Euclidean Proof of Pascal's Theorem.*

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Note on the expression for the area of a triangle in Cartesian Coordinates, and a general proof of the Addition Theorem in Trigonometry connected therewith.

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In the Euclidean theory of areas, where convex polygons alone are considered, there is no question as to the sign of an area. The element of area is the rectangle, and an area is signless, or always positive.

When an extension of the notion of an area is sought which will apply to any closed polygon noncrossed or crossed, a definition is given which, naturally, must be such as to affect in no way the meaning we have for a Euclidean area. The element of area is usually a Euclidean triangle, but the convention is made that the area of a triangle ABC is to be considered positive, if on tracing out the perimeter with the vertices in that order the area lies always to the left, but otherwise negative.

The area bounded by a closed broken line ABCD...KA is then *defined* as the algebraic sum of the triangles OAB, OBC,...OKA, O being any point in the plane.

It then follows that Area ABC...KA + Area AK...CBA is zero, and that if the polygon be convex the area is positive when the vertices are taken counterclockwise, and otherwise negative. It may also be proved that the position of O is immaterial.

In the Analytical Geometry we have the corresponding problem: Given the coordinates of the vertices of a polygon taken in order, can we express the area as a function of these coordinates? If O be taken at the origin, then it is clear that we can express the area provided we can determine the area of any triangle OPQ, where P and Q are any two points (x_1, y_1), (x_2, y_2).

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* Printed in the *Educational Times* for October.

https://doi.org/10.1017/S0013091500032508 Published online by Cambridge University Press