductible	100								
Limit	200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	26.29	31.42	31.85	30.19	27.72820	28.16825	27.94823	27.64775	27.84761
1	30.86	35.93	36.20	35.17	31.90438	32.03634	31.97036	31.88060	31.93604
2	31.96	36.52	36.80	35.73	32.32816	32.34222	32.33519	32.32464	32.33235
3	32.25	36.59	36.89	35.76	32.36032	32.36144	32.36088	32.35997	32.36069
5	32.35	36.60	36.91	35.76	32.36235	32.36236	32.36236	32.36235	32.36236
Deductible	200								
Limit	300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	14.13	19.32	19.73	18.13	15.60142	15.78502	15.69322	15.59322	15.61642
1	16.25	21.13	21.46	20.20	16.87891	16.89422	16.88657	16.87760	16.88120
2	16.76	21.33	21.67	20.35	16.94925	16.95008	16.94967	16.94914	16.94942
3	16.90	21.35	21.70	20.35	16.95215	16.95219	16.95217	16.95215	16.95216
5	16.95	21.36	21.70	20.35	16.95225	16.95225	16.95225	16.95225	16.95225
Deductible	300								
Limit	400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	9.43	14.03	14.46	12.79	10.61607	10.67877	10.64742	10.61419	10.61969
1	10.78	15.09	15.47	14.00	11.19875	11.20218	11.20047	11.19855	11.19913
2	11.10	15.20	15.58	14.07	11.22021	11.22034	11.22028	11.22020	11.22023
3	11.19	15.21	15.60	14.07	11.22081	11.22081	11.22081	11.22081	11.22081
5	11.22	15.21	15.60	14.07	11.22082	11.22082	11.22082	11.22082	11.22082

TABLE 5.3

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS,
$\lambda = 0.5$, $\alpha = 1.2$, $L = 0$, $c = 0$

Deductible 100 Limit 200									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	35.42	38.93	39.38	37.78	35.571	37.815	36.693	35.012	36.225
1	41.47	45.01	45.28	44.32	42.211	42.735	42.473	42.016	42.395
2	42.73	45.78	46.06	45.07	43.019	43.083	43.051	42.985	43.045
3	43.01	45.87	46.17	45.12	43.091	43.096	43.094	43.087	43.094
5	43.09	45.88	46.18	45.12	43.096	43.096	43.096	43.096	43.096
Deductible 200 Limit 300									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	9.31	13.17	13.59	11.99	10.19822	10.35706	10.27764	10.18356	10.22513
1	10.44	14.00	14.39	12.92	10.72204	10.72996	10.72600	10.72065	10.72432
2	10.67	14.07	14.47	12.96	10.73977	10.74003	10.73990	10.73970	10.73986
3	10.72	14.08	14.48	12.96	10.74021	10.74022	10.74022	10.74021	10.74021
5	10.74	14.08	14.48	12.95	10.74022	10.74022	10.74022	10.74022	10.74022
Deductible 300 Limit 400									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	3.91	6.47	6.92	5.17	4.39845	4.42014	4.40930	4.39707	4.40111
1	4.37	6.77	7.21	5.50	4.49525	4.49571	4.49548	4.49519	4.49535
2	4.46	6.79	7.24	5.51	4.49666	4.49666	4.49666	4.49666	4.49666
3	4.49	6.80	7.25	5.51	4.49667	4.49667	4.49667	4.49667	4.49667
5	4.50	6.80	7.25	5.51	4.49667	4.49667	4.49667	4.49667	4.49667

TABLE 5.4

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS, $\lambda = 2$, $\alpha = 2.5$, $L = 0$, $c = 0$

Deductible	100								
Limit	200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	61.67	62.74	63.16	61.65	59.10354	63.79292	61.44823	57.76533	60.66046
1	79.62	82.69	82.94	82.03	80.08163	82.68215	81.38189	79.12739	81.00793
2	84.47	87.02	87.20	86.54	85.14181	85.84695	85.49438	84.80093	85.41406
3	85.75	87.78	87.98	87.27	86.04843	86.16873	86.10858	85.96701	86.10128
5	86.16	87.92	88.14	87.35	86.19132	86.19283	86.19208	86.18918	86.19217
Deductible	200								
Limit	300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	18.12	22.91	23.32	21.78	19.38214	19.95334	19.66774	19.33008	19.47726
1	20.72	25.19	25.50	24.33	21.34185	21.39885	21.37035	21.33190	21.35799
2	21.29	25.43	25.75	24.52	21.47359	21.47743	21.47551	21.47263	21.47495
3	21.43	25.45	25.79	24.53	21.48017	21.48037	21.48027	21.48011	21.48025
5	21.48	25.46	25.79	24.52	21.48044	21.48044	21.48044	21.48044	21.48044
Deductible	300								
Limit	400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	7.73	11.55	11.99	10.31	8.60606	8.68901	8.64754	8.60080	8.61618
1	8.71	12.27	12.67	11.12	8.98219	8.98572	8.98396	8.98175	8.98296
2	8.92	12.33	12.74	11.15	8.99311	8.99321	8.99316	8.99309	8.99313
3	8.97	12.33	12.75	11.15	8.99334	8.99334	8.99334	8.99334	8.99334
5	8.99	12.34	12.75	11.15	8.99335	8.99335	8.99335	8.99335	8.99335

TABLE 5.5

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS, $\lambda = 5$, $a = 2.5$, $L = 0$, $c = 0$

Deductible 100
Limit 200

K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	94.92	92.28	92.28	92.27	90.20	94.36	92.28	88.41	92.07
1	160.26	159.23	159.43	158.72	155.21	163.48	159.34	151.84	158.50
2	193.34	194.85	195.03	194.41	191.51	198.79	195.15	188.35	194.30
3	207.36	209.32	209.39	209.14	207.51	211.40	209.45	205.53	209.04
5	214.56	215.40	215.44	215.28	214.94	215.36	215.15	214.59	215.13

Deductible 200
Limit 300

K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	41.32	45.32	45.79	44.01	41.779	44.340	43.059	41.551	42.188
1	50.44	55.25	55.50	54.55	51.823	52.471	52.147	51.714	51.999
2	52.81	56.94	57.18	56.29	53.475	53.584	53.530	53.449	53.513
3	53.45	57.20	57.46	56.49	53.679	53.693	53.686	53.675	53.685
5	53.68	57.24	57.51	56.50	53.701	53.701	53.701	53.701	53.701

Deductible 300
Limit 400

K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	18.67	23.89	24.30	22.73	20.1635	20.6169	20.3902	20.1351	20.2179
1	21.56	26.51	26.80	25.67	22.3198	22.3680	22.3439	22.3139	22.3302
2	22.24	26.81	27.12	25.92	22.4747	22.4781	22.4764	22.4741	22.4757
3	22.42	26.84	27.16	25.93	22.4830	22.4832	22.4831	22.4830	22.4831
5	22.48	26.85	27.17	25.92	22.4834	22.4834	22.4834	22.4834	22.4834

TABLE 5.6

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS,
$\lambda = 10$, $a = 2.5$, $L = 0$, $c = 0$

K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
Deductible 100									
Limit 200									
0	99.95	99.48	99.44	99.58	99.21	99.84	99.53	98.66	99.61
1	197.50	195.41	195.35	195.58	193.77	197.34	195.55	191.52	195.58
2	283.05	280.25	280.26	280.22	276.42	284.91	280.67	271.91	280.17
3	346.84	345.54	345.65	345.28	340.34	352.29	346.31	334.36	345.24
5	410.25	412.07	412.11	411.96	408.71	417.04	412.88	404.00	411.98
Deductible 200									
Limit 300									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	69.05	69.22	69.59	68.18	66.34	71.32	68.83	65.84	67.22
1	94.64	98.32	98.63	97.47	95.47	98.52	96.99	94.98	96.24
2	103.27	106.95	107.13	106.47	104.66	105.69	105.17	104.42	105.00
3	106.08	109.02	109.19	108.56	106.88	107.15	107.02	106.81	106.98
5	107.27	109.54	109.73	108.99	107.39	107.40	107.40	107.39	107.39
Deductible 300									
Limit 400									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	35.03	39.93	40.40	38.61	36.305	37.755	37.030	36.216	36.474
1	42.31	47.71	47.96	47.00	43.787	44.096	43.942	43.750	43.853
2	44.21	48.96	49.20	48.27	44.844	44.888	44.866	44.836	44.856
3	44.75	49.14	49.40	48.40	44.956	44.961	44.959	44.955	44.958
5	44.95	49.17	49.44	48.40	44.967	44.967	44.967	44.967	44.967

TABLE 5.7

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS, $\lambda = 0.5$, $\alpha = 1.2$, $L = 100$, $c = 0$

Deductible	100								
Limit	200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	4.57	4.51	4.34	4.98	4.18	3.87	4.02	4.23	4.09
1	5.68	5.10	4.95	5.54	4.60	4.17	4.39	4.68	4.48
2	5.97	5.18	5.04	5.58	4.63	4.19	4.41	4.71	4.51
3	6.05	5.19	5.05	5.57	4.63	4.19	4.41	4.71	4.51
5	6.07	5.19	5.05	5.57	4.63	4.19	4.41	4.71	4.51
Deductible	200								
Limit	300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	2.12	1.82	1.73	2.07	1.277	1.109	1.193	1.284	1.265
1	2.63	2.01	1.94	2.23	1.348	1.165	1.256	1.356	1.333
2	2.77	2.04	1.97	2.22	1.351	1.167	1.259	1.359	1.336
3	2.81	2.04	1.98	2.22	1.351	1.167	1.259	1.359	1.336
5	2.82	2.04	1.98	2.22	1.351	1.167	1.259	1.359	1.336
Deductible	300								
Limit	400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	1.34	1.06	1.01	1.22	0.583	0.523	0.553	0.584	0.579
1	1.67	1.17	1.12	1.29	0.604	0.542	0.573	0.606	0.601
2	1.75	1.18	1.14	1.28	0.605	0.542	0.573	0.607	0.601
3	1.78	1.18	1.14	1.28	0.605	0.542	0.573	0.607	0.601
5	1.79	1.18	1.14	1.28	0.605	0.542	0.573	0.607	0.601

TABLE 5.8

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS, $\lambda = 1$, $\alpha = 2.5$, $L = 100$, $c = 0$

Deductible	100								
Limit	200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	6.04	6.08	5.90	6.55	6.64	4.92	5.78	7.00	6.17
1	7.31	6.85	6.68	7.29	7.45	5.27	6.36	7.97	6.82
2	7.59	6.94	6.79	7.34	7.52	5.28	6.40	8.08	6.87
3	7.65	6.95	6.80	7.34	7.52	5.28	6.40	8.08	6.87
5	7.67	6.95	6.80	7.34	7.53	5.28	6.40	8.08	6.87
Deductible	200								
Limit	300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	1.12	0.83	0.80	0.94	0.52	0.37	0.45	0.54	0.50
1	1.36	0.90	0.88	0.98	0.54	0.38	0.46	0.56	0.51
2	1.41	0.91	0.89	0.97	0.54	0.38	0.46	0.56	0.52
3	1.42	0.91	0.89	0.97	0.54	0.38	0.46	0.56	0.52
5	1.43	0.91	0.89	0.97	0.54	0.38	0.46	0.56	0.52
Deductible	300								
Limit	400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	0.46	0.30	0.29	0.34	0.097	0.076	0.086	0.098	0.094
1	0.56	0.33	0.32	0.35	0.098	0.077	0.087	0.100	0.096
2	0.58	0.33	0.32	0.34	0.098	0.077	0.087	0.100	0.096
3	0.59	0.33	0.32	0.34	0.098	0.077	0.087	0.100	0.096
5	0.59	0.33	0.32	0.34	0.098	0.077	0.087	0.100	0.096

TABLE 5.9

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS, $\lambda = 2$, $\alpha = 2.5$, $L = 100$, $c = 0$

Deductible 100 Limit 200									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	4.57	4.51	4.34	4.98	4.18	3.87	4.02	4.23	4.09
1	5.68	5.10	4.95	5.54	4.60	4.17	4.39	4.68	4.48
2	5.97	5.18	5.04	5.58	4.63	4.19	4.41	4.71	4.51
3	6.05	5.19	5.05	5.57	4.63	4.19	4.41	4.71	4.51
5	6.07	5.19	5.05	5.57	4.63	4.19	4.41	4.71	4.51
Deductible 200 Limit 300									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	2.12	1.82	1.73	2.07	1.277	1.109	1.193	1.284	1.265
1	2.63	2.01	1.94	2.23	1.348	1.165	1.256	1.356	1.333
2	2.77	2.04	1.97	2.22	1.351	1.167	1.259	1.359	1.336
3	2.81	2.04	1.98	2.22	1.351	1.167	1.259	1.359	1.336
5	2.82	2.04	1.98	2.22	1.351	1.167	1.259	1.359	1.336
Deductible 300 Limit 400									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	1.34	1.06	1.01	1.22	0.583	0.523	0.553	0.584	0.579
1	1.67	1.17	1.12	1.29	0.604	0.542	0.573	0.606	0.601
2	1.75	1.18	1.14	1.28	0.605	0.542	0.573	0.607	0.601
3	1.78	1.18	1.14	1.28	0.605	0.542	0.573	0.607	0.601
5	1.79	1.18	1.14	1.28	0.605	0.542	0.573	0.607	0.601

TABLE 5.10

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS,
$\lambda = 5$, $\alpha = 2.5$, $L = 100$, $c = 0$

Deductible	100								
Limit	200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	65.34	66.95	67.15	66.44	65.01	69.12	67.06	63.43	66.43
1	98.42	102.58	102.75	102.13	101.31	104.43	102.87	99.94	102.23
2	112.43	117.04	117.11	116.87	117.31	117.04	117.17	117.13	116.97
3	117.75	121.79	121.83	121.70	123.03	120.34	121.68	123.90	121.77
5	120.26	123.44	123.50	123.29	125.17	121.11	123.14	126.85	123.34
Deductible	200								
Limit	300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	9.12	9.93	9.71	10.54	10.04	8.13	9.09	10.16	9.81
1	11.49	11.62	11.38	12.29	11.70	9.24	10.47	11.90	11.32
2	12.13	11.88	11.66	12.49	11.90	9.35	10.63	12.12	11.50
3	12.31	11.92	11.71	12.50	11.92	9.36	10.64	12.15	11.51
5	12.38	11.92	11.72	12.49	11.92	9.36	10.64	12.15	11.51
Deductible	300								
Limit	400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	2.89	2.62	2.50	2.95	2.16	1.75	1.95	2.18	2.11
1	3.57	2.91	2.81	3.20	2.31	1.86	2.09	2.34	2.26
2	3.75	2.95	2.86	3.20	2.32	1.87	2.09	2.35	2.27
3	3.80	2.95	2.87	3.20	2.32	1.87	2.09	2.35	2.27
5	3.82	2.95	2.87	3.20	2.32	1.87	2.09	2.35	2.27

TABLE 5.11

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS,
λ =10, a = 2.5, L = 100, c = 0

K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
Deductible 100									
Limit 200									
0	97.55	95.93	95.91	96.00	94.56	97.50	96.03	92.86	95.97
1	183.10	180.77	180.82	180.64	177.21	185.07	181.14	173.25	180.56
2	246.89	246.06	246.21	245.70	241.12	252.45	246.78	235.71	245.63
3	287.54	288.79	288.94	288.42	284.18	295.15	289.66	278.85	288.40
5	321.79	324.01	324.04	323.93	322.35	326.54	324.44	319.88	323.92
Deductible 200									
Limit 300									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	25.59	29.10	29.04	29.30	29.13	27.20	28.17	29.14	29.02
1	34.22	37.74	37.54	38.30	38.32	34.37	36.35	38.59	37.78
2	37.03	39.81	39.60	40.39	40.54	35.83	38.19	40.97	39.77
3	37.94	40.24	40.05	40.76	40.98	36.06	38.52	41.46	40.12
5	38.31	40.34	40.16	40.81	41.06	36.09	38.57	41.56	40.18
Deductible 300									
Limit 400									
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	7.27	7.78	7.56	8.39	7.48	6.34	6.91	7.53	7.38
1	9.18	9.02	8.80	9.65	8.54	7.13	7.84	8.62	8.38
2	9.71	9.21	9.01	9.78	8.65	7.21	7.93	8.74	8.48
3	9.87	9.24	9.04	9.79	8.66	7.21	7.94	8.75	8.49
5	9.93	9.24	9.05	9.78	8.66	7.21	7.94	8.75	8.49

TABLE 5.12

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS, $\lambda = 0.5$, $\alpha = 1.2$, $L = 200$, $c = 0$

Deductible	100								
Limit	200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	1.10	0.59	0.60	0.5576	0.42	0.31	0.36	0.44	0.40
1	1.39	0.67	0.69	0.59206	0.46	0.33	0.39	0.48	0.42
2	1.47	0.68	0.71	0.58855	0.46	0.33	0.39	0.48	0.43
3	1.49	0.68	0.71	0.58662	0.46	0.33	0.39	0.48	0.43
5	1.50	0.68	0.71	0.586	0.46	0.33	0.39	0.48	0.43
Deductible	200								
Limit	300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	0.51	0.19667	0.21	0.15	0.07	0.05586	0.06310	0.07154	0.06822
1	0.64	0.2189	0.24	0.15	0.07	0.05796	0.06560	0.07455	0.07096
2	0.68	0.22147	0.25	0.15	0.07	0.05803	0.06568	0.07465	0.07105
3	0.69	0.22177	0.25	0.15	0.07	0.05803	0.06569	0.07465	0.07105
5	0.70	0.22181	0.25	0.15	0.07	0.05803	0.06569	0.07465	0.07105
Deductible	300								
Limit	400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	0.32	0.11	0.12	0.07	0.02146	0.01816	0.01981	0.02165	0.02110
1	0.41	0.12	0.13	0.06	0.02205	0.01863	0.02034	0.02225	0.02168
2	0.43	0.12	0.14	0.06	0.02207	0.01864	0.02036	0.02227	0.02169
3	0.44	0.12	0.14	0.06	0.02207	0.01864	0.02036	0.02227	0.02169
5	0.44	0.12	0.14	0.06	0.02207	0.01864	0.02036	0.02227	0.02169

TABLE 5.13

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS, $\lambda = 1$, $\alpha = 2.5$, $L = 200$, $c = 0$

Deductible	100								
Limit	200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	1.26517	0.77162	0.78	0.74455	0.80789	0.34866	0.57828	0.96905	0.65004
1	1.54479	0.86172	0.89	0.79584	0.87970	0.36127	0.62049	1.07113	0.69867
2	1.60832	0.87178	0.90	0.79438	0.88469	0.36162	0.62316	1.07980	0.70147
3	1.623	0.87287	0.90	0.79298	0.88498	0.36162	0.62330	1.08041	0.70160
5	1.62725	0.87299	0.90	0.79256	0.88499	0.36163	0.62331	1.08045	0.70160
Deductible	200								
Limit	300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	0.23258	0.07	0.08	0.04008	0.01772	0.01007	0.0139	0.01905	0.01555
1	0.28685	0.08	0.09	0.03636	0.01817	0.01026	0.01422	0.01956	0.01590
2	0.3003	0.08	0.10	0.03478	0.01818	0.01026	0.01422	0.01957	0.01590
3	0.30375	0.08	0.10	0.03443	0.01818	0.01026	0.01422	0.01957	0.01590
5	0.30491	0.08	0.10	0.03435	0.01818	0.01026	0.01422	0.01957	0.01590
Deductible	300								
Limit	400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	0.10	0.02	0.03	0.01	0.00141	0.00095	0.00118	0.00147	0.00131
1	0.12	0.03	0.03	0.01	0.00142	0.00096	0.00119	0.00148	0.00132
2	0.13	0.03	0.03	0.01	0.00142	0.00096	0.00119	0.00148	0.00132
3	0.13	0.03	0.03	0.01	0.00142	0.00096	0.00119	0.00148	0.00132
5	0.13	0.03	0.03	0.01	0.00142	0.00096	0.00119	0.00148	0.00132

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TABLE 5.14

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS, $\lambda = 2$, $\alpha = 2.5$, $L = 200$, $c = 0$

Deductible 100	Limit 200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	4.85	4.33077	4.26	4.52	5.06	3.16	4.11	5.67	4.41
1	6.12	5.09722	5.04	5.24	5.97	3.49	4.73	6.84	5.09
2	6.46	5.21701	5.18	5.32	6.09	3.51	4.80	7.03	5.18
3	6.54	5.23424	5.20	5.33	6.11	3.51	4.81	7.06	5.18
5	6.57	5.23687	5.20	5.32	6.11	3.51	4.81	7.07	5.18

Deductible 200	Limit 300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	0.57	0.24	0.26	0.19	0.13	0.08	0.11	0.14	0.12
1	0.71	0.27	0.29	0.20	0.14	0.08	0.11	0.15	0.12
2	0.74	0.27	0.30	0.19	0.14	0.08	0.11	0.15	0.12
3	0.75	0.27	0.30	0.19	0.14	0.08	0.11	0.15	0.12
5	0.76	0.27	0.30	0.19	0.14	0.08	0.11	0.15	0.12

Deductible 300	Limit 400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	0.21	0.06	0.07	0.03	0.0109	0.0075	0.0092	0.0113	0.0102
1	0.27	0.07	0.08	0.03	0.0112	0.0076	0.0094	0.0116	0.0104
2	0.28	0.07	0.08	0.03	0.0112	0.0076	0.0094	0.0116	0.0104
3	0.29	0.07	0.08	0.03	0.0112	0.0076	0.0094	0.0116	0.0104
5	0.29	0.07	0.09	0.03	0.0112	0.0076	0.0094	0.0116	0.0104

TABLE 5.15

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS, $\lambda = 5$, $\alpha = 2.5$, $L = 200$, $c = 0$

Deductible	100								
Limit	200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	33.08	35.6228	35.60	35.69	36.30	35.31	35.81	36.51	35.80
1	47.09	50.087	49.96	50.42	52.30	47.92	50.11	53.70	50.54
2	52.41	54.8393	54.67	55.26	58.02	51.22	54.62	60.47	55.35
3	54.29	56.1651	56.01	56.56	59.72	51.88	55.80	62.75	56.63
5	55.13	56.5627	56.43	56.90	60.25	52.00	56.13	63.60	56.97
Deductible	200								
Limit	300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	2.37	1.69	1.68	1.74142	1.65	1.11	1.38	1.74	1.51
1	3.01	1.95	1.96	1.94238	1.86	1.22	1.54	1.96	1.69
2	3.19	1.99	2.00	1.95325	1.88	1.23	1.55	1.99	1.70
3	3.24	1.99	2.01	1.95088	1.88	1.23	1.55	1.99	1.70
5	3.26	2.00	2.01	1.94951	1.88	1.23	1.55	1.99	1.70
Deductible	300								
Limit	400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	0.68	0.30	0.31	0.25	0.155	0.110	0.132	0.160	0.145
1	0.86	0.33	0.36	0.26	0.163	0.115	0.139	0.169	0.153
2	0.90	0.34	0.37	0.25	0.164	0.115	0.139	0.169	0.153
3	0.92	0.34	0.37	0.25	0.164	0.115	0.139	0.170	0.153
5	0.92	0.34	0.37	0.25	0.164	0.115	0.139	0.170	0.153

TABLE 5.16

XL PURE PREMIUMS WITH REINSTATEMENTS UNDER VARIOUS DISTRIBUTION APPROXIMATIONS,
$\lambda = 10$, $a = 2.5$, $L = 200$, $c = 0$

Deductible	100								
Limit	200								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	85.56	84.8358	84.91	84.65	82.64	87.58	85.11	80.39	84.59
1	149.34	150.133	150.30	149.71	146.56	154.95	150.75	142.84	149.66
2	190.00	192.859	193.03	192.43	189.61	197.65	193.63	185.99	192.43
3	212.76	216.654	216.76	216.38	214.83	219.71	217.27	212.48	216.41
5	229.59	232.892	232.93	232.79	233.60	232.33	232.97	234.19	232.79
Deductible	200								
Limit	300								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	8.63	8.63	8.50	8.99961	9.19	7.17	8.18	9.44	8.76
1	11.45	10.70	10.57	11.0888	11.42	8.63	10.02	11.83	10.74
2	12.35	11.14	11.02	11.4655	11.85	8.85	10.35	12.32	11.10
3	12.63	11.22	11.11	11.5156	11.92	8.88	10.40	12.41	11.15
5	12.75	11.23	11.13	11.5177	11.93	8.88	10.41	12.42	11.16
Deductible	300								
Limit	400								
K	G	TG	TIG	Mixture	DF up	DF down	DF av	ROL	Sundt
0	1.91	1.25	1.24	1.26	1.056	0.792	0.924	1.086	1.004
1	2.44	1.43	1.45	1.39	1.169	0.865	1.017	1.205	1.105
2	2.60	1.46	1.48	1.40	1.179	0.870	1.024	1.216	1.114
3	2.64	1.46	1.49	1.39	1.179	0.870	1.025	1.216	1.114
5	2.66	1.46	1.49	1.39	1.179	0.870	1.025	1.216	1.114

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