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ON THE EXISTENCE OF EMBEDDED MINIMAL 2-SPHERES IN THE 3-SPHERE, ENDOWED WITH AN ARBITRARY METRIC

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In this thesis it is proved that in the 3-sphere endowed with any Riemannian metric (denoted by N), there exists an embedded, minimal 2-sphere.

Previously, the work of Sacks and Uhlenbeck [3] had shown that there exists a stationary (in general, not stable) immersion of S^2 into N. All other results concerning minimal immersions or embeddings of S^2 into a compact 3-manifold have excluded the case when the ambient space is topologically a 3-sphere.

First, by modifying the minimax techniques of Pitts [2], it is shown that there exists in N a stationary 2-varifold V which can be written as the (varifold) limit of embedded 2-spheres and which has certain local stability properties. Then, using these stability properties together with the recently developed regularity theorems of Almgren and Simon [1] and Schoen, Simon and Yau [4], one can prove that V has the form

$$V = \sum_{j=1}^{R} n_j \underline{\underline{v}}(M_j) \quad (n_j,\ R \in \{\text{integers}\})$$

where M_j $(j = 1, \ldots, R)$ are embedded, oriented, connected minimal surfaces in N and where $\underline{\underline{v}}(M_j)$ $(j = 1, \ldots, R)$ denotes the multiplicity

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one varifold associated with M_j .

Finally, each M_j is shown to be diffeomorphic to S^2 . The proof of this makes a more subtle use of the stability properties of V , together with the fact that V is the weak limit of embedded 2-spheres. Here it was necessary to make use of some previously unpublished results of Simon. The main results of the thesis will be published shortly in a joint paper with Simon.

References

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