or, writing $x = \dfrac{1}{z}$, and multiplying by z^{n-1},

$$(1 + p_1 z + \ldots + p_n z^n)\left(\frac{1}{1-\alpha_1 z} + \ldots + \frac{1}{1-\alpha_n z}\right)$$
$$= n + (n-1)p_1 z + \ldots + p_{n-1} z^{n-1}.$$

The left hand member is

$$\left(1 + p_1 z + \ldots + p_n z^n\right)\left(n + s_1 z + \ldots + s_n z^n + \frac{\alpha_1{}^{n+1} z^{n+1}}{1-\alpha_1 z} + \ldots + \frac{\alpha_n{}^{n+1} z^{n+1}}{1-\alpha_n z}\right),$$

that is,

$$(1 + p_1 z + \ldots + p_n z^n)\,(n + s_1 z + \ldots + s_n z^n)$$

together with a rational integral function of z containing z^{n+1} as a factor.

Hence, for $r = 0, 1, 2, \ldots, n$

the coefficient of z^r is the same in

$$(1 + p_1 z + \ldots + p_n z^n)\,(n + s_1 z + \ldots + s_n z^n)$$

and in

$$n + (n-1)p_1 z + \ldots + p_{n-1} z^{n-1},$$

from which Newton's formulae follow at once.

JOHN DOUGALL.

Arithmetical Solution of the Ages Problem.—The following solution of an old problem may be new to some readers.

"The ages of A and B are as 3 to 1, and in 15 years they will be as 2 to 1. Find their ages."

$$\frac{3}{1}\,,\quad \frac{2}{1}$$

$$\frac{3}{1}\,,\quad \frac{4}{2}$$

$$\frac{45}{15}\,,\quad \frac{60}{30}$$

In the first line the original and final ratios are written down as fractions. In the second line, without altering the value of the fractions, we make the difference of numerator and denominator the same in both cases. As a consequence of this, the difference of the numerators will now be the same as the difference of the denominators, viz., 1 in this case. We wish it to be 15, and therefore multiply by 15 all through to obtain the original and final ages.

Similarly with the general case, "two numbers are in the ratio of a to b. If n be added to each, the ratio becomes that of c to d. Find the original and final numbers."

$$\frac{a}{b}, \quad \frac{c}{d}$$

$$\frac{a(c-d)}{b(c-d)}, \quad \frac{c(a-b)}{d(a-b)}$$

$$\frac{a(c-d)\dfrac{n}{ad-bc}}{b(c-d)\dfrac{n}{ad-bc}}, \quad \frac{c(a-b)\dfrac{n}{ad-bc}}{d(a-b)\dfrac{n}{ad-bc}}.$$

The four numbers in the last line are the numbers required.

JOHN DOUGALL.

The Equivalence of certain Simultaneous Equations. —The object of this note will, perhaps, be best explained by first repeating the usual steps in the solution of two simultaneous equations in x, y, where one is linear and the other is of the second degree. Let the equations be

$$x-y+1=0, \qquad (1)$$

$$2x^2-y^2-7x+5y-1=0. \qquad (2)$$

From (1) we obtain $y=x+1$. Substituting $y=x+1$ in (2) we next obtain

$$x^2-4x+3=0, \qquad (3)$$

whence $\qquad x=1$ or 3.

Going back to (1), we find that

if $\quad x=1, \; y=2$;

if $\quad x=3, \; y=4$.

The solutions are therefore $(1, 2)$, $(3, 4)$.

https://doi.org/10.1017/S1757748900001298 Published online by Cambridge University Press