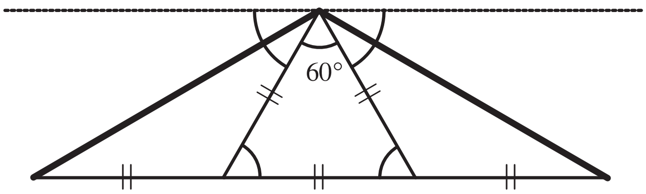
107.37 PWW: trisecting a line across an angle does not trisect the angle



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107.38 $c^2 = a^2 + bd$, a new proof of an extension of the Pythagorean theorem

In [1] the author provides a visual proof that generalises the typical rearrangement proof of the Pythagorean theorem. In the following diagram, we show a new proof of this generalisation. Indeed, with simple applications of the Pythagorean theorem, we have $c^2 = h^2 + (b + e)^2$, $h^2 = a^2 - e^2$ and $(b + e)^2 = bd + e^2$, where $d = b + 2e$. Therefore

$$c^2 = a^2 - e^2 + bd + e^2 = a^2 + bd.$$

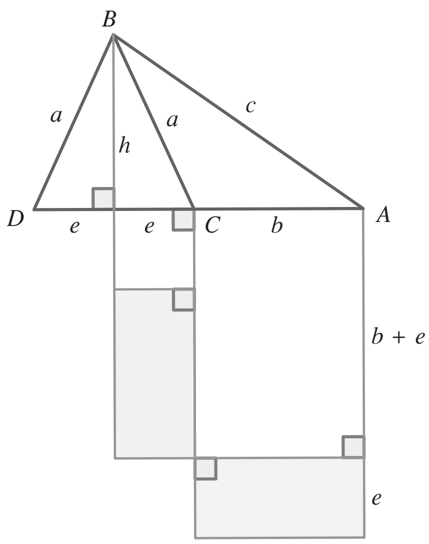

FIGURE 1

In other words, Figure 1 shows that:

Let ABC be an obtuse triangle, with C being the obtuse angle, under the usual convention that a, b, c are the lengths of the sides BC, CA, AB respectively. Let D be the point on AC produced such that BD has length a. Let d be the length of AD. Then $c^2 = a^2 + bd$.

In addition, the Figure shows that:

Let ABD be an acute triangle for which $c \geqslant a$, under the convention that a, d, c are the lengths of the sides BD, AD, AB respectively. Let C be the point on AD such that BC has length a. Let b be the length of AC. Then $c^2 = a^2 + bd$.

Since the relation $c^2 + a^2 + bd$ holds for both triangle ABC and ABD, it does not matter whether the angle opposite to the side AB is obtuse or acute. Furthermore, we note that $b = d$ if, and only if, the angle opposite to side c is a right angle.

Therefore, it seems desirable to formulate a synthetic statement that incorporates the previous ones and generalises the classic statement of the Pythagorean theorem. To this end, we remember that a line intersecting both a triangle's vertex and also the side that is opposite to that vertex is called *cevian* (from the Italian mathematician Giovanni Ceva (1647 – 1734), who proved a well-known theorem about cevians).

Using this definition, we can summarise the previous results as follows:

In any triangle, the square on the longest side is equal to the sum of the square on a second side with the rectangle whose dimensions are the third side and the projection of the first side on the third, obtained by the cevian congruent to the second side.

To conclude this Note, we need to clarify the second part of the statement given in [1, p. 521], because it contains a typo. Indeed, where it says: "$\cdot\cdot\cdot$ The diagrams also show that, given an acute triangle having sides of lengths a, b, d with $c \geqslant a$, …" we need to replace b with c. Moreover, as the reader can verify, the construction shown in [1] also works in the case $b \geqslant c$.

Therefore, we show below the correct statement and related diagrams, with some additional clarification:

Given an obtuse triangle having sides of lengths a, b, c with $c \geqslant a$, we have $c^2 = a^2 + bd$, where d is the length of the projection of c onto b, obtained by the cevian congruent to a. The diagrams also show that, given an acute triangle having sides of lengths a, c, d with $c \geqslant a$, we have $c^2 = a^2 + bd$, where b is the length of the projection of c onto d, obtained by the cevian congruent to a.

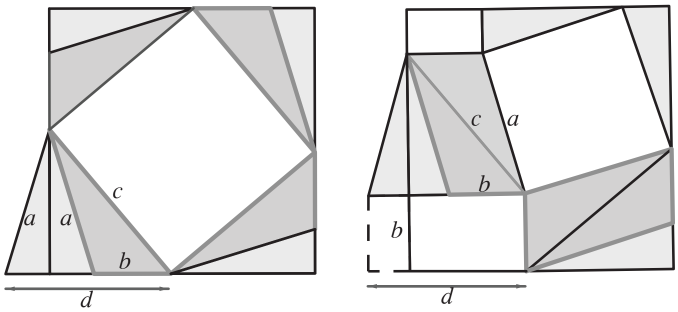

FIGURE 2

Acknowledgements

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Reference

1. F. Laudano, $c^2 = a^2 + bd$, a visual extension of the Pythagorean theorem, *Math. Gaz.* **105** (November 2021) pp. 520-521.

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107.39 Enomoto's problem in Wasan geometry

Japanese mathematics developed in the Edo era (1603-1868) is called *Wasan*. In this Note we consider a problem in Wasan geometry that appeared in a sangaku, which is a framed wooden board with geometric problems written on it. The figures of the problems were beautifully drawn in colour and the board was dedicated to a shrine or a temple. Today, sangaku is an iconic word for Wasan geometry. For a brief introduction of Wasan geometry, see [1]. In this Note, we consider the sangaku problem proposed by Enomoto (榎本信房) in 1807 [2], which can be stated as follows (see Figure 1):