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Erratum to 'Anosov Foliations and Cohomology'

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We regret to say that there is an error in the proof of theorem 1 of [**F**]. The algebras $E^*(U)$, $E^*(S)$ used there are not preserved by d, as claimed, save in the restricted case when U and S commute. A counterexample is the nilmanifold automorphism of [**AA**], Appendix 23. Thus the results of § 1 and § 2 are dubious, although the third section is unaffected.

As a partial correction, we consider the Lefschetz (zeta) function $\tilde{\zeta}$ of an Anosov automorphism $A: X \to X$ of a compact nilmanifold X and show that it behaves as if A were the Cartesian product of an expanding map and the 'inverse' of an expanding map. Let $\mathcal{N}$ be corresponding Lie algebra, S and U the stable and unstable subalgebras and $\alpha: \mathcal{N} \to \mathcal{N}$ the differential of A at the identity. As in [**F**], we can use Nomizu's theorem to identify the cohomology of X with the Lie algebra cohomology of $\mathcal{N}$ and we find

$$\tilde{\zeta} = \prod_i [\det I - t(\Lambda^i \alpha)]^{(-1)^{i+1}}.$$

Taking the divisor of this rational function we have, in the group ring $\mathbf{Z}\mathbf{C}^*$,

$$\operatorname{div}(\tilde{\zeta}) = -\prod_\lambda ([1]-[\lambda]^{-1}),$$

where λ runs over the eigenvalues of α with multiplicity. Grouping these eigenvalues into stable and unstable, we find

$$-\operatorname{div}(\tilde{\zeta}) = ([1]+\Sigma_s+(-1)^s[\lambda^{-1}\varepsilon_s]^{-1})([1]+\Sigma_u+(-1)^u[\lambda\varepsilon_u]^{-1}), \qquad (*)$$

where $\lambda^{-1}\varepsilon_s$, $\lambda\varepsilon_u$ are the eigenvalues of the Ruelle-Sullivan classes, $\varepsilon_s, \varepsilon_u \in \{\pm 1\}$, $\lambda = e^{h(A)}$, and where Σ_s, Σ_u are supported in the annuli $1<|z|<\lambda$, $\lambda^{-1}<|z|<1$ respectively. $(*)$ should be compared to the formula for the Lefschetz function $\tilde{\zeta}$ of a Cartesian product $f_1 \times f_2$ in terms of the Lefschetz functions $\tilde{\zeta}_1, \tilde{\zeta}_2$ of the factors:

$$-\operatorname{div}\tilde{\zeta} = (-\operatorname{div}\tilde{\zeta}_1)(-\operatorname{div}\tilde{\zeta}_2)$$

the formula for the Lefschetz function of an expanding map of degree d on a closed, oriented n-manifold:

$$-\operatorname{div}\tilde{\zeta} = [1]+\Sigma+(-1)^n[d], \operatorname{supp}\Sigma \subset \{1<|z|<|d|\}$$

and the formula for the Lefschetz function of the inverse f^{-1} of a diffeomorphism (or basic set)

$$-\operatorname{div}\tilde{\zeta}(f) = \Sigma \pm [\lambda] \quad \Rightarrow \quad -\operatorname{div}\tilde{\zeta}(f^{-1}) = \Sigma \pm [\lambda^{-1}].$$

One finds that $(*)$ is formally what would hold if $A = A_1 \times A_2^{-1}$ where A_1, A_2 are expanding maps (or expanding attractors) of degree $\varepsilon_u\lambda$, $\varepsilon_s\lambda$. Thus the divisor $-\text{div}\,\tilde{\zeta}$ factors according to the splitting $\mathcal{N} = S \oplus U$, even though the cohomology itself may not.

[AA] V. I. Arnold & A. Avez. *Ergodic Problems of Classical Mechanics* (Benjamin, 1968).
[F] D. Fried. Anosov foliations and cohomology. *Ergod. Th. & Dynam. Sys.* **6** (1986), 9–16.