





PHYSICS AND ASTRONOMY

NOVEL-RESULT

# Experimental monitoring of nonlinear wave interactions in crab orchard sandstone under uniaxial load

Alison E. Malcolm<sup>1,\*</sup> , Lauren Coates<sup>1</sup>, Kamal Moravej<sup>1</sup>, Andrey Melnikov<sup>1</sup> , Steve Butt<sup>2</sup> and Kristin M. Poduska<sup>3</sup> 

<sup>1</sup>Department of Earth Sciences, Memorial University of Newfoundland, St. John's, Newfoundland and Labrador A1B 3X5, Canada, <sup>2</sup>Faculty of Engineering, Memorial University of Newfoundland, St. John's, Newfoundland and Labrador A1B 3X5, Canada, and <sup>3</sup>Department of Physics and Physical Oceanography, Memorial University, St. John's, Newfoundland and Labrador A1B 3X7, Canada

\*Corresponding Author. Email: [amalcolm@mun.ca](mailto:amalcolm@mun.ca)

(Received 05 July 2022; Revised 01 November 2022; Accepted 02 November 2022)

## Abstract

When two waves interact within a rock sample, the interaction strength depends strongly on the sample's microstructural properties, including the orientation of the sample layering. The study that established this dependence on layering speculated that the differences were caused by cracks aligned with the layers in the sample. To test this, we applied a uniaxial load to similar samples of Crab Orchard Sandstone and measured the nonlinear interaction as a function of the applied load and layer orientation. We show that the dependence of the nonlinear signal changes on applied load is exponential, with a characteristic load of 11.4–12.5 MPa that is independent of sample orientation and probe wavetype (P or S); this value agrees with results from the literature, but does not support the cracks hypothesis.

**Keywords:** acoustics; nonlinear elasticity; rock properties

## Introduction

When low-amplitude waves interact with a material with a complicated structure, linear elasticity theory explains the resulting waveforms well. The system is well described by the linear wave equation, derived from Hooke's law and conservation of translational momentum and the wave speed is determined by the second-order elastic constants. However, as the wave amplitude gets larger, nonlinear effects become important. In particular, a large wave can change the properties of the material, resulting in speed changes of other waves traveling within the sample simultaneously. The magnitude and variability of these wavespeed changes are known to be sensitive to the microstructure of the material, but the details of that dependency are not completely understood. These phenomena are particularly important in rocks, as studied here.

We use wave mixing experiments wherein a strong PUMP wave perturbs the properties of a rock sample. We sense those perturbations by measuring the travelttime change of a smaller-amplitude probe wave. This is a type of PUMP/probe experiment, which is a classic (Hughes & Kelly, 1953), yet powerful, style of experiment that includes the relatively new dynamic acousto-elastic testing (DAET) method (Lott et al., 2016a; 2016b; 2017; Muir et al. 2020; Remillieux et al., 2017; Renaud et al., 2008; 2012; Rivière et al., 2013; 2015; Sens-Schönfelder & Eulenfeld, 2019). In DAET, a resonant mode (PUMP) is excited in the sample, and then analyzed with a high-frequency probe wave. Instead of resonant modes, we use transient waves (Gallot et al., 2015). Earlier work suggests that the nonlinear response depends on the

sample's layering orientation (TenCate et al., 2016). Modeling for this particular experiment is a challenge because the sample experiences two dynamic forces (PUMP, probe) and one static force (press); the closest existing models consider only the PUMP and probe (Gallot et al., 2015; Rusmanugroho et al., 2020).

We aim to separate the crack-induced signals from the intrinsic anisotropy by running PUMP/probe experiments repeatedly on a layered rock sample under different uniaxial loads. We use two samples of the same Crab Orchard Sandstone—with different layering orientations, as used by TenCate et al. (2016). Applying a load to the samples, we look for differences in the evolution of the signals with load that correlate with layering orientation; these kinds of correlations would be evidence supporting cracks as the driving mechanism. Viswanathan et al. (2022) give a thorough literature overview related to crack behavior 2022.

Ours is the first instance of this particular experimental configuration being performed under applied load, and the first to look at the orientation dependence of the responses; some data presented herein were part of a conference presentation by Hayes et al. (2018). It builds on: classical nonlinear resonance studies under different loads and saturation conditions (Zinszner et al., 1997), pressure-dependent DAET studies (Rivière et al., 2016), and experiments that monitor velocity changes with different confining pressures (Simpson et al., 2021). These earlier works suggest an exponential decrease in nonlinearity with increasing load, with a characteristic pressure  $\sim 10$  MPa for sandstones (Rivière et al., 2016) and  $\sim 1$  MPa for rocks from an active fault zone (Simpson et al., 2021).

## Theory

Our experiments measure the change,  $\Delta M$ , one wave makes in the elastic modulus,  $M$ , sensed by another wave (S-wave probe has  $M = \mu$ ; P-wave probe has  $M = \lambda + 2\mu$ , where  $\mu$  is the shear modulus and  $\lambda$  the Lamé parameter). The modulus time dependence has already been studied (Gallot et al., 2015). We focus on how  $\Delta M$  changes with applied load,  $P$ , with experiments at different values of  $P$ . We use the maximum of  $\Delta M$  over time  $t$  to recover  $\Delta M(P)$ , and fit our data to an exponential model (Rivière et al., 2016),

$$\frac{\Delta M}{M}(P) = Ae^{-\frac{P}{P_0}}, \quad (1)$$

where  $P_0$  is the characteristic load.

## Sample description

We examine two samples of Crab Orchard Sandstone (COS) from Cumberland, Tennessee, which is beige, fine-grained, and cross-bedded with subrounded grain shapes and no preferred grain alignment. The rock is compositionally and texturally mature (composition: 80% quartz, 10% orthoclase, 9% cement (clays and micas), 1% mica). Though the bedding layers have thicknesses in millimeter range, this alignment is not visible in sub-mm-scale scanning-electron micrographs. Sample parameters (length in each dimension, density, P- and S-wave velocities, and  $x$ - and  $y$ -anisotropy) are listed in Table 1. Sample 1 has horizontal layers, and sample 2 has vertical layers. Pictures of the samples and scanning electron microprobe images are given in Figure S.1 in the Supplementary Material.

## Methods

Figure 1 shows our experimental setup and Table 2 summarizes the experimental parameters. We use an established experimental setup (Gallot et al., 2015; Khajehpour Tadavani et al., 2020; TenCate et al., 2016) and place it inside a hydraulic press. Using this setup, we report two types of data.

**Table 1.** Physical sample parameters

	$L_x$ (mm)	$L_y$ (mm)	$L_z$ (mm)	$\rho$	$V_{Px}$	$V_{Py}$	$V_{Sx}$	$V_{Sy}$	$\gamma_P$	$\gamma_S$
Sample 1	126	155	52	2.4	3.2	3.05	2.24	2.22	5.1%	0.85%
Sample 2	125	154	52	2.5	3.27	3.23	2.25	2.19	1.1%	2.5%

Note. Dimensions were measured with calipers. Velocities were measured via probe transducers by measuring the P- and S-wave traveltimes across the sample in all three dimensions.  $L_j$  is the length along the  $j$ th axis;  $V_{Mj}$  is the velocity of wave mode M (P or S) propagating in direction  $j$ ;  $\gamma_M$  is the M-mode anisotropy.

The first data are velocities and anisotropies as a function of applied load (shown in Figure 2). These are computed from measured traveltimes of the corresponding wave. Anisotropy is defined as the percentage difference in velocities between the two horizontal directions. Note that there is a difference in frequency between velocities measured along the  $x$ - and  $y$ -directions.

The second data are from PUMP/probe experiments that use a high-amplitude S-wave PUMP signal (propagating along the  $x$ -direction with polarization in the  $y$ -direction) to perturb the rock. We measure the resulting traveltime delay using two different probes: a P-wave (propagating and polarized along the  $y$ -direction), and an S-wave probe (propagating along the  $y$ -direction with polarization in the  $z$ -direction). These delays are measured by cross-correlating signals recorded with and without the PUMP wave. A single traveltime delay measurement gives one data point on the plots in Figure 3. Each measurement (each point on the  $x$ -axis) represents a different transmission delay between when the PUMP wave is sent into the rock sample and when the probe wave is sent into the rock sample. As such, it measures traveltime delays caused by different phases of the traveling PUMP wave. Further experimental details, including rationales for frequency choices and travel time delay details, are discussed in Section S.1 in the Supplementary Material, and detailed parameter settings are given in Section S.2. in the Supplementary Material.

This design is similar to DAET, except that our PUMP wave is a propagating S-wave, not a resonance mode. A detailed experimental protocol is available here: <https://doi.org/10.17504/protocols.io.14egn71zyv5d/v1>. We use a probe signal that is two orders-of-magnitude weaker in strain ( $\sim 10^{-8}$ ) relative to the PUMP ( $10^{-6}$ ). Details of this strain measurement appear in Section S.3. in the Supplementary Material.

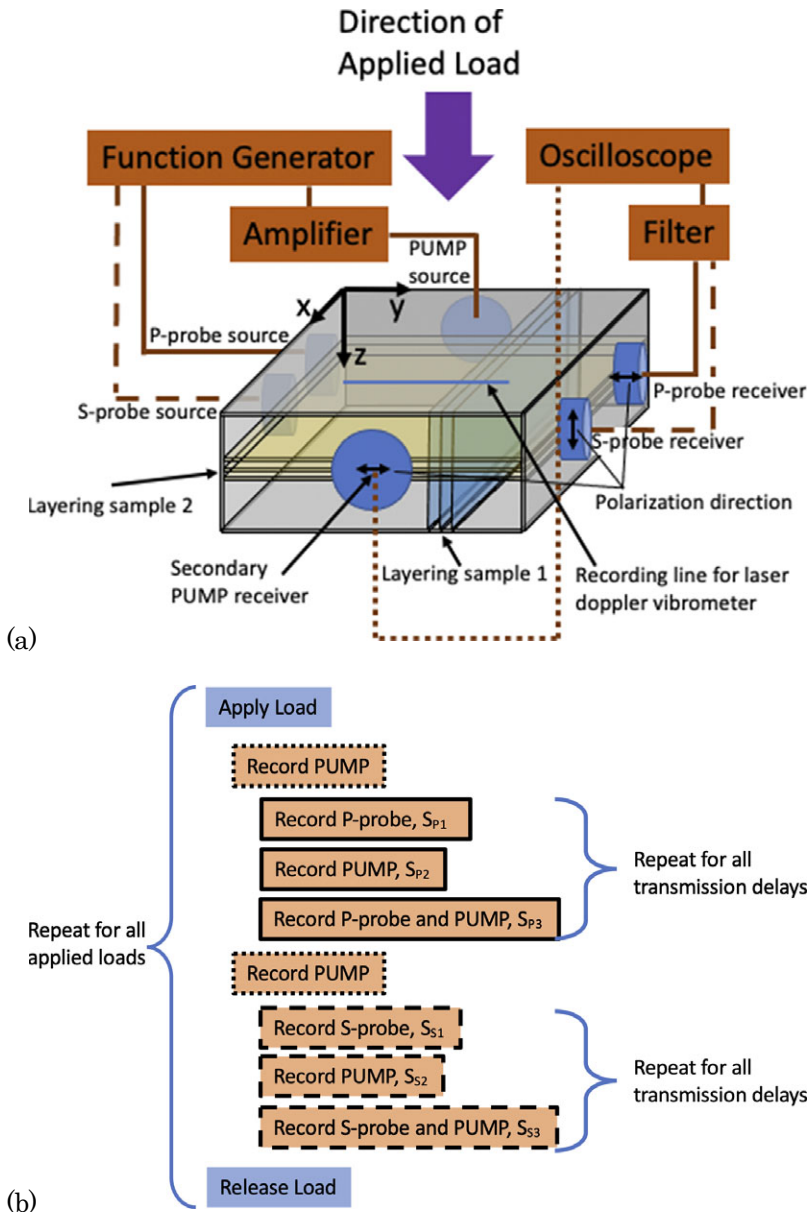
### Loading protocols

We repeated our experiments at 5–6 uniaxial loads for each sample and probe type. A hydraulic press provided the load (Figure 1). The sample, along with spacers, was placed in the cell between two stainless steel plates to promote uniform load distribution. The press pistons applied a constant force with a sequence of hydraulics, with the applied load being this force divided by the sample area. We applied the load in steps: raising the force to have a 1 MPa load on the sample and collecting data for both the P and S probes, then releasing the force, then raising the force to 2 MPa and recording the next data set, continuing up to 15 MPa for sample 1 and 18 MPa for sample 2. The additional load for sample 2 was necessary because of the reversal between 10 and 15 MPa. Although the steel plates help to distribute the strain uniformly throughout the sample, we do not expect the strain to be uniform throughout. We do expect it to be distributed similarly at different loads and among different samples.

## Results

### Velocities and amplitudes

As a precursor to the nonlinear wave mixing data, we first assessed changes in velocity, anisotropy, and PUMP amplitude with applied load (Figure 2).



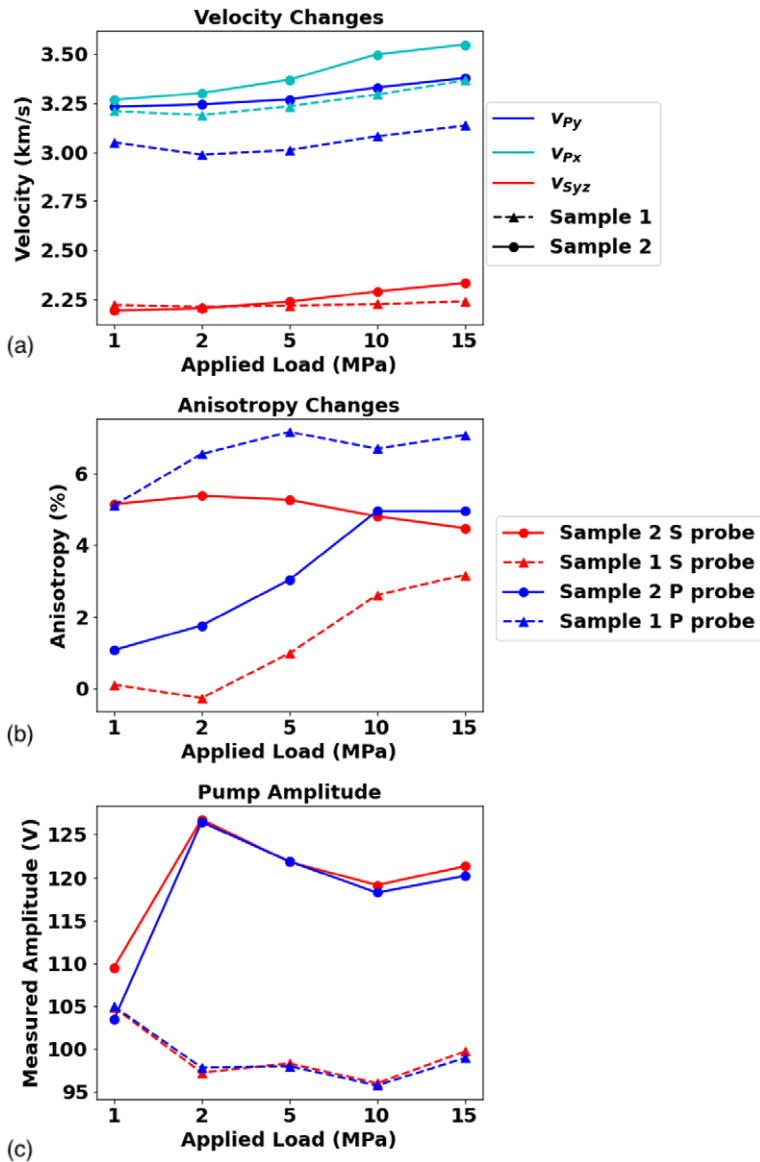
**Figure 1.** (a) The experimental setup, including the coordinate system to be used later. Sample dimensions and physical properties are given in Table 2. In all experiments, the PUMP source is connected to the function generator and amplifier. Solid lines denote connections for P-probe experiments; dashed lines correspond to S-probe experiments; dotted lines correspond to PUMP recording only. When we record the PUMP waveform to analyze it, we use the receiver setup indicated with the dotted lines, otherwise, all signals are recorded on the corresponding probe receiver. The polarization directions are noted on each receiver for convenience; the corresponding sources have the same polarizations. (b) Summary of experimental protocols. The line style on the boxes (solid, dashed, or dotted) indicates the receiver setup, as described for (a)

We measured the travel times of four waves from which we obtained four velocities:  $v_{yy}$  (P-probe),  $v_{yz}$  (S-probe),  $v_{xy}$  (S-PUMP), and  $v_{xx}$  (P-wave generated by S-PUMP transducer). We use a standard method for these measurements (Yurikov et al., 2019), which is summarized in Section S.1 in the Supplementary Material. In Figure 2a, all measured velocities increase as a function of applied load, except for a slight decrease for sample 1 velocities at low loads.

**Table 2.** Summary of experimental parameters

Wave	Transducer resonance	Driving frequency	Cycles	Polarization direction	Propagation direction	Amp (V)	$\approx$ strain	$\lambda$ (mm)
PUMP	100 kHz	90 kHz	4	<i>y</i>	<i>x</i>	10	$10^{-6}$	24
P-probe	1 MHz	1 MHz	1	<i>y</i>	<i>y</i>	0.1	$10^{-8}$	3.6
S-probe	1 MHz	1 MHz	1	<i>z</i>	<i>y</i>	0.1	$10^{-8}$	2.2

Abbreviations: amp, amplitude (peak-to-peak voltage) of the input signal before going through the (50×) amplifier;  $\lambda$ , wavelength.



**Figure 2.** Comparison of (a) velocity, (b) anisotropy, and (c) recorded PUMP amplitude with applied load. (a) All measured velocities increase as a function of applied load, except for a slight decrease for sample 1 velocities at low loads. (b) Anisotropy is most significant for P-waves in sample 1, as expected. All measures of anisotropy increase slightly and then plateau or decrease at higher applied loads. Nevertheless, all are within the errors of the estimated velocities. (c) PUMP amplitude differences are quite consistent on the same sample (with different probes), but evolve quite differently as a function of load between the two samples. Overall, the amplitude changes are 9–20% of the average PUMP amplitude. The legend in (b) also applies to (c).

*Nonlinear responses*

For each sample and applied load, we performed two kinds of nonlinear wave-mixing experiments: P-wave probe, and S-wave probe. Figure 3 shows measured travel time delays (in ns) as a function of the transmission delay time (in  $\mu\text{s}$ ) between when the PUMP and probe waves were initiated.

Figure 3 shows two clear frequency components in the time delay versus transmission delay data (as reported in similar experiment designs (Gallot et al., 2015; TenCate et al., 2016)). The first component follows the total envelope of the PUMP wave pulse, while the second higher-frequency component matches the period of the PUMP wave (90 kHz).

The component due to the PUMP envelope explains why there is a net rise in time delay with transmission delay for only some PUMP/probe combinations (cf. Figure 3(a) and (b)). The probe senses the increasing or decreasing part of the PUMP envelope depending on sample geometry and the relative

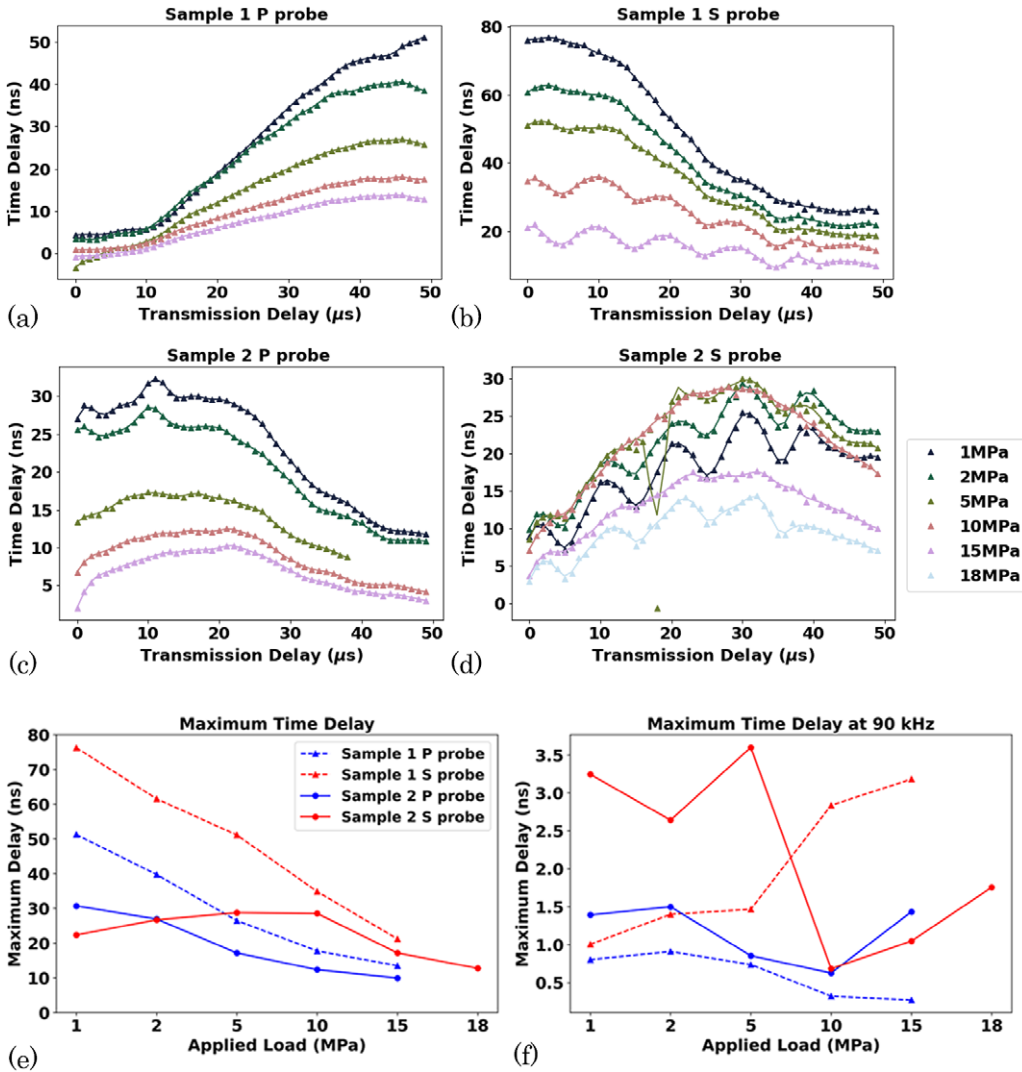


Figure 3. Time delay versus transmission delay time data for different applied loads. (a) Sample 1 (vertical layers) with a P-probe, (b) sample 1 with an S-probe, (c) sample 2 (horizontal layers) with a P-probe, and (d) sample 2 with an S-probe. Note that, with the exception of the data in (d), the delay time decreases with applied load. (e) The maximum delay time as a function of applied load. (f) The maximum of the 90 kHz signal component as a function of applied load.

locations of the PUMP and probe transducers. Thus, (a) shows the onset of the PUMP/probe interaction, whereas (b) shows the tail-end of the interaction as the PUMP pulse passes out of the interaction region in the center of the sample. In this envelope part of the time delay versus transmission delay data, others have found changes with sample orientation (TenCate et al., 2016). We discuss this orientation dependence further in Section S.6 in the Supplementary Material.

For the second, higher-frequency (90 kHz) component, we filtered the travel time delay data (Butterworth bandpass filter, corner frequencies 50 and 150 kHz), and then recorded the maximum. However, there is no consistent trend in this 90 kHz component; previous work has also shown this component to be independent of sample orientation (TenCate et al., 2016). What controls the signal at 90 kHz remains an open question.

In summary, the envelope of the travel time delays decreases as a function of applied load for all experiments, except for the S-probe in sample 2.

### Linking modulus to applied load

Since we measured traveltimes (and thus modulus) change for many different transmission delays, we reduced these data to a single number as a function of applied load. To do this, we extracted the maximum traveltimes delay (and thus change in modulus) for each applied load, and fit the resulting data sets to this simple model (Figure 4). For sample 2 with the S-probe, there is no modulus change before 10 MPa; thus, the fit includes only 10, 15, and 18 MPa. The characteristic load for each probe and sample type are shown in the insets of Figure 4; these are consistent within our experimental errors. The values agree with those recovered on sandstones (Rivière et al., 2016), but they are different from those recovered for metamorphic rocks (Simpson et al., 2021).

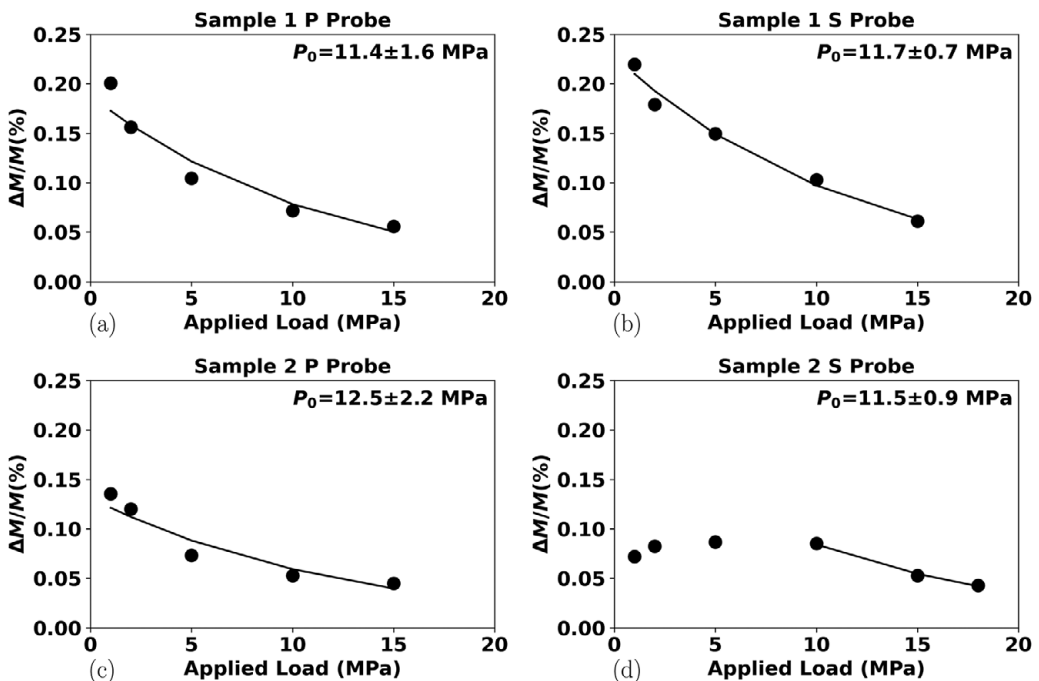


Figure 4. Fits to the model in equation (1) for (a) sample 1 (vertical layers) with a P-probe, (b) sample 1 with an S-probe, (c) sample 2 (horizontal layers) with a P-probe, and (d) sample 2 with an S-probe. For all cases, the characteristic load  $P_0$  (insets) is the same within error.



## Conclusions

We present a data set showing the evolution of the nonlinear interaction of different wave types as a function of applied uniaxial load. We find a characteristic load that is consistent with literature results for other samples measured with different experimental configurations. We observe no dependence on layer orientation in the response to load. Although we do observe dependence of the nonlinear response on the direction of layering, we cannot exclude differences between samples as the cause of this difference. This indicates that the controlling mechanism may not be cracks, but another structure aligned with the layering.

**Open peer review.** To view the open peer review materials for this article, please visit <http://doi.org/10.1017/exp.2022.24>.

**Supplementary materials.** To view supplementary material for this article, please visit <http://doi.org/10.1017/exp.2022.24>.

**Data availability statement.** Data are openly available via the Memorial Dataverse repository Malcolm and Poduska (2021): Malcolm, Alison; Poduska, Kristin M., 2021, "Nonlinear Wave Mixing as a Function of Applied Load," <https://doi.org/10.5683/SP3/CZXST1>, Scholars Portal Dataverse, V1. The experimental method is published on protocols.io at <https://www.protocols.io/edit/pump-probe-experiment-cbvxsxn7n>.

**Acknowledgments.** Computational support from ComputeCanada and ACENET is gratefully acknowledged.

**Author contributions.** A.E.M., L.C., and S.B. designed the study. L.C. and K.M. carried out the experiments in a lab managed by S.B. K.M.P., A.E.M., A.M. processed the data, discussed and wrote up the results.

**Funding statement.** This work is supported by Chevron and with grants from the Natural Sciences and Engineering Research Council of Canada (NSERC) Industrial Research Chair Program and Discovery Grants program, as well as InnovateNL.

**Conflict of interest.** The authors have no conflicts of interest to declare.

## References

- Gallot, T., Malcolm, A. E., Szabo, T. L., Brown, S., Burns, D., & Fehler, M. (2015). Characterizing the nonlinear interaction of S- and P-waves in a rock sample. *Journal of Applied Physics*, **117**, 034902.
- Hayes, L., Malcolm, A., Moravej, K., & Butt, S. (2018). Nonlinear interactions of P and S waves under uniaxial stress. In *POMA proceedings of the 21st International Symposium on Nonlinear Acoustics*. Vol. 34. No. 1. Acoustical Society of America.
- Hughes, D., & Kelly, J. (1953). Second-order elastic deformation of solids. *Physical Review*, **89**, 1940.
- Khajehpour Tadavani, S., Poduska, K. M., Malcolm, A. E., & Melnikov, A. (2020). A non-linear elastic approach to study the effect of ambient humidity on sandstone. *Journal of Applied Physics*, **128**, 244902.
- Lott, M., Payan, C., Garnier, V., Vu, Q. A., Eiras, J. N., Remillieux, M. C., Le Bas, P.-Y., & Ulrich, T. (2016a). Three-dimensional treatment of nonequilibrium dynamics and higher order elasticity. *Applied Physics Letters*, **108**, 141907.
- Lott, M., Remillieux, M. C., Garnier, V., Le Bas, P.-Y., Ulrich, T. J., & Payan, C. (2017). Nonlinear elasticity in rocks: A comprehensive three-dimensional description. *Physical Review Materials*, **1**, 023603.
- Lott, M., Remillieux, M. C., Le Bas, P.-Y., Ulrich, T. J., Garnier, V., & Payan, C. (2016b). From local to global measurements of nonclassical nonlinear elastic effects in geomaterials. *Journal of the Acoustical Society of America*, **140**, EL231.
- Malcolm, A., & Poduska, K. M. (2021). Nonlinear wave mixing as a function of applied load. Scholars Portal Dataverse V1.
- Muir, T. G., Cormack, J. M., Slack, C. M., & Hamilton, M. F. (2020). Elastic softening of sandstone due to a wideband acoustic pulse. *Journal of the Acoustical Society of America*, **147**, 1006–1014.
- Remillieux, M. C., Ulrich, T. J., Goodman, H. E., & Ten Cate, J. A. (2017). Propagation of a finite-amplitude elastic pulse in a bar of Berea sandstone: A detailed look at the mechanisms of classical nonlinearity, hysteresis, and nonequilibrium dynamics. *Journal of Geophysical Research: Solid Earth*, **122**, 8892–8909.
- Renaud, G., Calle, S., Remenieras, J. P., & Defontaine, M. (2008). Exploration of trabecular bone nonlinear elasticity using time-of-flight modulation. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, **55**, 1497–1507.
- Renaud, G., Le Bas, P.-Y., & Johnson, P. A. (2012). Revealing highly complex elastic nonlinear (anelastic) behavior of Earth materials applying a new probe: Dynamic acoustoelastic testing. *Journal of Geophysical Research*, **117**, B06202.
- Rivière, J., Pimienta, L., Scuderi, M., Candela, T., Shokouhi, P., Fortin, J., Schubnel, A., Marone, C., & Johnson, P. A. (2016). Frequency, pressure, and strain dependence of nonlinear elasticity in Berea Sandstone. *Geophysical Research Letters*, **43**, 3226–3236.
- Rivière, J., Renaud, G., Guyer, R. A., & Johnson, P. A. (2013). Pump and probe waves in dynamic acousto-elasticity: Comprehensive description and comparison with nonlinear elastic theories. *Journal of Applied Physics*, **114**, 054905.



- Rivière, J., Shokouhi, P., Guyer, R. A., & Johnson, P. A. (2015). A set of measures for the systematic classification of the nonlinear elastic behavior of disparate rocks. *Journal of Geophysical Research: Solid Earth*, **120**, 1587–1604.
- Rusmanugroho, H., Darijani, M., & Malcolm, A. (2020). A numerical model for the nonlinear interaction of elastic waves with cracks. *Wave Motion*, **92**, 102444.
- Sens-Schönfelder, C., & Eulenfeld, T. (2019). Probing the in situ elastic nonlinearity of rocks with earth tides and seismic noise. *Physical Review Letters*, **122**, 138501.
- Simpson, J., van Wijk, K., and Adam, L. (2021). Spatial dependence of dynamic nonlinear rock weakening at the alpine fault, New Zealand. *Geophysical Research Letters*, **48**, e2021GL093862.
- TenCate, J. A., Malcolm, A. E., Feng, X., & Fehler, M. C. (2016). The effect of crack orientation on the nonlinear interaction of a P wave with an S wave. *Geophysical Research Letters*, **43**, 6146–6152.
- Viswanathan, H. S., Ajo-Franklin, J., Birkholzer, J. T., Carey, J. W., Guglielmi, Y., Hyman, J., Karra, S., Pyrak-Nolte, L., Rajaram, H., Srinivasan, G., & Tartakovsky, D. M. (2022). From fluid flow to coupled processes in fractured rock: Recent advances and new frontiers. *Reviews of Geophysics*, **60**, e2021RG000744.
- Yurikov, A., Nourifard, N., Pervukhina, M., & Lebedev, M. (2019). Laboratory ultrasonic measurements: Shear transducers for compressional waves. *The Leading Edge*, **38**, 392–399.
- Zinszner, B., Johnson, P., & Rasolofosaon, P. (1997). Influence of change in physical state on elastic nonlinear response in rock: Significance of effective pressure and water saturation. *Journal of Geophysical Research*, **102**, 8105–8120.

---

**Cite this article:** Malcolm AE, Coates L, Moravej K, Melnikov A, Butt S, Poduska KM (2023). Experimental monitoring of nonlinear wave interactions in crab orchard sandstone under uniaxial load. *Experimental Results*, **4**, e10, 1–14. <https://doi.org/10.1017/exp.2022.24>

# Peer Reviews

**Reviewing editor:** Prof. Stefano Camera, PhD

Universita degli Studi di Torino, Physics, Via Pietro Giuria, 1, Torino, Italy, 10124

Minor revisions requested.

doi:10.1017/exp.2022.24.pr1

## Review 1: Experimental Monitoring of Nonlinear Wave Interactions Under Uniaxial Load

**Reviewer:** Kasper Van Wijk 

Date of review: 03 August 2022

© The Author(s), 2023. Published by Cambridge University Press. This is an Open Access article, distributed under the terms of the Creative Commons Attribution licence (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted re-use, distribution and reproduction, provided the original article is properly cited.

**Conflict of interest statement.** Reviewer declares none.

### Comment

Comments to the Author: This is an excellent report on new experiments to improve our understanding of nonlinearity in earth materials, which could be important for earthquake rupture processes and volcanic eruptions, for example, and should be published.

It tests the hypothesis that microfractures are a dominant factor in said nonlinearity, but the conclusions of the experiments suggest other factors may be at play. The assumption in this paper, and the 2016 Ten Cate et al. paper in GRL is that the fractures in the samples are aligned with bedding. This orientation is a finding by Benson et al, 2005. However, the experiments here do not see the correlation between aligned fractures and nonlinearity. Could it be the fractures are not preferentially aligned after all? Figure 2 in this paper does not clearly show the opening of these fractures with load in the linear (p-wave) velocities and total anisotropy, either. Assuming the orientation of the cracks is along the bedding, would the authors not expect in sample 1 the linear P-wave velocity to decrease and anisotropy to increase with load? At least more than in sample 2? The biggest increase in anisotropy is sample 2, P probe. Could it be the bedding is controlling the anisotropy and fractures are not aligned (with the bedding) to explain the consistency in the nonlinear experiments?

Final question is about the total amount of nonlinearity: While the characteristic load is constant, sample 1 appears to be more nonlinear (up to 0.2 percent) than sample 2 (less than 0.15 percent). Is there more to the difference between the samples than the orientation of the cut, or is this difference in total nonlinearity negligible?

### Score Card

#### Presentation



Is the article written in clear and proper English? (30%)

5/5

Is the data presented in the most useful manner? (40%)

4/5

Does the paper cite relevant and related articles appropriately? (30%)

5/5

## Context



Does the title suitably represent the article? (25%)

5/5

Does the abstract correctly embody the content of the article? (25%)

5/5

Does the introduction give appropriate context? (25%)

5/5

Is the objective of the experiment clearly defined? (25%)

5/5

## Analysis



Does the discussion adequately interpret the results presented? (40%)

3/5


Is the conclusion consistent with the results and discussion? (40%)

3/5

Are the limitations of the experiment as well as the contributions of the experiment clearly outlined? (20%)

4/5

## Review 2: Experimental Monitoring of Nonlinear Wave Interactions Under Uniaxial Load

Reviewer: Dr. H. Haeri 

Date of review: 17 August 2022

© The Author(s), 2023. Published by Cambridge University Press. This is an Open Access article, distributed under the terms of the Creative Commons Attribution licence (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted re-use, distribution and reproduction, provided the original article is properly cited.

**Conflict of interest statement.** Reviewer declares none.

### Comment

Comments to the Author: 1-What is the novelty of the manuscript? It should be clearly expressed in the last paragraph of the introduction section of the paper.

2-The model dimensions should be specified.

3-Only the important findings and observations should be given. Conclusion part should be checked.

4-Enrich the literature review in introduction.

5-The literature review must be improved by discussing the following works:

<https://doi.org/10.1002/2017JB014773>

<https://doi.org/10.1007/s11223-017-9872-6>

<https://doi.org/10.1007/s00603-017-1310-3>

### Score Card

#### Presentation



Is the article written in clear and proper English? (30%)

5/5

Is the data presented in the most useful manner? (40%)

4/5

Does the paper cite relevant and related articles appropriately? (30%)

3/5

#### Context



Does the title suitably represent the article? (25%)

4/5

Does the abstract correctly embody the content of the article? (25%)

4/5

Does the introduction give appropriate context? (25%)

4/5

Is the objective of the experiment clearly defined? (25%)

3/5

#### Analysis



Does the discussion adequately interpret the results presented? (40%)

4/5


Is the conclusion consistent with the results and discussion? (40%)

4/5

Are the limitations of the experiment as well as the contributions of the experiment clearly outlined? (20%)

4/5

## Review 3: Experimental Monitoring of Nonlinear Wave Interactions Under Uniaxial Load

Reviewer: Dr. Anna Pandolfi 

Date of review: 01 September 2022

© The Author(s), 2023. Published by Cambridge University Press. This is an Open Access article, distributed under the terms of the Creative Commons Attribution licence (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted re-use, distribution and reproduction, provided the original article is properly cited.

**Conflict of interest statement.** Reviewer declares none.

### Comment

Comments to the Author: However, the presentation is based on all information reported in previous works and therefore is impossible to understand the meaning and the relevance of the experiment.

1. Title, abstract and introduction should explicitly mention that the material considered is rock.
2. The introduction should clearly lead the reader towards the materials of interest; as it is,
3. The indication of a precise load in the abstract is useless (and deprecated), because it has no possibility to be related to any particular material.
4. Page 1. The experiment under consideration has to be explained before making comments about the previous studies.
5. Page 2. An experiment cannot be “nonlinear elastic”. The material may behave as nonlinear elastic, according to the behavior that is observed. Rephrase.
6. Theory. The short paragraph is insufficient to explain what you are investigating. What is the definition of elastic modulus that is considered in (1)?
7. Sample description. A picture of the two samples is missing, to comply with the description.
8. Table 1. The meaning and the relevance within the experiment of the variables listed in the table is not reported in the text.
9. Even if referred through a hyperlink, the experiment must be succinctly reported, for the sake of clarity in reading the manuscript.
10. Figure 2. Velocity, Anisotropy, Measured amplitude. These important definition, indispensable to read the plots, must reside in the body of the paper, not in the supplementary material.

### Score Card

#### Presentation



Is the article written in clear and proper English? (30%)

1/5

Is the data presented in the most useful manner? (40%)

5/5

Does the paper cite relevant and related articles appropriately? (30%)

2/5

#### Context



Does the title suitably represent the article? (25%)

4/5

Does the abstract correctly embody the content of the article? (25%)

4/5

Does the introduction give appropriate context? (25%)

4/5

Is the objective of the experiment clearly defined? (25%)

4/5

## Analysis



Does the discussion adequately interpret the results presented? (40%)

---

5/5

Is the conclusion consistent with the results and discussion? (40%)

---

5/5

Are the limitations of the experiment as well as the contributions of the experiment clearly outlined? (20%)

---

3/5