

SPATIO-TEMPORAL MODELLING FOR NONSTATIONARY POINT REFERENCED DATA

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Spatial and spatio-temporal phenomena are commonly modelled as Gaussian processes via the geostatistical model [4]. The geostatistical model has the benefit of modelling the spatial dependence structure using covariance functions. Most commonly, the covariance functions impose an assumption of spatial stationarity on the process. That means the covariance between observations at particular locations depends only on the distance between the locations [1]. It has been widely recognised that most, if not all, processes manifest spatially nonstationary covariance structure [6]. If the study domain is small in area or there is not enough data to justify more complicated nonstationary approaches, then stationarity may be assumed for the sake of mathematical convenience [3]. However, relationships between variables can vary significantly over space, and a ‘global’ estimate of the relationships may obscure interesting geographical phenomena [2, 3, 7].

In this thesis, we consider three different approaches for accounting for nonstationarity in both spatial and spatio-temporal processes. We first propose partitioning the spatial or spatio-temporal data into subregions using the K-means algorithm based on a set of appropriate geographic features. This allows for the fitting of separate stationary covariance functions to the smaller subregions to account for local differences in covariance across the study region. Second, we extend the concept of covariance network regression to model the covariance matrix of both spatial and spatio-temporal processes. The resulting covariance estimates are found to be more flexible in accounting for spatial autocorrelation than standard stationary approaches. The third approach involves developing a geographic random forest methodology that uses a neighbourhood structure for each location based on the K-means algorithm.

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We find that clustering based on geographic measures such as longitude and latitude ensure that observations that are too far away to have any influence on the observations near the locations where a local random forest is fitted are not selected to form the neighbourhood.

In addition to developing flexible methods to account for nonstationarity, we develop a pivotal discrepancy measure approach for goodness-of-fit testing of spatio-temporal geostatistical models [5]. We find that partitioning the pivotal discrepancy measures increases the power of the test.

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