# 2-CATEGORICAL FAM CONSTRUCTIONS

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This thesis provides constructions for 2-categories of families indexed by 1-categories, which are shown to share properties with the Fam construction for ordinary categories. In particular, we describe the free completion of a 2-category under oplax colimits of oplax functors from 1-categories as such a 2-category of families. Submonads of this free cocompletion with objects given by pseudofunctors or by strict functors are described and shown to be instances of higher-dimensional analogues of Weber's *familial 2-monads*. A related $(2, 1)$-categorical Fam construction is shown to be both a free cocompletion and a higher-dimensional familial monad. Alongside these monads we consider related notions of familial functors and show that left extensions along such functors have a weak lifting property for lax transformations, as well as 2-natural transformations. This extension property endows the free oplax cocompletion with a lax-Gray functorial structure.

In the course of this investigation we provide a characterisation of coalgebras for oplax-transformation classifiers on presheaves as the saturation of the class of weights for conical oplax colimits. From this characterisation we construct the free completion under this class of colimits, of which our 2-categorical Fam constructions are all subfunctors. We also consider representable notions of 2-fibrations, and conclude that horizontally split 2-fibrations are the appropriate Cat-representable notion, whereas general 2-fibrations are Gray-representable.

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