

# The Luminosity Problem: Testing Theories of Star Formation

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**Abstract.** Low-mass protostars are less luminous than expected. This luminosity problem is important because the observations appear to be inconsistent with some of the basic premises of star formation theory. Two possible solutions are that stars form slowly, which is supported by recent data, and/or that protostellar accretion is episodic; current data suggest that the latter accounts for less than half the missing luminosity. The solution to the luminosity problem bears directly on the fundamental problem of the time required to form a low-mass star. The protostellar mass and luminosity functions provide powerful tools both for addressing the luminosity problem and for testing theories of star formation. Results are presented for the collapse of singular isothermal spheres, for the collapse of turbulent cores, and for competitive accretion.

## 1. The Luminosity Problem

Why don't protostars shine more brightly? In a seminal paper, Kenyon *et al.* (1990) identified this luminosity problem, developed an approach to treat it (the protostellar luminosity function), and proposed almost all the solutions to the problem that have subsequently been studied. The luminosity problem is simple to state: The accretion luminosity of a protostar is

$$L_{\text{acc}} = f_{\text{acc}} \frac{Gm\dot{m}}{r_*} = 3.9f_{\text{acc}} \left( \frac{m}{0.25M_{\odot}} \right) \left( \frac{2R_{\odot}}{r_*} \right) \left( \frac{\dot{m}}{10^{-6} M_{\odot} \text{ yr}^{-1}} \right) L_{\odot}, \quad (1.1)$$

where  $f_{\text{acc}}$  is the fraction of the accretion energy that goes into radiation (Kenyon *et al.* took  $f_{\text{acc}} = 1$ ),  $m$  is the protostellar mass, and  $\dot{m}$  is the accretion rate. Stahler (1988) has calculated the protostellar radius for a variety of masses and accretion rates, including the thermostatic effects of deuterium burning; we find that  $r_* \simeq 2R_{\odot}$  is a typical value for the harmonic mean radius. By comparing the number of embedded sources with the number of T Tauri stars, Kenyon *et al.* (1990) inferred a star formation time  $t_f \simeq (0.1 - 0.2)$  Myr in Taurus-Auriga; to build up a star of average mass ( $0.5 M_{\odot}$ ) in this time requires an accretion rate  $\dot{m} \simeq (2.5 - 5) \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ . On average, the mass of a protostar will be about half its final stellar mass, which implies that the typical luminosity of a protostar that will become a star of average mass is  $L \sim (10 - 20)L_{\odot}$ . (The median luminosity of the average mass star is comparable to the median luminosity—see §3 below.) The median luminosity in the Kenyon *et al.* sample is  $1.6L_{\odot}$ . One statement of the luminosity problem is that the median protostellar luminosity is observed to be about an order of magnitude less than the expected value.

Kenyon *et al.* (1990) also provided an alternative description of the luminosity problem: Identifying the peak in the observed luminosity function of embedded sources at  $0.3 L_{\odot}$

as the luminosity of the lowest mass stars, which they took to be  $0.1 M_{\odot}$  (keep in mind that this was prior to the discovery of brown dwarfs), and estimating the radius of these protostars as  $r_* \sim 1 R_{\odot}$ , they inferred a mass accretion rate of only  $10^{-7} M_{\odot} \text{ yr}^{-1}$ . However, very general arguments (Stahler, Shu, & Taam 1980) indicate that the accretion rate due to gravitational collapse should be of order

$$\dot{m} \sim \frac{(c_s^2 + v_A^2 + \sigma_{\text{turb}}^2)^{3/2}}{G}, \quad (1.2)$$

$$\geq \frac{c_s^3}{G} = 1.4 \times 10^{-6} \left( \frac{T}{10 \text{ K}} \right)^{3/2} M_{\odot} \text{ yr}^{-1}. \quad (1.3)$$

There are solutions that give higher accretion rates than this, such as the Larson (1969)-Penston (1969) solution for the collapse of an isothermal sphere of constant density, but there are no solutions that give lower accretion rates, at least prior to the time that the accretion is affected by the outer boundary (Henriksen *et al.* 1997). Since the observed temperature in molecular clouds is  $T \sim 10 \text{ K}$ , this again leads to an order of magnitude discrepancy between observation and theory.

Results from the recent *Spitzer* c2d survey of five nearby star-forming molecular clouds (Evans *et al.* 2009; the survey does not include Taurus-Auriga) confirm that protostars have low luminosities. Evans *et al.* (2009) classified the young stellar objects (YSOs) in the traditional class system, in which Class 0 represents protostars with envelope masses greater than the protostellar mass; Class I represents embedded objects with masses greater than the envelope mass; Flat Spectrum objects represent a transitional class; and Class II corresponds to T Tauri stars. Dunham *et al.* (2010) analyzed this sample and found an extinction-corrected median luminosity of  $1.5 L_{\odot}$ . Including  $350 \mu\text{m}$  data for many of the brighter sources, they obtained an extinction-corrected mean of  $5.3 L_{\odot}$ .

The luminosity problem is thus well established observationally. It is fundamental because observations appear difficult to reconcile with some of the the basic premises of star formation theory, that stars form via gravitational collapse (eq. 1.3) and that they radiate the binding energy in the process (eq. 1.1). As the discussion above illustrates, there are two related aspects to the problem: the observed luminosity appears to be less than that theoretically expected, and there is an excess of very low-luminosity sources.

## 2. Proposed Solutions

Kenyon *et al.* (1990) proposed a number of solutions to the luminosity problem, of which the two major ones are slow accretion and episodic accretion (discussed below). They also suggested that brown dwarfs could alleviate the luminosity problem by providing sources with luminosities less than they considered. Since their work predated the discovery of brown dwarfs, they did not include them. It is now known that brown dwarfs constitute about 20% of stars (Andersen *et al.* 2008), and they do permit some sources to have lower luminosities. We include brown dwarfs in the models described below.

There is an additional effect that reduces the luminosity problem: The hydromagnetic outflows observed from protostars extract kinetic energy from the accreting gas (McKee & Ostriker 2007), reducing the energy radiated. If half the energy lost by the disk is mechanical energy and this in turn is half the total potential energy, then  $f_{\text{acc}} \simeq \frac{3}{4}$  (Offner & McKee 2010).

### 2.1. Episodic Accretion

Kenyon *et al.* (1990) suggested that the accretion onto the protostar (as opposed to infall onto a circumstellar disk) could be episodic, so that much of the protostellar mass would

be accreted in short periods of time, with high luminosities. Such brief, high-luminosity accretion events could be associated with FU Ori outbursts, which have inferred accretion rates of  $\dot{m} \sim 10^{-4} M_{\odot} \text{ yr}^{-1}$ . Kenyon *et al.* (1990) pointed out that the 150 YSOs in Taurus Auriga that formed in the last  $10^6$  yr correspond to a total accretion rate of  $0.75 \times 10^{-4} M_{\odot} \text{ yr}^{-1}$ , comparable to that of a single FU Ori object; thus, a significant fraction of the mass of a protostar might be acquired during FU Ori events.

Hartmann & Kenyon (1996) refined this argument: They cited a rate of star formation within 1 kpc of the Sun of  $(5-10) \times 10^{-3} M_{\odot} \text{ yr}^{-1}$ , which is similar to the recent estimate of  $8 \times 10^{-3} M_{\odot} \text{ yr}^{-1}$  by Fuchs *et al.* (2009). At that time there were 5-9 known FU Ori objects within 1 kpc; if each were to accrete at a rate of  $10^{-4} M_{\odot} \text{ yr}^{-1}$ , then protostars could gain about 10% of their mass this way. They pointed out that this was a lower limit, since more FU Ori objects would most likely be discovered. That has indeed occurred: Greene *et al.* (2008) count a total of 22 known FU Ori objects, of which 18 are within 1 kpc. Rounding off, we infer that 20 objects accreting at  $10^{-4} M_{\odot} \text{ yr}^{-1}$  would account for 25% of the mass accreted by protostars within 1 kpc.

To elaborate on this result, consider a simple model for episodic accretion, in which protostars are in a high-accretion state for a total time  $\Delta t_{\text{high}}$  and in a low-accretion state for the rest of the time, which is effectively the star formation time,  $\Delta t_{\text{low}} \simeq t_f$ . The fraction of the mass accreted during a high state is

$$F_{\text{high}} = \frac{\dot{m}_{\text{high}} \Delta t_{\text{high}}}{\langle \dot{m}_f \rangle}. \quad (2.1)$$

We shall adopt parameters appropriate for FU Ori outbursts, but in fact all episodes of high accretion could be included in  $F_{\text{high}}$ . The total time spent in the high state is

$$\Delta t_{\text{high}} = \frac{\mathcal{N}_{p, \text{high}}}{\dot{\mathcal{N}}_*}, \quad (2.2)$$

where  $\mathcal{N}_{p, \text{high}}$  is the number of protostars in a high state in some volume of the Galaxy and  $\dot{\mathcal{N}}_*$  is the star formation rate there. For a mean stellar mass of  $0.5 M_{\odot}$ , the local star formation rate cited above corresponds to a birthrate within 1 kpc of the Sun of 0.016 stars  $\text{yr}^{-1}$ . If the number of FU Ori objects within 1 kpc is about 20, then  $\Delta t_{\text{high}} = 1250$  yr. Inserting Hartmann & Kenyon's estimate that  $\dot{m}_{\text{high}} \simeq 10^{-4} M_{\odot} \text{ yr}^{-1}$ , we recover the result given above,  $F_{\text{high}} \simeq 0.25$ . We also note that the duty cycle of FU Ori outbursts is small:  $\Delta t_{\text{high}}/t_f \sim 10^3/(10^{5-6}) < 0.01$ . This small value is consistent with the absence of any known FU Ori sources in the Evans *et al.* (2009) sample.

There are two uncertain numbers that entered into this estimate of  $F_{\text{high}}$ , the number of FU Ori objects,  $\mathcal{N}_{p, \text{high}}$ , and the accretion rate,  $\dot{m}_{\text{high}}$ . Undoubtedly, more such objects will be discovered within 1 kpc of the Sun in the future; however, it should be noted that several of the bursting objects, such as L1551 IRS5, have luminosities, and therefore accretion rates, an order of magnitude less than the average (Hartmann & Kenyon 1996). Further study will also refine the observed value of the mean accretion rate. Hartmann & Kenyon (1996) infer an accretion rate  $\dot{m}_{\text{high}} = 1.9 \times 10^{-4} M_{\odot} \text{ yr}^{-1}$  and a stellar radius  $r_* = 5.9 R_{\odot}$  for FU Ori itself. However, after the rapid accretion ceases, the star will shrink back to its original size, releasing a comparable amount of energy. A mean value  $\dot{m}_{\text{high}} \simeq 1 \times 10^{-4} M_{\odot} \text{ yr}^{-1}$  thus seems reasonable in this case.

Hartmann & Kenyon (1996) also noted that given 5 known outbursts in 60 years and a star formation rate of 0.01 stars  $\text{yr}^{-1}$  within 1 kpc of the Sun implies that there are about 10 bursts per object. The more recent data cited above lead to a similar conclusion. Since  $\Delta t_{\text{high}} \simeq 1250$  yr, this means that a typical outburst lasts about 100 yr, consistent with the observationally inferred lifetime (Hartmann & Kenyon 1996).

Finally, we note that a potential problem with the FU-outburst solution to the luminosity problem is that these outbursts appear to be located preferentially in regions of low star-formation rates (Greene *et al.* 2008).

## 2.2. Slow Accretion ( $\dot{m} \lesssim 10^{-6} M_{\odot} \text{ yr}^{-1}$ )

As an alternative solution to the luminosity problem, Kenyon *et al.* (1990) suggested two ways to increase the ages of the protostars and therefore reduce the inferred accretion rate: (1) If the star formation rate decreased substantially over the lifetime of the T Tauri stars, then the observed number of Class I sources would require a greater lifetime; or (2) if the lifetime of the T Tauri stars were larger, then the protostellar lifetime would increase proportionately. They rejected the first possibility since it is difficult to understand how star formation can decelerate in 1 Myr when it is in a region with a crossing time of 10 Myr. The second possibility is more plausible, since the estimated lifetime of T Tauri stars has increased in the intervening 20 years. In their recent analysis of protostellar lifetimes, Evans *et al.* (2009) concluded that the lifetime of Class II sources is about 2 Myr, which would imply a Class I lifetime of  $t_f \simeq (0.2 - 0.4)$  Myr in Taurus Auriga. The larger of these two values is favored by data in the c2d survey, for which Evans *et al.* (2009) found a Class I lifetime of 0.44 Myr. The total protostellar lifetime,  $t_f$ , must also include the time spent in the Class 0 phase, which they found to be  $\simeq 0.1$  Myr, so that the total inferred mean lifetime of protostars in these clouds is  $\langle t_f \rangle = 0.54$  Myr. This long protostellar lifetime, together with the correction for non-radiative energy losses ( $f_{\text{acc}} = \frac{3}{4}$ ) and the correction for unseen outbursts (also a factor of  $\frac{3}{4}$ ) effectively resolves the luminosity problem: The accretion rate for a star of average mass is then  $\dot{m} \simeq 10^{-6} M_{\odot} \text{ yr}^{-1}$ , which corresponds to a typical accretion luminosity from equation (1.1) of about  $2.2 L_{\odot}$ , comparable to the observed median luminosity of  $1.5 L_{\odot}$ .

What about the other aspect of the luminosity problem, which is that the inferred accretion rates are much less than expected theoretically? There are two physical effects that reduce the accretion rate below that in standard models, namely, protostellar outflows and the finite size of the protostellar envelope. Bontemps *et al.* (1996) found that protostellar outflow rates vary as the envelope mass and showed that this could be understood if accretion rates declined exponentially with time:  $\dot{m} = \dot{m}_0 \exp(-t/t_*)$  (see also Myers *et al.* 1998), where the decay time is  $t_* = m_f/\dot{m}_0$  and  $m_f$  is the final protostellar mass (ie., the initial stellar mass). This exponential decline in the accretion rate does not capture the reduction due to the initial stage of protostellar outflows (Shibata & Uchida 1985). It is thus plausible when these effects are included, current theories can be consistent with the inferred low accretion rates. However, this will be possible only if the high accretion rates associated with an initial Larson-Penston accretion phase (Henriksen *et al.* 1997, Schmeja & Klessen 2004) do not release a significant amount of radiative energy.

## 3. The Protostellar Luminosity Function

### 3.1. Previous Work

Any solution of the luminosity problem must explain the distribution of protostellar luminosities—the protostellar luminosity function (PLF). Kenyon *et al.* (1990) introduced the PLF and considered two cases: (1) the standard isothermal sphere (IS) case (Shu 1977), in which the accretion rate is independent of mass so that the formation time,  $t_f$ , is proportional to the final mass,  $m_f$ ; and (2) a case in which the accretion rate is proportional to  $m_f$ , so that  $t_f$  is independent of  $m_f$ . Fletcher and Stahler (1994a,b)

extended this work in the IS case by determining both the protostellar mass function (PMF) and the PLF for pre-main-sequence stars.

Dunham *et al.* (2010) carried out a detailed analysis of the joint distribution of protostellar luminosities and bolometric temperatures. Interestingly, they adopted an accretion rate of  $4.6 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ , much higher than implied by the protostellar lifetimes inferred by Evans *et al.* (2009). In one series of models, they included outflows, which entrained envelope material and resulted in a core star-formation efficiency of 0.3-0.5. The outflows were not intrinsically collimated, as in the study by Matzner & McKee (2000), so that outflow cavities eventually expanded to an opening angle of 90 deg, terminating accretion. The outflows led to substantial variations in the observed bolometric luminosity and temperature as functions of the inclination angle. Despite the reduction in the accretion rate, the mean luminosities were still too high. The best agreement with observation was obtained by assuming that most of the mass was accreted in FU Ori-type events with accretion rates of  $10^{-4} M_{\odot} \text{ yr}^{-1}$ , although the resulting models spent much more time with bolometric temperatures above  $10^3 \text{ K}$  than is observed. These models required  $F_{\text{high}} \sim 0.8$  (Dunham, private communication). This is consistent with observation only if the number of FU Ori sources within 1 kpc of the Sun were about 3 times higher than currently known (see eq. 2.2).

### 3.2. The PMF and the PLF

We follow McKee & Offner (2010) and Offner & McKee (2010) in describing the protostellar mass function (PMF) and the protostellar luminosity function (PLF). Let  $\psi(m_f)d \ln m_f$  be the fraction of stars born in the mass range  $dm_f$ ;  $\psi$  is thus the IMF. The PMF is the present-day mass function of the protostars, and it must be consistent with the IMF when the protostellar mass reaches its final value,  $m_f$ . We denote the fraction of protostars in the mass range  $dm$  that will have final masses in the mass range  $dm_f$  by  $\psi_{p2}(m, m_f)d \ln m d \ln m_f$ ; it is the IMF weighted by the time the protostar spends at a given mass,

$$\psi_{p2}(m, m_f) = \left[ \frac{(m/\dot{m}(m, m_f))}{\langle t_f \rangle} \right] \psi(m_f), \tag{3.1}$$

where  $\langle t_f \rangle$  is the average protostellar lifetime. We have assumed that the accretion rate is a function of  $m$  and  $m_f$ , which can be readily generalized to allow for high and low accretion states. The PMF is the integral of this function over all possible values of the final mass, ranging from the lower bound on the IMF,  $m_{\ell}$ , to the upper bound,  $m_u$ , subject to the constraint  $m \leq m_f$ :

$$\psi_p(m) = \int_{\max(m_{\ell}, m)}^{m_u} \psi_{p2} d \ln m_f. \tag{3.2}$$

It follows that the fraction of protostars in the mass range  $dm$  and the luminosity range  $dL$  is

$$\Psi_{p2}(L, m) d \ln m d \ln L = \psi_{p2}(m, m_f) d \ln m d \ln m_f. \tag{3.3}$$

The PLF is then

$$\Psi_p(L) = \int_{m_{\text{min}}}^{m_{\text{max}}} d \ln m \Psi_{p2}(L, m), \tag{3.4}$$

$$= \int_{m_{\text{min}}}^{m_{\text{max}}} d \ln m \frac{\psi_{p2}[m, m_f(L, m)]}{|\partial \ln L / \partial \ln m_f|}, \tag{3.5}$$

where the limits of integration are such that  $m_\ell \leq m_f \leq m_u$  and  $m \leq m_f$ . The PMF and PLF depend on the history of the mass accretion rate,  $\dot{m}(m, m_f)$  (eq. 3.1) and therefore on the theory of star formation.

## 4. Testing Theories of Star Formation

### 4.1. Accretion Histories

We consider three different theories of star formation, plus one variant:

\* **Isothermal Sphere** (IS, Shu 1977)—Inside-out collapse of singular isothermal sphere:

$$\dot{m} = \dot{m}_{\text{IS}} = 1.54 \times 10^{-6} (T/10\text{K})^{3/2} M_\odot \text{ yr}^{-1}. \quad (4.1)$$

The formation time is proportional to the mass,  $t_f \propto m$ , so this accretion model is valid only for low-mass stars (Shu, Adams & Lizano 1987).

\* **Turbulent Core** (TC, McKee & Tan 2002, 2003)—Inside-out collapse of a turbulent core:

$$\dot{m} = \dot{m}_{\text{TC}} \left( \frac{m}{m_f} \right)^{1/2} m_f^{3/4}, \quad \text{with } \dot{m}_{\text{TC}} \propto \Sigma^{3/4}. \quad (4.2)$$

The accretion rate increases with both the protostellar mass,  $m$ , and the final mass,  $m_f$ . This model was developed for high-mass star formation, and is equivalent to the accretion rate in the Hennebelle & Chabrier (2008) theory of the IMF.

\* **Two-Component Turbulent Core** (2CTC)—Begins as isothermal accretion and evolves to turbulent accretion (similar to the TNT model of Myers & Fuller 1992):

$$\dot{m} = \left[ \dot{m}_{\text{IS}}^2 + \dot{m}_{\text{TC}}^2 \left( \frac{m}{m_f} \right) m_f^{3/2} \right]^{1/2}. \quad (4.3)$$

\* **Competitive Accretion** (CA, Zinnecker 1982, Bonnell *et al.* 1997)—Stars form in a common gas reservoir, usually accreting at the tidally limited Bondi rate,  $\dot{m} \propto m^{2/3}$ , and all having the same formation time:

$$\dot{m} = \dot{m}_{\text{CA}} \left( \frac{m}{m_f} \right)^{2/3} m_f, \quad \text{with } \dot{m}_{\text{CA}} \propto 1/(\text{free-fall time}). \quad (4.4)$$

### 4.2. Comparing the PMF and PLF with Observation

We adopt a truncated Chabrier (2005) IMF:

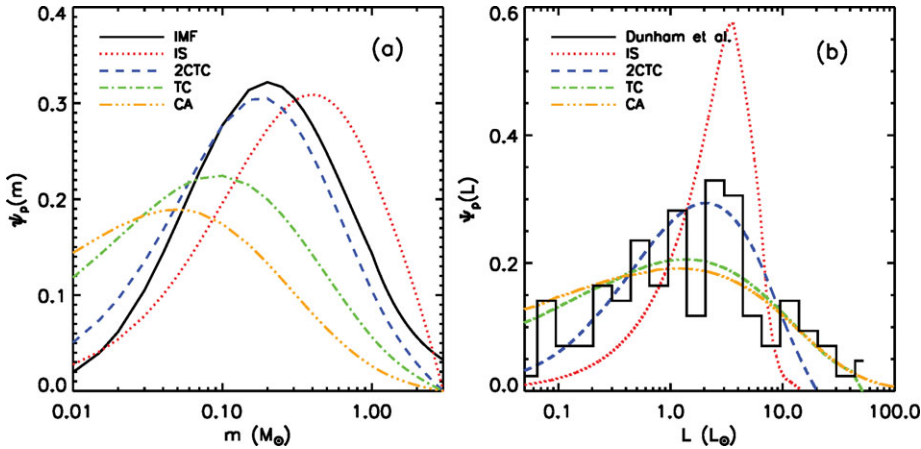
$$\psi(m_f) \propto \exp - \left[ \frac{\log^2(m_f/0.2)}{2 \times 0.55^2} \right] \quad (m_f \leq 1 M_\odot) \quad (4.5)$$

$$\propto m_f^{-1.35} \quad (1 M_\odot < m_f \leq m_u) \quad (4.6)$$

The Evans *et al.* (2009) sample of Class II YSOs has about 400 objects. In this sample, 9 YSOs are expected with masses exceeding  $3M_\odot$  if  $m_u \gg 3M_\odot$  (i.e., stars with masses much greater than  $3M_\odot$  are possible). Since none are seen, we adopt an upper cutoff  $m_u = 3M_\odot$  for the IMF.

Figure 1a illustrates the PMF (eq. 3.2) estimated with each of the accretion histories. As discussed by McKee & Offner (2010), the models predict very different mass distributions. For example, in the isothermal sphere case more massive protostars spend a





**Figure 1.** Left: The PMF,  $\psi_p(m)$ , for the four accretion models and the Chabrier IMF. Right: The PLF,  $\Psi_p(L)$ , for the same models assuming no tapering of the accretion rate, with  $F_{\text{high}} = 0.25$  and  $\langle t_f \rangle = 0.56$  Myr with the data from Dunham *et al.* (2010). Both panels adopt a cluster upper mass  $m_u = 3 M_\odot$ .

longer time accreting and consequently weight the distribution towards higher masses. In contrast, for competitive accretion, where all the protostars share the same protostellar lifetime, the significantly larger number of low-mass protostars shifts the PMF towards lower masses. Unfortunately, since we can't directly measure the protostellar masses, Figure 1a is not sufficient to observationally discriminate between the models.

For a more direct comparison, Offner & McKee (2010) calculate the PLF using the predicted mass functions in combination with a stellar evolution model (see Offner *et al.* 2009 for details). Figure 1b shows the PLF for each accretion model plotted with the extinction corrected luminosities from Dunham *et al.* (2010). The model curves assume that 25% of the mass is accreted during unseen bursts ( $F_{\text{high}} = 0.25$ ) and  $\langle t_f \rangle = 0.56$  Myr. This is equivalent to applying both the slow accretion and variable accretion solutions and thus likely represents a lower bound on the predicted luminosities. For comparison with Dunham *et al.* 2010, we adopt an upper bolometric luminosity uncertainty of 50% and use the uncorrected bolometric luminosities to set a lower error bound.

The distributions can be characterized by the mean, median, and standard deviation. For  $m_u = 3 M_\odot$  and  $L_{\text{min}} = 0.05 L_\odot$ , the fiducial models have means in the range  $2.5 L_\odot$  (2CTC) -  $3.6 L_\odot$  (CA), all a factor of  $\sim 1.5$ -2 below the observed value:  $5.3^{+2.6}_{-1.9} L_\odot$ . We find that the mean luminosities fall in a narrower range,  $2.6 L_\odot$  (2CTC)-  $3.4 L_\odot$  (IS), when the accretion rates taper off towards the end of the protostellar lifetime:

$$\dot{m} = \dot{m}_0(m, m_f) \left[ 1 - \left( \frac{t}{t_f} \right) \right], \tag{4.7}$$

where  $\dot{m}_0(m, m_f)$  is the untapered accretion rate. (Foster & Chevalier 1993 found that the accretion rate tapered off at late times in their 1D calculations, and Myers *et al.* 1998 included an exponential tapering in their models.) Only the non-tapered CA and tapered IS models fall within the uncertainty, suggesting that the adopted lifetime may be too high by as much as a factor of 2. In contrast, the fiducial medians, which range from  $0.8 L_\odot$  (CA) to  $2.5 L_\odot$  (IS), are in better agreement with the observed median of  $1.5^{+0.7}_{-0.4} L_\odot$ . The median of the TC ( $0.9 L_\odot$ ) and 2CTC ( $1.4 L_\odot$ ) models are within error and remain so even for a lifetime reduced by a factor up to 2.4 and 1.6, respectively. The observed standard deviation of  $\log L$ ,  $0.7^{+0.2}_{-0.1}$ , is consistent with the 2CTC (0.6), TC (0.7) and CA

(0.8) cases. The outcome of the comparison is sensitive to the values of  $F_{\text{high}}$  and  $f_{\text{acc}}$ , in addition to  $\langle t_f \rangle$ , and all have significant uncertainty. The parameter dependence may be reduced by comparing to the ratio of the median to the mean luminosity,  $0.3 \frac{+0.2}{-0.1}$ . This rules out the IS model.

## 5. Conclusions

The luminosity problem in low-mass star formation can be resolved if low-mass stars form slowly, over a period  $\sim 0.5$  Myr, as suggested by the results of Evans *et al.* (2009). Indeed, if some of the accretion energy is released mechanically ( $1 - f_{\text{acc}} \simeq 1/4$ ) and in unseen FU Ori outbursts ( $F_{\text{high}} \simeq 1/4$ ), then for most models the star formation time must be somewhat less than 0.5 Myr to be consistent with the observed protostellar luminosities. Different theories of star formation predict different protostellar mass functions, which are currently inaccessible to direct observation, and protostellar luminosity functions, which have been observed. The latter serves as an important metric to discriminate between theories of star formation.

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## References

- Andersen, M., Meyer, M. R., Greissl, J., & Aversa, A. 2008, *ApJL*, 683, L183  
 Bonnell, I. A., Bate, M. R., Clarke, C. J., & Pringle, J. E. 1997, *MNRAS*, 285, 201  
 Bonnell, I. A., Bate, M. R., Clarke, C. J., & Pringle, J. E. 2001, *MNRAS*, 323, 785  
 Bontemps, S., Andre, P., Terebey, S., & Cabrit, S. 1996, *A&A*, 311, 858  
 Foster, P. N. & Chevalier, R. A. 1993, *ApJ*, 416, 303  
 Hartmann, L., Cassen, P., & Kenyon, S. J. 1997, *ApJ*, 475, 770  
 Hennebelle, P. & Chabrier, G. 2008, *ApJ*, 684, 395  
 Henriksen, R., Andre, P., & Bontemps, S. 1997, *A&A*, 323, 549  
 Larson, R. B. 1969, *MNRAS*, 145, 271  
 McKee, C. F. & Ostriker, E. C. 2007, *ARAA*, 45, 565  
 McKee, C. F. & Tan, J. C. 2002, *Nature*, 416, 59  
 McKee, C. F. & Tan, J. C. 2003, *ApJ*, 585, 850  
 Myers, P. C., Adams, F. C., Chen, H. & Schaff, E. 1998, *ApJ*, 492, 703  
 Myers, P. C. & Fuller, G. A. 1992, *ApJ*, 396, 631  
 Offner, S. S. R., Klein, R. I., McKee, C. F., & Krumholz, M. R. 2009, 703, 131.  
 Ostriker, E. C. & Shu, F. H. 1995, *ApJ*, 447, 813  
 Penston, M. V. 1969, *MNRAS*, 144, 425  
 Schmeja, S. & Klessen, R. S. 2004, *A&A*, 419, 405  
 Shibata, K. & Uchida, Y. 1985, *PASJ*, 37, 31  
 Shu, F. H. 1977, *ApJ*, 214, 488  
 Stahler, S. W. 1988, *ApJ*, 332, 804  
 Tan, J. C. & McKee, C. F. 2004, *ApJ*, 603, 383  
 Zinnecker, H. 1982, *Annals of the New York Academy of Sciences*, 395, 226