# 25

# Dynamical Theory of Observation

## 25.1 Introduction

In this chapter we discuss how quantized detector network (QDN) theory can be extended to cover the creation and decommissioning of apparatus, from the perspective of observers and their laboratories. This extension will be referred to as *extended* QDN. It allows us to discuss bit power sets, laboratories, the universal register, contextual vacua, and the creation of quantized detector apparatus. This extended formalism is used to describe the Elitzur–Vaidman bomb-tester experiment and the Hardy paradox experiment.

A central concept running through this book is that of the *observer*, the enigmatic "*I*" of *I think therefore I am*. The problem is that, despite the many triumphs of quantum mechanics (QM), the physics of the observer and observation is still not well understood.

Regardless of how observers are defined and whether classical or quantum principles are involved, physicists generally believe that classical information in some form is extracted from systems under observation (SUOs) in actual physics experiments. In all branches of science, their language reflects this belief: experimentalists talk of measuring an electron's spin or the mass of a new particle, and so on.

The conceptual issues in quantum mechanics such as wave–particle duality, quantum interference, and nonlocality gave the first indication that all might not be well with this perspective. We need only to look at the photon concept to appreciate some of the problems with the idea that photons are particles (Paul, 2004). There are experiments where a photon (that is, a signal) is detected from a crystal, but the atom "from whence it came" cannot be identified, suggesting that a cooperative process is involved, rather like the phonon concept in the physics of crystals.

The problem we have with the photon-as-particle concept is the gap in logic. If the particle interpretation is taken literally, then obvious questions about the physical structure or its equivalent of such particles spring up. Does such

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a particle have a "surface"? If it has a surface, what is that surface made of? These and other simple (minded) questions quickly lead to the conclusion that the particle concept is a convenient and economical objectification of context. In other words, a useful illusion.

Some particle concepts appear better in this respect than others. For instance, fermion number is conserved in astrophysical processes, but photon number is not. Photons from the sun cannot be said to have traveled from the deep interior, whereas we could say so about neutrinos.

So much for the signals that observers detect. What about the observers themselves? It is surely too great an assumption to think of them as a mere auxiliary phenomenon in physics. Without observers, physics is a vacuous subject.

What seems to be missing is a comprehensive dynamical theory of observation that would treat observers and SUOs more on the same footing. In such a theory, observers would be subject to the same laws of physics as the SUOs that they were observing. Such a theory would be capable of accounting for the creation and annihilation of observers and their apparatus, as well as states of SUOs, because in the real world, that is what goes on and nothing lasts forever.

We do not have such a theory. That may come only once the physics of emergence has been much better understood. However, we can make a start with what we have at present in what we think may be a useful direction (Jaroszkiewicz, 2010). In this chapter, we develop an extension of the binary truth value concepts explored in earlier chapters, aimed at describing the possible states of apparatus in a more general form than having just two states, *yes* or *no*.

## 25.2 Power Bits

We have up to this point identified the two possible normal signal states, *ground* and *signal*, of a functioning binary detector as the two elements of a bit, denoted $\mathbf{0}$ and $\mathbf{1}$, respectively. As we have mentioned, however, bits are not vector spaces and there seems to be no meaning to the addition of bit state $\mathbf{0}$ to bit state $\mathbf{1}$, or even of the multiplication of a bit state by a real or complex number.

There is in fact a way of defining bit state addition, of a kind, in terms of set theory. We recall that the *power set* $\mathcal{P}(S)$ of a set is the set of all possible subsets of $S$ including the empty set $\emptyset$ and $S$ itself. Recall also that the number of elements in the power set of a set of cardinality $c$ is $2^c$. Therefore, we expect the power set $\mathcal{P}(B)$ of a bit $B$ to have four distinct, nonempty elements. These are

$$\mathcal{P}(B) = \{\underset{\sim}{\mathbf{0}}, \underset{\sim}{\mathbf{1}}, \underset{\sim}{\mathbf{2}}, \underset{\sim}{\mathbf{3}}\}, \tag{25.1}$$

where we define

$$\underset{\sim}{\mathbf{0}} \equiv \{\mathbf{0}\}, \quad \underset{\sim}{\mathbf{1}} \equiv \{\mathbf{1}\}, \quad \underset{\sim}{\mathbf{2}} \equiv \{\mathbf{0}, \mathbf{1}\}, \quad \underset{\sim}{\mathbf{3}} \equiv \{\emptyset\}. \tag{25.2}$$

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In this scheme, the set $\{\emptyset\}$ is a nontrivial element of $\mathcal{P}(B)$ and counts as one element of the power set. We shall refer to the power set of a bit as a *power bit*.

In this chapter, we shall work in terms of the elements of $\mathcal{P}(B)$ rather than with the elements of $B$ itself, identifying elements $\underset{\sim}{\mathbf{0}}$ and $\underset{\sim}{\mathbf{1}}$ of $\mathcal{P}(B)$ as synonymous with bit states $\mathbf{0}$ and $\mathbf{1}$ of $B$, respectively. The value of using $\mathcal{P}(B)$ rather than $B$ itself is that the elements of the former are sets, so we can use the set properties of *union* and *intersection* to make propositions about power bits equivalent to those in the algebra of sets (Howson, 1972).

**Interpretation**

Before we proceed further, however, we need to resolve the following problem: the power set $\mathcal{P}(B)$ of a bit $B$ appears to have too many elements for a physical interpretation. Logic suggests that only the elements $\underset{\sim}{\mathbf{0}}$ and $\underset{\sim}{\mathbf{1}}$ of the power set are actually needed in an experiment. What could the elements $\underset{\sim}{\mathbf{2}}$ and $\underset{\sim}{\mathbf{3}}$ represent?

It has been our experience that many such questions in QDN are answered by looking at the situation in question and identifying any hidden assumptions. Consider an experimentalist who is going into a laboratory in order to determine the signal status of a certain detector. Even before they could get an answer, one glaring fact has to be true: that the detector actually *exists.*

Now whether or not a detector actually exists in a laboratory ("existence" being defined as a detector actually being found and recognized as such by an observer coming into that laboratory) is surely an empirical question, the answer to which the observer cannot assume is "*yes, it exists*," *before* they enter the laboratory. This then is where one of our two extra elements of the power set $\mathcal{P}(B)$ finds a natural interpretation. We interpret the element $\underset{\sim}{\mathbf{3}} \equiv \{\emptyset\}$ as a state of physical nonexistence, in the laboratory, of the detector in question. We shall call it the *void state* of a detector.

It will undoubtedly seem strange to assign a state in our formalism to a condition of nonexistence of a detector. But if we want to construct a theory where apparatus can be created or destroyed, then that is precisely what we have to do.

As for the remaining element, $\underset{\sim}{\mathbf{2}} \equiv \{\mathbf{0}, \mathbf{1}\}$, of the power set $\mathcal{P}(\mathcal{B})$, the ambiguity in the elements it has ($\mathbf{0}$ and $\mathbf{1}$) fits in nicely with an obvious physical condition that a detector could be in: a state where the apparatus exists physically but is not operating normally, to the extent that it is not registering either a ground state or a signal state. For instance, such a detector could be faulty. Or it could have been deliberately taken off-line (or decommissioned). We shall call such a condition of a detector the *faulty state*, the *off-line state*, or the *decommissioned state*.

Given the above four states of a detector, we can find four natural power bit questions, denoted $\widetilde{\boldsymbol{i}}$, $i = 0, 1, 2, 3$. These are explained in Table 25.1, along with the answers they induce when asked of each of the four power bit states.

Table 25.1 *The extended QDN questions and answers*

| **Question in words** | **Symbol** | $\underset{\sim}{\boldsymbol{0}}$ | $\underset{\sim}{\boldsymbol{1}}$ | $\underset{\sim}{\boldsymbol{2}}$ | $\underset{\sim}{\boldsymbol{3}}$ |
|---|---|---|---|---|---|
| Is this detector in its ground state? | $\widetilde{\boldsymbol{0}}$ | 1 | 0 | 0 | 0 |
| Is this detector in its signal state? | $\widetilde{\boldsymbol{1}}$ | 0 | 1 | 0 | 0 |
| Is this detector off-line? | $\widetilde{\boldsymbol{2}}$ | 0 | 0 | 1 | 0 |
| Is this detector in its void state? | $\widetilde{\boldsymbol{3}}$ | 0 | 0 | 0 | 1 |

We may summarize these answers by the rule

$$\widetilde{\boldsymbol{i}}\underset{\sim}{\boldsymbol{j}} = \delta^{ij}, \qquad 0 \leqslant i, j \leqslant 3. \tag{25.3}$$

## 25.3 Power Bit Operators

A *power bit operator* is any mapping from the power set $\mathcal{P}(B)$ back into the power set. Given an element $\underset{\sim}{\boldsymbol{i}}$ of $\mathcal{P}(B)$ and a power bit operator $\underset{\cdots}{\boldsymbol{O}}$, then we denote the value of the operator's action on $\underset{\sim}{\boldsymbol{i}}$ by $\underset{\cdots}{\boldsymbol{O}}\,\underset{\sim}{\boldsymbol{i}}$. There is a total of $4^4 = 256$ different bit operators and only a few will be of use to us.

## 25.4 Matrix Representation

A useful way of representing power bit operators is via matrices. The elements $\underset{\sim}{\boldsymbol{i}}$ of $\mathcal{P}(B)$ may be represented by column matrices $[\underset{\sim}{\boldsymbol{i}}]$ given by

$$\underset{\sim}{\boldsymbol{0}} \rightleftharpoons [\underset{\sim}{\boldsymbol{0}}] \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \underset{\sim}{\boldsymbol{1}} \rightleftharpoons [\underset{\sim}{\boldsymbol{1}}] \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \text{and so on.} \tag{25.4}$$

We represent the action of bit operator $\underset{\cdots}{\boldsymbol{O}}$ on $\underset{\sim}{\boldsymbol{i}}$ by the action of a *power bit matrix* $[\underset{\cdots}{\boldsymbol{O}}]$ on a column matrix $[\underset{\sim}{\boldsymbol{i}}]$, such that

$$\underset{\cdots}{\boldsymbol{O}}\,\underset{\sim}{\boldsymbol{i}} \rightleftharpoons [\underset{\cdots}{\boldsymbol{O}}][\underset{\sim}{\boldsymbol{i}}] \equiv [\underset{\cdots}{\boldsymbol{O}}\,\underset{\sim}{\boldsymbol{i}}]. \tag{25.5}$$

In this matrix representation the dual elements $\widetilde{\boldsymbol{i}}$ are represented by the row matrices

$$\begin{aligned} \widetilde{\boldsymbol{0}} &\rightleftharpoons \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad \widetilde{\boldsymbol{1}} \rightleftharpoons \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \\ \widetilde{\boldsymbol{2}} &\rightleftharpoons \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \quad \widetilde{\boldsymbol{3}} \rightleftharpoons \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{25.6}$$

This is consistent with the question and answer relations (25.3).

We can use the power bit matrix representation to define the operational meaning of the dyadics $\underset{\sim}{\boldsymbol{i}}\widetilde{\boldsymbol{j}}$. These can then serve as formal basis elements in

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the expansion of power bit operators. Given a power bit operator defined by (25.5), we can write it as the formal (dyadic) expression $\underset{\dots}{\boldsymbol{O}} = \sum_{i=0}^{3} \underset{\dots}{\boldsymbol{O}}\,\underset{\sim}{\boldsymbol{i}}\,\widetilde{\boldsymbol{i}}$.

Products of power bit operators are defined in the natural way: given power bit operators $\underset{\dots}{\boldsymbol{M}}$, $\underset{\dots}{\boldsymbol{N}}$, we define their "product" $\underset{\dots}{\boldsymbol{M}}\,\underset{\dots}{\boldsymbol{N}}$ by its action on any element $\underset{\sim}{\boldsymbol{i}}$ of the power set $\mathcal{P}(B)$ according to the rule $(\underset{\dots}{\boldsymbol{M}}\,\underset{\dots}{\boldsymbol{N}})\underset{\sim}{\boldsymbol{i}} \equiv \underset{\dots}{\boldsymbol{M}}\{\underset{\dots}{\boldsymbol{N}}\,\underset{\sim}{\boldsymbol{i}}\}$. This product rule is associative but not commutative, which can be seen from the matrix representation.

## 25.5 Special Operators

In the following, we have to take into account that a bit power set does not contain a *zero* element, because such a set is not a vector space. The void element $\underset{\sim}{\boldsymbol{3}}$ is not physically meaningless quantity, either: it represents a definite state in a laboratory. Therefore, the bit operators we discuss in this section cannot map elements of a bit power set to any state other than $\underset{\sim}{\boldsymbol{0}}$, $\underset{\sim}{\boldsymbol{1}}$, $\underset{\sim}{\boldsymbol{2}}$, or $\underset{\sim}{\boldsymbol{3}}$.

The operators we define below have been constructed on the basis of what seems reasonable to us. The following bit operators seem physically reasonable, practically useful, and unavoidable in the kind of formalism we are aiming for.

### Identity $\underset{\dots}{\boldsymbol{I}}$

This operator maps every element back into itself, i.e., $\underset{\dots}{\boldsymbol{I}}\,\underset{\sim}{\boldsymbol{i}} = \underset{\sim}{\boldsymbol{i}}$. Its matrix elements are given by the Kronecker delta, that is, $[\underset{\dots}{\boldsymbol{I}}]_{ij} = \delta_{ij}$.

### Annihilator $\underset{\dots}{\boldsymbol{Z}}$

This operator maps any element $\underset{\sim}{\boldsymbol{i}}$ of the power set $\mathcal{P}(B)$ into the void state $\underset{\sim}{\boldsymbol{3}} \equiv \{\emptyset\}$, that is, $\underset{\dots}{\boldsymbol{Z}}\,\underset{\sim}{\boldsymbol{i}} = \underset{\sim}{\boldsymbol{3}}$, $i = 0, 1, 2, 3$. Its matrix representation is

$$[\underset{\dots}{\boldsymbol{Z}}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}. \tag{25.7}$$

The annihilator has a fundamental role in our theory: it represents the process of destroying and removing from the laboratory all traces of an already physically existing detector.

### The Power Bit Signal Projection Operators $\underset{\dots}{\boldsymbol{P}}$ and $\underset{\dots}{\widehat{\boldsymbol{P}}}$

These operators have the action

$$\begin{aligned} &\underset{\dots}{\boldsymbol{P}}\underset{\sim}{\boldsymbol{0}} = \underset{\sim}{\boldsymbol{0}}, \quad \underset{\dots}{\boldsymbol{P}}\underset{\sim}{\boldsymbol{1}} = \underset{\sim}{\boldsymbol{3}}, \quad \underset{\dots}{\boldsymbol{P}}\underset{\sim}{\boldsymbol{2}} = \underset{\sim}{\boldsymbol{3}}, \quad \underset{\dots}{\boldsymbol{P}}\underset{\sim}{\boldsymbol{3}} = \underset{\sim}{\boldsymbol{3}}, \\ &\underset{\dots}{\widehat{\boldsymbol{P}}}\underset{\sim}{\boldsymbol{0}} = \underset{\sim}{\boldsymbol{3}}, \quad \underset{\dots}{\widehat{\boldsymbol{P}}}\underset{\sim}{\boldsymbol{1}} = \underset{\sim}{\boldsymbol{1}}, \quad \underset{\dots}{\widehat{\boldsymbol{P}}}\underset{\sim}{\boldsymbol{2}} = \underset{\sim}{\boldsymbol{3}}, \quad \underset{\dots}{\widehat{\boldsymbol{P}}}\underset{\sim}{\boldsymbol{3}} = \underset{\sim}{\boldsymbol{3}}, \end{aligned} \tag{25.8}$$

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so their matrix representations are

$$[\underset{\cdots}{\boldsymbol{P}}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}, \quad [\underset{\cdots}{\widehat{\boldsymbol{P}}}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}. \tag{25.9}$$

These operators are idempotent, namely, $\underset{\cdots}{\boldsymbol{P}}\underset{\cdots}{\boldsymbol{P}} = \underset{\cdots}{\boldsymbol{P}}$, $\underset{\cdots}{\widehat{\boldsymbol{P}}}\underset{\cdots}{\widehat{\boldsymbol{P}}} = \underset{\cdots}{\widehat{\boldsymbol{P}}}$ and *power bit orthogonal*, namely, $\underset{\cdots}{\boldsymbol{P}}\underset{\cdots}{\widehat{\boldsymbol{P}}} = \underset{\cdots}{\widehat{\boldsymbol{P}}}\underset{\cdots}{\boldsymbol{P}} = \underset{\cdots}{\boldsymbol{Z}}$. In this context, the annihilator $\underset{\cdots}{\boldsymbol{Z}}$ plays the role of a zero element.

### The Power Bit Signal Creation and Annihilation Operators $\underset{\cdots}{\boldsymbol{A}}$, $\underset{\cdots}{\widehat{\boldsymbol{A}}}$

These operators are defined principally by their action on the normal signal states $\underset{\sim}{\mathbf{0}}$ and $\underset{\sim}{\mathbf{1}}$:

$$\begin{aligned} &\underset{\cdots}{\boldsymbol{A}}\underset{\sim}{\mathbf{0}} = \underset{\sim}{\mathbf{3}}, \quad \underset{\cdots}{\boldsymbol{A}}\underset{\sim}{\mathbf{1}} = \underset{\sim}{\mathbf{0}}, \quad \underset{\cdots}{\boldsymbol{A}}\underset{\sim}{\mathbf{2}} = \underset{\sim}{\mathbf{3}}, \quad \underset{\cdots}{\boldsymbol{A}}\underset{\sim}{\mathbf{3}} = \underset{\sim}{\mathbf{3}}, \\ &\underset{\cdots}{\widehat{\boldsymbol{A}}}\underset{\sim}{\mathbf{0}} = \underset{\sim}{\mathbf{1}}, \quad \underset{\cdots}{\widehat{\boldsymbol{A}}}\underset{\sim}{\mathbf{1}} = \underset{\sim}{\mathbf{3}}, \quad \underset{\cdots}{\widehat{\boldsymbol{A}}}\underset{\sim}{\mathbf{2}} = \underset{\sim}{\mathbf{3}}, \quad \underset{\cdots}{\widehat{\boldsymbol{A}}}\underset{\sim}{\mathbf{3}} = \underset{\sim}{\mathbf{3}}, \end{aligned} \tag{25.10}$$

which gives the matrix representations

$$[\underset{\cdots}{\boldsymbol{A}}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}, \quad [\underset{\cdots}{\widehat{\boldsymbol{A}}}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}. \tag{25.11}$$

These operators are *power bit nilpotent*, that is, $\underset{\cdots}{\boldsymbol{A}}\underset{\cdots}{\boldsymbol{A}} = \underset{\cdots}{\widehat{\boldsymbol{A}}}\underset{\cdots}{\widehat{\boldsymbol{A}}} = \underset{\cdots}{\boldsymbol{Z}}$, with $\underset{\sim}{\boldsymbol{Z}}$ once again playing the role of a zero element.

The product rules for the operators $\underset{\cdots}{\boldsymbol{P}}, \underset{\cdots}{\widehat{\boldsymbol{P}}}, \underset{\cdots}{\boldsymbol{A}}$, and $\underset{\cdots}{\widehat{\boldsymbol{A}}}$ are given in Table 25.2. Comparison with Table 4.1 shows that these tables are isomorphic, provided the zero operator $\boldsymbol{Z}$ in Table 4.1 is identified with the annihilator $\underset{\cdots}{\boldsymbol{Z}}$ in Table 25.2.

### Construction Operator $\underset{\cdots}{\boldsymbol{C}}$

This operator acts on every element $\underset{\sim}{\boldsymbol{i}}$ of power bit set and sets it to the signal ground state in readiness for observation, i.e., $\underset{\cdots}{\boldsymbol{C}}\underset{\sim}{\boldsymbol{i}} = \underset{\sim}{\mathbf{0}}$, $i = 0, 1, 2, 3$. There are

Table 25.2 *The product table for the special operators*

| | $\underset{\cdots}{\boldsymbol{P}}$ | $\underset{\cdots}{\widehat{\boldsymbol{P}}}$ | $\underset{\cdots}{\boldsymbol{A}}$ | $\underset{\cdots}{\widehat{\boldsymbol{A}}}$ |
|---|---|---|---|---|
| $\underset{\cdots}{\boldsymbol{P}}$ | $\underset{\cdots}{\boldsymbol{P}}$ | $\underset{\cdots}{\boldsymbol{Z}}$ | $\underset{\cdots}{\boldsymbol{A}}$ | $\underset{\cdots}{\boldsymbol{Z}}$ |
| $\underset{\cdots}{\widehat{\boldsymbol{P}}}$ | $\underset{\cdots}{\boldsymbol{Z}}$ | $\underset{\cdots}{\widehat{\boldsymbol{P}}}$ | $\underset{\cdots}{\boldsymbol{Z}}$ | $\underset{\cdots}{\widehat{\boldsymbol{A}}}$ |
| $\underset{\cdots}{\boldsymbol{A}}$ | $\underset{\cdots}{\boldsymbol{Z}}$ | $\underset{\cdots}{\boldsymbol{A}}$ | $\underset{\cdots}{\boldsymbol{Z}}$ | $\underset{\cdots}{\boldsymbol{P}}$ |
| $\underset{\cdots}{\widehat{\boldsymbol{A}}}$ | $\underset{\cdots}{\widehat{\boldsymbol{A}}}$ | $\underset{\cdots}{\boldsymbol{Z}}$ | $\underset{\cdots}{\widehat{\boldsymbol{P}}}$ | $\underset{\cdots}{\boldsymbol{Z}}$ |

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two scenarios. If a detector is in its void state, then it does not exist, so the action of the construction operator represents the physical construction of that detector, set up in its ground state in the laboratory, prior to any experiment. It is assumed that facilities exist in the laboratory for this. Alternatively, if the detector already exists, then the construction operator resets it to its ground state if it is normal or repairs it and sets it to its ground state if it is faulty.

The construction operator is represented by the matrix

$$[\underset{\dots}{\boldsymbol{C}}] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{25.12}$$

**Decommissioning Operator $\underset{\dots}{\boldsymbol{D}}$**

This operator represents the action of decommissioning an already existing detector, setting it into its decommissioned state $\underset{\sim}{\mathbf{2}}$. This operator does *not* reset states $\underset{\sim}{\mathbf{0}}$, $\underset{\sim}{\mathbf{1}}$, or $\underset{\sim}{\mathbf{2}}$ to the void state $\underset{\sim}{\mathbf{3}}$ because in the real world, there will invariably be some remaining information in the form of *debris* that will inform the observer that apparatus has been decommissioned. This is an important feature of our discussion toward the end of this chapter of the Elitzur–Vaidman bomb-tester experiment and Hardy's paradox experiment.

The decommissioning operator is represented by the matrix

$$[\underset{\dots}{\boldsymbol{D}}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{25.13}$$

## 25.6 The Laboratory

In extended QDN it is assumed that an observer exists in a physical environment referred to as the *laboratory*, $\Lambda$. This will have the facilities for the construction or introduction of apparatus consisting of a number of detectors. At any given stage $\Sigma_n$, the observer will associate a state known as a *generalized labstate* to the collection of detectors at that time. This state could be a pure state or a mixed state. We shall restrict our attention in this chapter to pure labstates for reasons of space.

Extended QDN is based on what really happens in laboratories. Suppose an observer walks into a laboratory, intending to look at a given detector. There are the following four classical scenarios.

**No Detector**

The detector in question does not exist. The observer will see this; it is an empirical fact that has physical significance. This absence is represented by extended labstate $\underset{\sim}{\mathbf{3}}$.

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**Faulty Detector**
The detector in question exists physically, but is faulty, off-line, or decommissioned. This situation is represented by extended labstate $\underset{\sim}{\mathbf{2}}$.

**Ground State**
The detector in question exists physically, is in perfect working order, and is in its ground state. This is represented by extended labstate $\underset{\sim}{\mathbf{0}}$.

**Signal State**
The detector in question exists physically, is in perfect working order, and is in its signal state. This is represented by extended labstate $\underset{\sim}{\mathbf{1}}$.

## 25.7 The Universal Register

The power set approach to detectors allows us to think of an absence of a detector in $\Lambda$ as an observable fact that can be representable mathematically. The state corresponding to an absent detector is represented by the element $\underset{\sim}{\mathbf{3}}$ of its associated power set. We can do this for any number of absent detectors. Therefore, we can represent a complete absence of any detectors whatsoever by an infinite collection of such elements.

**Remark 25.1** This is one of the very few places in QDN that we permit an infinity to enter the discussion; it costs us nothing in material terms and is a purely theoretical construct that nevertheless represents a very real state of existence of a laboratory, *before* any detectors have been constructed in it.

Such a condition corresponds to an observer without any apparatus, i.e., an empty laboratory. We denote this labstate by the symbol $\underset{\sim}{\emptyset}$ and call it the *information void*, or just the *void*. It represents a potential for the existence of any detector or detectors, relative to a given observer.

If the observer's laboratory $\Lambda$ is in its void state $\underset{\sim}{\emptyset}$, that does not mean that the laboratory $\Lambda$ or the observer do not exist, or that there are no systems under observation in $\Lambda$. It means simply that the observer has no current means of acquiring any information. An empty laboratory devoid of any detectors is a physically meaningful concept, but one with no interesting empirical content.

The information void can be thought of as one element in an infinite set called the *universal register* $\mathbf{\Omega}$, the Cartesian product of an infinite number of bit power sets. We write

$$\underset{\sim}{\emptyset} \equiv \prod_{\alpha}^{\infty} \underset{\sim}{\mathbf{3}}^{\alpha}, \in \mathbf{\Omega} \equiv \prod_{\alpha}^{\infty} \mathcal{P}(B^{\alpha}), \tag{25.14}$$

where the index $\alpha$ could in principle be discrete, continuous, or a combination of both.

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The cardinality of the universal register is a measure of how many power sets could belong to it. That depends on the imagination, and may be assumed to be infinity. Precisely what sort of Cantorian cardinality it should be is not clear, but that is a vacuous concept anyway. If we thought in terms of $\Lambda$ sitting in continuous space, then we expect at least the cardinality, $\mathfrak{c}$, of the continuum. However, that is a metaphysical statement, because there would not be enough energy in the universe to create a continuum of detectors.[1] What helps us immeasurably here is that real observers can only ever deal with finite numbers of detectors. We can generally ignore all nonexistent potential detectors, but keep in mind that an absence of a detector can be significant in some circumstances, such as in the Renniger thought experiment (Renniger, 1953).

The product notation in (25.14) reflects the relationship between collections of power sets and the tensor products of qubit spaces that we encounter in the quantum version of this approach, discussed later. In these products, ordering is not significant, since labels keep track of the various terms. An arbitrary classical labstate $\underset{\sim}{\boldsymbol{\Psi}}_n$ in the universal register $\boldsymbol{\Omega}_n$ at stage $\Sigma_n$ will be of the form $\prod_\alpha^\infty \underset{\sim}{\boldsymbol{\psi}}{}_n^\alpha$, where $\underset{\sim}{\boldsymbol{\psi}}{}_n^\alpha$ is one of the four elements of $\mathcal{P}(B_n^\alpha) \equiv \{\underset{\sim}{\boldsymbol{0}}{}_n^\alpha, \underset{\sim}{\boldsymbol{1}}{}_n^\alpha, \underset{\sim}{\boldsymbol{2}}{}_n^\alpha, \underset{\sim}{\boldsymbol{3}}{}_n^\alpha\}$.

Operators acting on universal register states will be denoted in blackboard bold font with three dots below, and act as follows. If $\underset{\dots}{\boldsymbol{O}}{}_n^\alpha$ is a power bit operator acting on elements of $\mathcal{P}(B_n^\alpha)$, then $\underset{\dots}{\mathbb{O}}{}_n \equiv \prod_\alpha^\infty \underset{\dots}{\boldsymbol{O}}{}_n^\alpha$ acts on an arbitrary classical state $\underset{\sim}{\boldsymbol{\Psi}}_n \equiv \prod_\alpha^\infty \underset{\sim}{\boldsymbol{\psi}}{}_n^\alpha$ in $\boldsymbol{\Omega}_n$ according to the rule

$$\underset{\dots}{\mathbb{O}}{}_n \underset{\sim}{\boldsymbol{\Psi}}_n = \left\{\prod_\alpha^\infty \underset{\dots}{\boldsymbol{O}}{}_n^\alpha\right\}\left\{\prod_\beta^\infty \underset{\sim}{\boldsymbol{\psi}}{}_n^\beta\right\} \equiv \prod_\alpha^\infty \underset{\dots}{\boldsymbol{O}}{}_n^\alpha \underset{\sim}{\boldsymbol{\psi}}{}_n^\alpha. \tag{25.15}$$

For every classical register state $\underset{\sim}{\boldsymbol{\Psi}} \equiv \prod_\alpha^\infty \underset{\sim}{\boldsymbol{\psi}}{}^\alpha$ there will be a corresponding dual register state $\widetilde{\boldsymbol{\Psi}} \equiv \prod_\alpha^\infty \widetilde{\boldsymbol{\psi}}{}^\alpha$, where $\widetilde{\boldsymbol{\psi}}{}^\alpha$ is dual to $\underset{\sim}{\boldsymbol{\psi}}{}^\alpha$. Classical register states including the void satisfy the orthonormality condition

$$\widetilde{\boldsymbol{\Phi}}\underset{\sim}{\boldsymbol{\Psi}} \equiv \left\{\prod_\alpha^\infty \widetilde{\boldsymbol{\phi}}{}^\alpha\right\}\prod_\beta^\infty \underset{\sim}{\boldsymbol{\psi}}{}^\beta = \prod_\alpha^\infty \widetilde{\boldsymbol{\phi}}{}^\alpha \underset{\sim}{\boldsymbol{\psi}}{}^\alpha = \prod_\alpha^\infty \delta^{\phi^\alpha \psi^\alpha}. \tag{25.16}$$

Classical register states $\underset{\sim}{\boldsymbol{\Phi}}$, $\underset{\sim}{\boldsymbol{\Psi}}$ that differ in at least one bit power set element therefore satisfy the rule $\widetilde{\boldsymbol{\Phi}}\underset{\sim}{\boldsymbol{\Psi}} = 0$.

## 25.8 Contextual Vacua

In conventional classical mechanics or Schrödinger–Dirac quantum mechanics, empty space is generally not represented by any specific mathematical object.

[1] In other words, the Universe could not observe itself completely.

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In relativistic quantum field theory (RQFT), however, empty space is represented by the *vacuum*, a normalized vector in an infinite-dimensional Hilbert space. It has physical properties such as zero total momentum, zero total electric charge, and so on, which although bland are physically significant attributes nevertheless. In RQFT, particle states are represented by the application of particle creation operators to the vacuum.

In our approach we encounter an analogous concept. Starting with an information void $\underset{\sim}{\emptyset}_n$, we represent the construction of a collection of detectors in the laboratory $\Lambda_n$ at stage $\Sigma_n$ by the application of a corresponding number of construction operators $\underset{\cdots}{\mathbb{C}}{}^i_n$ acting on that information void.

**Example 25.2** The construction of detector $i_n$ in its signal ground state in a previously empty laboratory is represented by the element $\underset{\cdots}{\mathbb{C}}{}^i_n \underset{\sim}{\emptyset}_n$ of the universal register $\mathbf{\Omega}_n$ at stage $\Sigma_n$, where

$$\underset{\cdots}{\mathbb{C}}{}^i_n \equiv \left\{ \prod_{j \neq i}^{\infty} \underset{\cdots}{\boldsymbol{I}}{}^j_n \right\} \underset{\cdots}{\boldsymbol{C}}{}^i_n. \tag{25.17}$$

More generally, an extended labstate $\underset{\sim}{\mathbf{0}}{}^{[r]}_n$ consisting of a number $r$ of detectors each in its signal ground state at stage $\Sigma_n$ is given by

$$\underset{\sim}{\mathbf{0}}{}^{[r]}_n \equiv \underset{\cdots}{\mathbb{C}}{}^1_n \underset{\cdots}{\mathbb{C}}{}^2_n \cdots \underset{\cdots}{\mathbb{C}}{}^r_n \underset{\sim}{\emptyset}_n, \tag{25.18}$$

where without loss of generality we label the detectors involved from 1 to $r$. Such a state will be said to be a *rank-$r$ ground state*, or *contextual vacuum state*.

We can now draw an analogy between the vacuum of RQFT and the rank-$r$ ground states in our formalism. The physical three-dimensional space of conventional physics would correspond to a contextual vacuum of extremely large rank, *if* physical space were relevant to the experiment. This would be the case for discussions involving particle scattering or gravitation, for example. For many experiments, however, such as the Stern–Gerlach experiment and quantum optics networks, physical space would be considered part of the relative external context and therefore could be ignored for the purposes of those experiments. It all depends on what the observer is trying to do.

In the real world there is more than one observer, so a theory of observation should take account of that fact. That is readily done in extended QDN. For example, the mutual contextual vacuum for two distinct observers $A$ and $B$ for which some commonality of time had been established would be represented by an element in $\mathbf{\Omega}_n$ of the form

$$\underset{\sim}{\mathbf{0}}{}^{A,B}_n \equiv \underset{\cdots}{\mathbb{C}}{}^{A,1}_n \underset{\cdots}{\mathbb{C}}{}^{A,2}_n \cdots \underset{\cdots}{\mathbb{C}}{}^{A,r_A}_n \underset{\cdots}{\mathbb{C}}{}^{B,1}_n \underset{\cdots}{\mathbb{C}}{}^{B,2}_n \cdots \underset{\cdots}{\mathbb{C}}{}^{B,r_B}_n \underset{\sim}{\emptyset}_n, \tag{25.19}$$

and so on. If subsequent dynamics was such that the detectors of observer $A$ never sent signals to those of observer $B$ and vice versa, then to all intents and

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purposes we could discuss each observer as if we were primary and they were separate secondary observers. If on the other hand some signals did pass between them, then that would be equivalent to having only one secondary observer.

If no commonality of time or other context has been established between primary observers, then there can be no physical meaning to (25.19). This is an important point in cosmology, where there are frequent discussions about multiple universe "bubbles" beyond the limits of observation. The mere fact that astronomers have received light from extremely distant galaxies establishes a context between the signal sources in those galaxies and the detectors used by the astronomers and validates the use of general relativity (GR) all the way to those regions of spacetime. If, on the other hand, no such signals have been received, then there is no such context. Therefore, relative to astronomers today, the universe beyond the horizon of observation can be meaningfully represented only by an information void, not a spatial vacuum. Something may be there, but we should not discuss it as if we had access to any form of information about it, such as its spacetime structure.

Much the same concern must be raised about black hole physics. That requires careful analysis of the contextual relation between observers outside the critical (Schwarzschild) radius and those who were assumed to be inside it, if any such context can be found. An event horizon is a boundary between observers on one side and observers on the other. For all intents and purposes, observers on one side of such a horizon have to regard those on the other side as in an information void.

This raises deep questions and concerns about the use of coordinate patches in general relativity and black hole physics that may be related to the problems of "quantizing gravity." We have asserted throughout this book that quantum mechanics is much more like a theory of entitlement than about the structure of SUOs. With this in mind, we should always ask the question: how do we *know* this or that? Consider GR. It is standard to postulate a line element without asking what could have been the source of that spacetime structure, or whether there are physically enigmatic possibilities, such as closed time-like curves, as in Gödel's spacetime (Gödel, 1949). There may be event horizons in our solutions to Einstein's equations. Our point is this: what empirical entitlement do we have to discuss two regions of spacetime separated by an event horizon, such as happens in the case of the Schwarzschild metric? Contextual completeness should tell us that such a scenario is vacuous. Trying to quantize any theory under such circumstances seems to be asking for trouble.

## 25.9 Experiments

Long before any experiment can begin, an observer starts off with a laboratory $\Lambda$ in its void state $\underset{\sim}{\emptyset}$. Then at some stage long before any runs can be taken, specific apparatus consisting of a finite number $r$ of detectors has to be constructed in $\Lambda$. We will assume without loss of generality that these are all functioning

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normally and in their ground state, so the labstate at that point is given by the equivalent of the right-hand side of (25.18). All of this is necessary before state preparation.

### Modules

We have so far discussed only the detectors. Similar comments can be made about the modules that sit in the information void. The difference is that labstates are directly associated with detectors, both real or virtual, whereas modules are almost invariably classical in their representation, at least in the formulation of QDN that we have discussed in this book. Therefore, we have not invested much effort in constructing a dynamical theory of modules, although that is expected in a more complete theory.

There is an intriguing relationship here with Feynman diagrams in RQFT. Consider quantum electrodynamics (QED) and the simplest Feynman diagram for electron–proton scattering, Figure 25.1. In QED, the particle source and final state detectors are taken to be at remote past and future times, respectively, and propagation is in the information void (the vacuum). However, the quantum fields are interacting in QED. This particular Feynman diagram can be interpreted as a process where a virtual module (the photon, represented by the wavy line in Figure 25.1) is created not by the observer, but by the quantum process itself. This module has many of the characteristics of a beam splitter, discussed in Chapter 11. Because this module is created via the quantum dynamics in the information void, it is reasonable to classify the experiment as type T3, as defined in Chapter 21.

Care has to be taken not to take Feynman diagrams too literally. They represent individual terms in infinite perturbation expansions or in asymptotic expansions of complete amplitudes. Those complete amplitudes will satisfy all the requirements of scattering matrix theory (S-matrix) (Eden et al., 1966), and they will behave as virtual quantum modules.

### The Contextual Register

According to what we said earlier, external context involving detectors in their void state $\underset{\sim}{\mathbf{3}}$ can be ignored; they do not exist in any physical sense. Therefore, we need only discuss those detectors that are in states $\underset{\sim}{\mathbf{0}}$, $\underset{\sim}{\mathbf{1}}$, or $\underset{\sim}{\mathbf{2}}$. A further simplification is that in real experiments, observers generally filter out observations

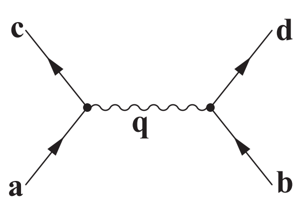


Figure 25.1. Lowest order Feynman diagram for electron–proton scattering.

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from faulty detectors (assuming these have been identified) by postselecting only those labstates that contain the *normal bit states* $\underset{\sim}{\mathbf{0}}$ or $\underset{\sim}{\mathbf{1}}$. We shall confine our attention to such normal labstates until we deal with applications to quantum mechanics.

Given this condition, we can restrict our discussion at any given stage $\Sigma_n$ to the *physical register* $\underset{\sim}{\boldsymbol{\mathcal{R}}}_n$, a subset of the universal register $\boldsymbol{\Omega}_n$. If there are $r_n$ working detectors in the laboratory at stage $\Sigma_n$, then $\underset{\sim}{\boldsymbol{\mathcal{R}}}_n$ consists of $2^{r_n}$ normal states. This is the extended QDN version of the classical registers $\mathcal{R}_n$ discussed in earlier chapters. We may represent these normal states with the extended version of the computational basis and signal basis representations discussed previously, For example, a typical normal extended basis labstate is of the form

$$\underset{\sim}{\boldsymbol{i}}_n \equiv \underset{\sim}{\boldsymbol{i}}_n^1 \underset{\sim}{\boldsymbol{i}}_n^2 \dots \underset{\sim}{\boldsymbol{i}}_n^{r_n}, \tag{25.20}$$

where $i^j = 0$ or $1$ for $j = 1, 2, \dots, r_n$, and

$$i_n \equiv i_n^1 + 2i_n^2 + \dots + 2^{r_n - 1} i_n^{r_n}. \tag{25.21}$$

The physical register $\underset{\sim}{\boldsymbol{\mathcal{R}}}_n$ represents all those detectors in the laboratory $\Lambda$ at stage $\Sigma_n$ that exist relative to the observer and are not faulty.

Virtually all of the concepts, such as signality, encountered with binary registers are encountered with physical registers, with some enhancements that are particular to enhanced QDN. One such is that the power bit operators $\underset{\dots}{\boldsymbol{A}}$ and $\underset{\dots}{\widehat{\boldsymbol{A}}}$ defined in (25.10) are not adjoints of each other, whereas the bit operators $\boldsymbol{A}$ and $\widehat{\boldsymbol{A}}$ defined in (4.13) are mutual adjoints. It was in anticipation of that fact that we chose the hat (circumflex) notation for $\widehat{\boldsymbol{A}}$ and not $\boldsymbol{A}^\dagger$.

### Physical Register Signal Operators

Given a rank-$r$ physical register $\underset{\sim}{\boldsymbol{\mathcal{R}}}_n$ we define the physical register signal operators

$$\underset{\dots}{\mathbb{A}}_n^i \equiv \left\{ \prod_{j \neq i}^{\infty} \underset{\dots}{\boldsymbol{I}}_n^j \right\} \underset{\dots}{\boldsymbol{A}}_n^i, \quad \underset{\dots}{\widehat{\mathbb{A}}}_n^i \equiv \left\{ \prod_{j \neq i}^{\infty} \underset{\dots}{\boldsymbol{I}}_n^j \right\} \underset{\dots}{\widehat{\boldsymbol{A}}}_n^i, \qquad 1 \leqslant i \leqslant r. \tag{25.22}$$

There are several interesting processes that can be described by these extended register operators. For example, an application of the operator $\underset{\dots}{\mathbb{A}}_n^i$ to the rank-$r$ contextual vacuum $\underset{\sim}{\mathbf{0}}_n^{[r]} \equiv \underset{\dots}{\mathbb{C}}_n^1 \underset{\dots}{\mathbb{C}}_n^2 \dots \underset{\dots}{\mathbb{C}}_n^r \underset{\sim}{\boldsymbol{\emptyset}}_n$ gives a rank-$(r-1)$ contextual vacuum:

$$\underset{\dots}{\mathbb{A}}_n^i \underset{\dots}{\mathbb{C}}_n^1 \underset{\dots}{\mathbb{C}}_n^2 \dots \underset{\dots}{\mathbb{C}}_n^r \underset{\sim}{\boldsymbol{\emptyset}}_n = \underset{\dots}{\mathbb{C}}_n^1 \underset{\dots}{\mathbb{C}}_n^2 \dots \underset{\dots}{\mathbb{C}}_n^{i-1} \underset{\dots}{\mathbb{C}}_n^{i+1} \dots \underset{\dots}{\mathbb{C}}_n^r \underset{\sim}{\boldsymbol{\emptyset}}_n. \tag{25.23}$$

This is a nonzero labstate in $\boldsymbol{\Omega}_n$ but is not an element of the original physical register $\underset{\sim}{\boldsymbol{\mathcal{R}}}_n^{[r]}$. What has happened is analogous to the convention qubit register result $\mathbb{A}_n^i \mathbf{0}_n = 0$. In the extended QDN case, we do not get zero but the equivalent

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of it: the action of $\underset{\cdots}{\mathbb{A}}{}^i_n$ on the signal ground state (contextual vacuum) $\underset{\sim}{\mathbf{0}}{}^{[r]}_n$ of $\underset{\sim}{\mathcal{R}}{}^{[r]}$ maps it into the ground state of a different physical register, one of rank $r-1$, a state orthogonal to every state in $\underset{\sim}{\mathcal{R}}{}^{[r]}$.

## 25.10 Quantization

The signal operators and the corresponding projection operators $\underset{\cdots}{\mathbb{P}}{}^i_n, \underset{\cdots}{\widehat{\mathbb{P}}}{}^i_n$ can be used to discuss all the quantum processes covered in this book, and will be used in the discussions on the bomb-tester and Hardy paradox below. An additional feature we make use of is that we can now factor into the mathematical representation the construction operators $\underset{\cdots}{\mathbb{C}}{}^i_n$ and the decommissioning operators $\underset{\cdots}{\mathbb{D}}{}^i_n$ explicitly.

In the next two sections we show how the extended QDN formalism describes experiments where the apparatus changes in one way or another during the experiment. In particular, the faulty/off-line state plays an important role in these experiments.

## 25.11 The Elitzur–Vaidman Bomb-Tester Experiment

In this experiment, a stockpile of active (A) and dud (D) bombs is analyzed, one by one, in order to find as many unexploded active bombs as possible. The approach follows that discussed in Elitzur and Vaidman (1993). The stage diagram shown in Figure 25.2 is a modified Mach–Zehnder interferometer. $B^1$ and $B^2$ are beam splitters, $M$ is a mirror, and $T$ is the actual bomb-testing triggering device that explodes the bomb if it is an active bomb *and* a signal could have been detected at $2_1$.[2]

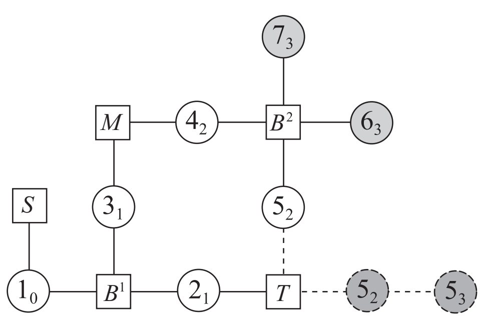


Figure 25.2. The bomb-tester network.

[2] One of the reasons this experiment is significant and to some extent baffling is the strange mix of classical and quantum counterfactuality. Analyzing it in terms of *which-path* information seems the best approach, because such information is not controversial and its extent can be established directly from a stage diagram in general.

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The process goes as follows. Source $S$ prepares a single photon state at $1_0$, which is then passed into beam splitter $B^1$. Transmitted channel $2_1$ feeds onto the bomb tester module $T$ while reflected channel $3_1$ is deflected by mirror $M$ toward beam splitter $B^2$.

The action at module $T$ is critical and is based on a specific assumption that sits on the knife edge between the classical and quantum worlds. For each run of the experiment, a bomb is taken from the untested stockpile of bombs (which includes active and dud bombs) and is placed in contact with module $T$. If the bomb is a dud (D), then $2_1$ is not prevented from being deflected onto $5_2$, and thence onto beam splitter $B^2$, where it can interfere with $4_2$.

If on the other hand the bomb is active (A), then either channel $3_1$ is realized *and* the bomb does not explode, or channel $3_1$ is not realized, and then certainly the bomb explodes. In this latter eventuality, channel $5_2$ is blocked.

This experiment involves a randomly changing apparatus network, because the bomb being tested in a given run is replaced by a new, randomly active or dud bomb from the unused stockpile. In addition, the network is self-intervening in the case of an active bomb coinciding with realized channel $2_1$. This experiment is a combination of T2 and T3 experiments (these are discussed in Chapter 21). The experiment looks like T2 in that the probability of a bomb being active or dud in each run introduces an explicit time dependence in the experiment as a whole, but that probability is determined by factors external to the apparatus. It has also the characteristics of a T3 experiment, in that beam splitter $B^1$ has random outcomes, and these determine whether module $T$ is triggered. Therefore, we may classify this experiment as T4.

There are two distinct networks to consider separately, because whether the bomb is active or dud is part of relative external context and not a quantum process requiring extended QDN. Therefore, we deal with each of the networks separately, using a pure labstate description for each initially. Then we use a density matrix approach to consider the combined experiment.

Our formulation of this scenario differs in two respects from those considered previously in this book. First, we shall use extended QDN. Second, we shall introduce *persistence*, in that all real and virtual detectors will be created before stage $\Sigma_0$ and persist to the final stage.

### Creation of the Apparatus

Prior to the creation of the apparatus, well before stage $\Sigma_0$, the signal state of the laboratory $\Lambda$ is its void state $\underset{\sim}{\emptyset}$ in the universal register $\mathbf{\Omega}$.[3] We may take the apparatus to be created in its ground state (equivalent to a contextual vacuum)

[3] The choice of the words *its* and *the* in this sentence is deliberate. A void state is contextual, whereas we can assume that there is just one enormous universal register that describes the totality of possible contextual existence and nonexistence that could ever be imagined, under all possible circumstances. This latter assumption costs us nothing.

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before stage $\Sigma_0$, that is, before the start of each run, so we would normally expect the apparatus to exist and be in its quiescent, no-signal state,

$$\underset{\sim}{\mathbf{0}}_0 \equiv \underset{\cdots}{\mathbb{C}}{}^1_0 \underset{\cdots}{\mathbb{C}}{}^2_0 \underset{\cdots}{\mathbb{C}}{}^3_0 \underset{\cdots}{\mathbb{C}}{}^4_0 \underset{\cdots}{\mathbb{C}}{}^5_0 \underset{\cdots}{\mathbb{C}}{}^6_0 \underset{\cdots}{\mathbb{C}}{}^7_0 \underset{\sim}{\boldsymbol{\emptyset}}_0. \tag{25.24}$$

This deserves some explanation, because sites $1_0, 2_1, 3_1, 4_2$, and $5_2$ are not real detectors. So the question arises, what does it mean to "create" them?

The answer is given by Schwinger, quoted in Chapter 24. The modules $S$, $B^1$, $B^2$, $M$, and $T$ really do "exist" relative to the observer, and it is that very existence that creates, as it were, the context for virtual detector sites $1_0, 2_1, 3_1, 4_2$, and $5_2$ to be meaningful. We may refer to this as "*Schwinger counterfactual reality.*" It is not metaphysics but actually part and parcel of the process of running any experiment, as we have commented before. The observer would have had to place real detectors in those sites in order to calibrate the given modules, *before* the actual runs had started (and then taken those calibration detectors away). Otherwise, there could be no degree of confidence in what the observer believed about the labstates being studied.

In addition to the creation of real and virtual detector sites, the preparation device $S$ prepares a photon signal in $1_0$, so the initial extended labstate $\underset{\sim}{\boldsymbol{\Psi}}_0$ is given by $\underset{\sim}{\boldsymbol{\Psi}}_0 \equiv \underset{\cdots}{\widehat{\mathbb{A}}}{}^1_0 \underset{\sim}{\mathbf{0}}_0$. This is the initial extended labstate for each of the two networks we now consider.

In the following, it is useful to label the respective transmission coefficient and reflection coefficient associated with each beam splitter separately, in order to see how information is flowing through the networks. We work in extended QDN throughout.

### *Runs with a Dud Bomb*

**Stage $\Sigma_0$ to Stage $\Sigma_1$**

By inspection of Figure 25.2 we have

$$\underset{\sim}{\boldsymbol{\Psi}}_1 \equiv \underset{\cdots}{\mathbb{U}}_{1,0} \underset{\sim}{\boldsymbol{\Psi}}_0 = (t^1 \underset{\cdots}{\widehat{\mathbb{A}}}{}^2_1 + ir^1 \underset{\cdots}{\widehat{\mathbb{A}}}{}^3_1) \underset{\sim}{\mathbf{0}}_1. \tag{25.25}$$

**Stage $\Sigma_1$ to Stage $\Sigma_2$**

In this scenario, the bomb is a dud, so any signal entering module $T$ from port $2_1$ is transmitted directly to $5_2$. Taking the mirror $M$ not to change any phase, we then have

$$\underset{\sim}{\boldsymbol{\Psi}}_2 \equiv \underset{\cdots}{\mathbb{U}}_{2,1} \underset{\sim}{\boldsymbol{\Psi}}_1 = (t^1 \underset{\cdots}{\widehat{\mathbb{A}}}{}^5_2 + ir^1 \underset{\cdots}{\widehat{\mathbb{A}}}{}^4_2) \underset{\sim}{\mathbf{0}}_2. \tag{25.26}$$

**Stage $\Sigma_2$ to Stage $\Sigma_3$**

Using

$$\underset{\cdots}{\mathbb{U}}_{3,2} \underset{\cdots}{\widehat{\mathbb{A}}}{}^5_2 \underset{\sim}{\mathbf{0}}_2 = (t^2 \underset{\cdots}{\widehat{\mathbb{A}}}{}^7_3 + ir^2 \underset{\cdots}{\widehat{\mathbb{A}}}{}^6_3) \underset{\sim}{\mathbf{0}}_3, \tag{25.27}$$

$$\underset{\cdots}{\mathbb{U}}_{3,2} \underset{\cdots}{\widehat{\mathbb{A}}}{}^4_2 \underset{\sim}{\mathbf{0}}_2 = (t^2 \underset{\cdots}{\widehat{\mathbb{A}}}{}^6_3 + ir^2 \underset{\cdots}{\widehat{\mathbb{A}}}{}^7_3) \underset{\sim}{\mathbf{0}}_3, \tag{25.28}$$

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we end up with the final state outcome probabilities

$$\Pr(6_3|\underset{\sim}{\boldsymbol{\Psi}}_0) = |t^1r^2 + r^1t^2|^2, \quad \Pr(7_3|\underset{\sim}{\boldsymbol{\Psi}}_0) = |t^1t^2 - r^1r^2|^2. \tag{25.29}$$

The significance of this result is that for matched symmetric beam splitters, for which $t^1 = r^1 = t^2 = r^2 = 1/\sqrt{2}$, the prediction is that $\Pr(7_3|\underset{\sim}{\boldsymbol{\Psi}}_0) = 0$. The conclusion, that for symmetrical beam splitters a dud bomb never coincides with a positive signal in detector $7_3$, is critical to the point of this experiment.

### ***Runs with an Active Bomb***

In this scenario, the bomb is active and could explode if triggered at module $T$. We shall take the decommissioning of $5_2$ as a marker of that explosion: the observer will be able to see real debris in the laboratory if the bomb explodes.

Everything is the same as in the previous scenario with a dud bomb, up to stage $\Sigma_1$.

**Stage $\boldsymbol{\Sigma_1}$ to Stage $\boldsymbol{\Sigma_2}$**

In this scenario, the bomb explodes and decommissions the device $T$ if a signal enters from $2_1$. Hence we may write

$$\underset{\sim}{\boldsymbol{\Psi}}_2 \equiv \underset{\dots}{\mathbb{U}}_{2,1}\underset{\sim}{\boldsymbol{\Psi}}_1 = (t^1\underset{\dots}{\mathbb{D}}{}^5_2 + ir^1\underset{\dots}{\widehat{\mathbb{A}}}{}^4_2)\underset{\sim}{\mathbf{0}}_2. \tag{25.30}$$

**Stage $\boldsymbol{\Sigma_2}$ to Stage $\boldsymbol{\Sigma_3}$**

The debris created by any bomb exploding at stage $\Sigma_2$ will certainly be transmitted on to stage $\Sigma_3$, so we have

$$\begin{aligned} \underset{\dots}{\mathbb{U}}_{3,2}\underset{\dots}{\mathbb{D}}{}^5_2\underset{\sim}{\mathbf{0}}_2 &= \underset{\dots}{\mathbb{D}}{}^5_3\underset{\sim}{\mathbf{0}}_3, \\ \underset{\dots}{\mathbb{U}}_{3,2}\underset{\dots}{\widehat{\mathbb{A}}}{}^4_2\underset{\sim}{\mathbf{0}}_2 &= (t^2\underset{\dots}{\widehat{\mathbb{A}}}{}^6_3 + ir^2\underset{\dots}{\widehat{\mathbb{A}}}{}^7_3)\underset{\sim}{\mathbf{0}}_3, \end{aligned} \tag{25.31}$$

giving the final state

$$\underset{\sim}{\boldsymbol{\Psi}}_3 = t^1\underset{\dots}{\mathbb{D}}{}^5_3\underset{\sim}{\mathbf{0}}_3 + ir^1(t^2\underset{\dots}{\widehat{\mathbb{A}}}{}^6_3 + ir^2\underset{\dots}{\widehat{\mathbb{A}}}{}^7_3)\underset{\sim}{\mathbf{0}}_3. \tag{25.32}$$

We conclude that in this scenario, the following probabilities hold:

$$\begin{aligned} &\text{probability of an explosion :} && (t^1)^2, \\ &\text{probability of no explosion and signal in } 6_3 \text{ :} && (r^1t^2)^2, \\ &\text{probability of no explosion and signal in } 7_3 \text{ :} && (r^1r^2)^2. \end{aligned} \tag{25.33}$$

In the symmetric case, we conclude that the probability of no explosion of an active bomb *and* a signal in detector $7_3$ is $\frac{1}{4}$.

It is this outcome that allows the observer to find an unexploded, active bomb, because detector $7_3$ does not trigger in the symmetric beam splitter scenario when the bomb in question is a dud.

### ***Random Testing***

Unfortunately, the observer does not know before each run whether a particular bomb is active or a dud. Consider a sequence of runs such that there is a (classical)

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probability $\omega^A$ of encountering an active bomb and a probability $\omega^D \equiv 1 - \omega^A$ of a dud. In this case we take a density matrix approach. At the $n$th stage, for $n = 0, 1, 2, 3$, we define the extended density matrix

$$\underset{\dots}{\rho}_n \equiv \omega^A \underset{\sim}{\mathbf{\Psi}}{}^A_n \widetilde{\mathbf{\Psi}}{}^A_n + \omega^D \underset{\sim}{\mathbf{\Psi}}{}^D_n \widetilde{\mathbf{\Psi}}{}^D_n. \tag{25.34}$$

Here $\underset{\sim}{\mathbf{\Psi}}{}^A_n$ and $\underset{\sim}{\mathbf{\Psi}}{}^D_n$ are the extended labstates for the active and dud scenarios, respectively, at stage $\Sigma_n$. It could be thought that this must be incorrect at stage $\Sigma_0$, because the observer has no information as to which scenario is in place. But in fact, that information could be established in principle *before* stage $\Sigma_1$ by, for example, triggering the bomb directly. It would explode in the active case and that would inform the observer that the initial state should have been $\underset{\sim}{\mathbf{\Psi}}{}^A_n$ in the now aborted run. A similar remark holds for the dud case.

The overall probability of triggering detector $7_3$ is given by

$$\Pr(7_3) = Tr\left\{ \underset{\dots}{\rho}_3 \underset{\dots}{\widehat{\mathbb{P}}}{}^7_3 \right\}, \tag{25.35}$$

where $\underset{\dots}{\widehat{\mathbb{P}}}{}^7_3$ is the signal projection operator for detector $7_3$. We find

$$\Pr(7_3) = \omega^D (t^1 t^2 - r^1 r^2)^2 + \omega^A (r^1 r^2)^2. \tag{25.36}$$

In the symmetric case, we conclude that whenever detector $7_3$ registers a signal, the bomb is active and can be stockpiled accordingly. A single sweep of the original stockpile of active and dud bombs will identify one-quarter of the active bombs without exploding them (the rest go up in smoke). Moreover, whenever detector $6_3$ has triggered, there has been no explosion, but the observer cannot be sure which scenario has occurred. Therefore, that bomb can be retested.

In this way, the observer should be able to find up to one-third of the active bombs, the rest having exploded.

This thought experiment has been the motivation for real experiments that simulate in some way the action of module $T$ in the above experiment, a class of experiment known as *interaction-free measurement* (Kwiat et al., 1994).

## 25.12 The Hardy Paradox Experiment

The Elitzur–Vaidman bomb-tester experiment discussed above may be interpreted as a simplified form of double-slit experiment, where the screen has only two sites and one of the slits can be blocked off or not, depending on whether a bomb is active or dud. This blocking off occurs in a classical way, because the uncertainty as to whether the bomb is active or dud is not intrinsic to the nature of the bomb but reflects the observer's ignorance of the nature of the bomb.

A variant of the bomb-tester experiment is known as the Hardy paradox experiment (Hardy, 1992). In this variant, the blocking-off of a slit occurs in an intrinsically random way because it is governed by quantum processes, contrasted

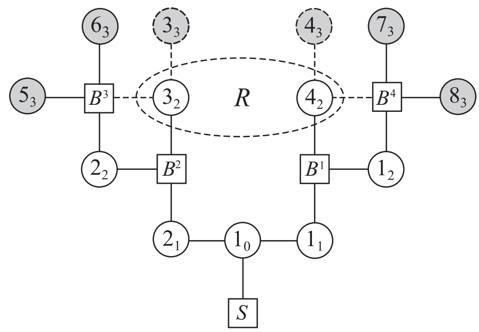


Figure 25.3. The Hardy paradox experiment.

to the bomb-tester experiment analyzed above, where the observer's ignorance about the nature of the bomb plays a role.

The Hardy paradox experiment shown in Figure 25.3 consists of an electron–positron pair $1_1$, $2_1$ produced by a source $S$. These particles are then deflected by magnets (not shown) to pass through the equivalent of two coupled Mach–Zehnder-type networks, as shown in Figure 25.3. Output ports $3_2$ and $4_2$ of beam splitters $B^2$ and $B^1$, respectively, are arranged to intersect in a confined region, $R$, where electron–positron annihilation may occur.

There are two possibilities if both particles are in region $R$ at the same stage: either (1) they annihilate into a photon pair, or (2) they pass onto their respective beam splitters $B^3$ and $B^4$ as shown.

**Case (1)**

In case (1) and in conventional terminology, the annihilation effectively blocks off one of the entry ports in each of those beam splitters. This is equivalent to a virtual module in interaction region $R$, which would otherwise be a true information void. In this case, the photons produced by the annihilation are detected in $3_3$ and $4_3$.

**Case (2)**

In case (2), the virtual detectors $3_2$ and $4_2$ transmit their amplitudes onto beam splitters $B^3$ and $B^4$, respectively, as shown in Figure 25.3. Interference occurs in each of these beam splitters. In the symmetric case, a signal will be detected at $6_3$ and not in $5_4$, and a signal will be detected in $7_3$ and not in $8_3$.

The Hardy paradox experiment is intrinsically a pure quantum experiment and we can discuss it via pure labstates alone. An important point is that electron–positron annihilation is a well-known quantum process that occurs in nature, whereas the detonation mechanism of the Elitzur–Vaidman bomb-tester is left unspecified.

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As with the bomb-tester experiment above, we start our analysis of the Hardy paradox experiment by first defining the contextual vacuum $\underset{\sim}{\mathbf{0}}_0 \equiv \underset{\cdots}{\mathbb{C}}{}^1_n \underset{\cdots}{\mathbb{C}}{}^2_n \dots \underset{\cdots}{\mathbb{C}}{}^8_n \underset{\sim}{\boldsymbol{\emptyset}}_0$ and then the initial state is given by $\underset{\sim}{\boldsymbol{\Psi}}_0 \equiv \underset{\cdots}{\widehat{\mathbb{A}}}{}^1_0 \underset{\sim}{\mathbf{0}}_0$.

The dynamics then goes as follows.

**Stage $\boldsymbol{\Sigma_0}$ to Stage $\boldsymbol{\Sigma_1}$**

The electron–positron pair produced by source $S$ is split by magnetic fields ready to be deflected into beam splitters $B^1$ and $B^2$ as shown, giving extended labstate $\underset{\sim}{\boldsymbol{\Psi}}_1 \equiv \underset{\cdots}{\widehat{\mathbb{A}}}{}^1_1 \underset{\cdots}{\widehat{\mathbb{A}}}{}^2_1 \underset{\sim}{\mathbf{0}}_1$.

**Stage $\boldsymbol{\Sigma_1}$ to Stage $\boldsymbol{\Sigma_2}$**

$$\underset{\sim}{\boldsymbol{\Psi}}_2 \equiv \underset{\cdots}{\mathbb{U}}_{2,1} \underset{\sim}{\boldsymbol{\Psi}}_1 = (t^1 \underset{\cdots}{\widehat{\mathbb{A}}}{}^1_2 + ir^1 \underset{\cdots}{\widehat{\mathbb{A}}}{}^4_2)(t^2 \underset{\cdots}{\widehat{\mathbb{A}}}{}^2_2 + ir^2 \underset{\cdots}{\widehat{\mathbb{A}}}{}^3_2) \underset{\sim}{\mathbf{0}}_2. \tag{25.37}$$

**Stage $\boldsymbol{\Sigma_2}$ to Stage $\boldsymbol{\Sigma_3}$**

The only novel factor concerns potential electron–positron annihilation, involving $3_2$ and $4_2$. We will explore the possibility of partial annihilation, by taking the evolution for this term to be given by

$$\underset{\cdots}{\mathbb{U}}_{3,2} \underset{\cdots}{\widehat{\mathbb{A}}}{}^4_2 \underset{\cdots}{\widehat{\mathbb{A}}}{}^3_2 \underset{\sim}{\mathbf{0}}_2 = \alpha \underset{\cdots}{\mathbb{D}}{}^4_3 \underset{\cdots}{\mathbb{D}}{}^3_3 \underset{\sim}{\mathbf{0}}_3 + \beta (t^4 \underset{\cdots}{\widehat{\mathbb{A}}}{}^8_3 + ir^4 \underset{\cdots}{\widehat{\mathbb{A}}}{}^7_3)(t^3 \underset{\cdots}{\widehat{\mathbb{A}}}{}^5_3 + ir^3 \underset{\cdots}{\widehat{\mathbb{A}}}{}^6_3) \underset{\sim}{\mathbf{0}}_3, \tag{25.38}$$

where $|\alpha|^2 + |\beta|^2 = 1$. These coefficients reflect the fact that electron–positron annihilation is a quantum process and does not always occur. Note the introduction of the destruction operators here, signifying that photons have been produced (these can be regarded as the debris from the annihilation of the virtual electron–positron detectors at $3_3$ and $4_3$).

The relevant information when fed into program MAIN gives the following correlations for the symmetric beam splitter case:

$$\begin{aligned}
&\Pr(3_3, 4_3|1_0) = \tfrac{1}{4}|\alpha|^2, \\
&\Pr(5_3, 7_3|1_0) = \Pr(5_3, 8_3|1_0) = \Pr(6_3, 8_3|1_0) = \tfrac{1}{16}|1-\beta|^2, \\
&\Pr(6_3, 7_3|1_0) = \tfrac{1}{16}|3+\beta|^2.
\end{aligned} \tag{25.39}$$

When $\beta$ is set to one, corresponding to no annihilation in region $R$, the prediction is that the only outcome is that both $6_3$ and $7_3$ show a signal. However, when annihilation is permitted, $\alpha \neq 0$ and the outcomes show characteristics of interaction-free measurement. This is deduced from conservation of electric charge. The argument goes as follows. If a signal is detected at, say, $5_3$, then annihilation cannot have occurred, because signality is conserved in this experiment; annihilation would give a signal at $3_3$ and at $4_3$, and so a signal elsewhere would be ruled out. But a signal in $5_3$ arises only if something is affecting the signal from $3_2$, and the only factor that could affect that could be $4_2$. Therefore, any signal in $5_3$ is an "interaction-free" detection of the particle at $4_2$. A similar remark holds for any signal observed at $8_3$.

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## 25.13 Implications and Comments

The application of our extended QDN formalism to the Elitzur–Vaidman bomb-tester and Hardy paradox experiments demonstrates that the concept of faulty or decommissioned states has physical significance.

An important point about this chapter is that the phenomena discussed show that specific reductionist details of interactions seem not to be relevant to the overall picture. For instance, in the bomb-tester experiment we have not said anything at all about what sort of module $T$ really is. Likewise, we have not done any specific quantum electrodynamical calculation in the Hardy paradox. What seems important is the architecture of the apparatus involved, and that is an emergent aspect of these experiments.

Such considerations are what have led us to the conclusion that quantum mechanics is really a theory of contextuality. That the rules are unfamiliar and counterintuitive in many cases is a commentary not on reality, but on the inadequacy and limitations of our classical conditioning.