

## Orbital Evolution of Small Particles Ejected from Martian Satellites

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**Abstract.** The dynamical behavior of dust particles ejected from Martian satellites is examined. On the basis of numerical simulations and analytical estimation for the orbital evolution of particles, we have found that there exists two kinds of resonance orbits around Mars. Particles ejected from Phobos and Deimos are influenced by these orbital resonances, and form dust rings with an asymmetrical structure along the orbit of the satellite, i.e., dust rings with a thin disk-like shape for Phobos and with a vertically extended structure for Deimos.

### 1. Introduction

Although imaging data taken by the Viking Orbiter 2 showed no evidence of dust rings in backscattered light (Duxbury & Ocampo 1988), many authors suspect the existence of diffuse dust rings around Mars. Since Mars has two small satellites, Phobos and Deimos, the ejecta particles produced by impacts of interplanetary meteoroids onto the surface of satellites can supply the ring materials along the orbits of the satellites (Davis et al. 1981, Banaskiewicz & Ip 1991). The dynamics of small particles around Mars is mainly influenced by the gravities of the Sun, Mars and the parent satellite as well as solar radiation pressure and the Lorentz force. Horányi et al. (1990, 1991) showed that solar radiation pressure and Lorentz forces are dominant on particles smaller than  $1\mu\text{m}$  and their lifetimes against collision with Mars are less than a few tens of days. On the other hand, for particles larger than  $1\mu\text{m}$  (about  $10^{-12}\text{g}$  for a spherical silicate), solar radiation pressure still affects the dynamics of ring particles while the Lorentz force is insignificant around Mars.

Recent studies of the dynamics of dust particles around Mars indicate more complex features of the dust rings (e.g. Juhász & Horányi 1995, Krivov et al. 1995, Hamilton 1996, Ishimoto 1996) than that of previous work. The main topic common to these studies is the effect of planetary oblateness combined with solar radiation pressure. Although the value of Mars' oblateness is smaller than that of the outer planets, large secular variations of orbital eccentricities and inclinations occur for ring particles when we take into account the oblateness perturbation. Here the role of planetary oblateness in dynamical evolution of ring particles will be mentioned briefly.

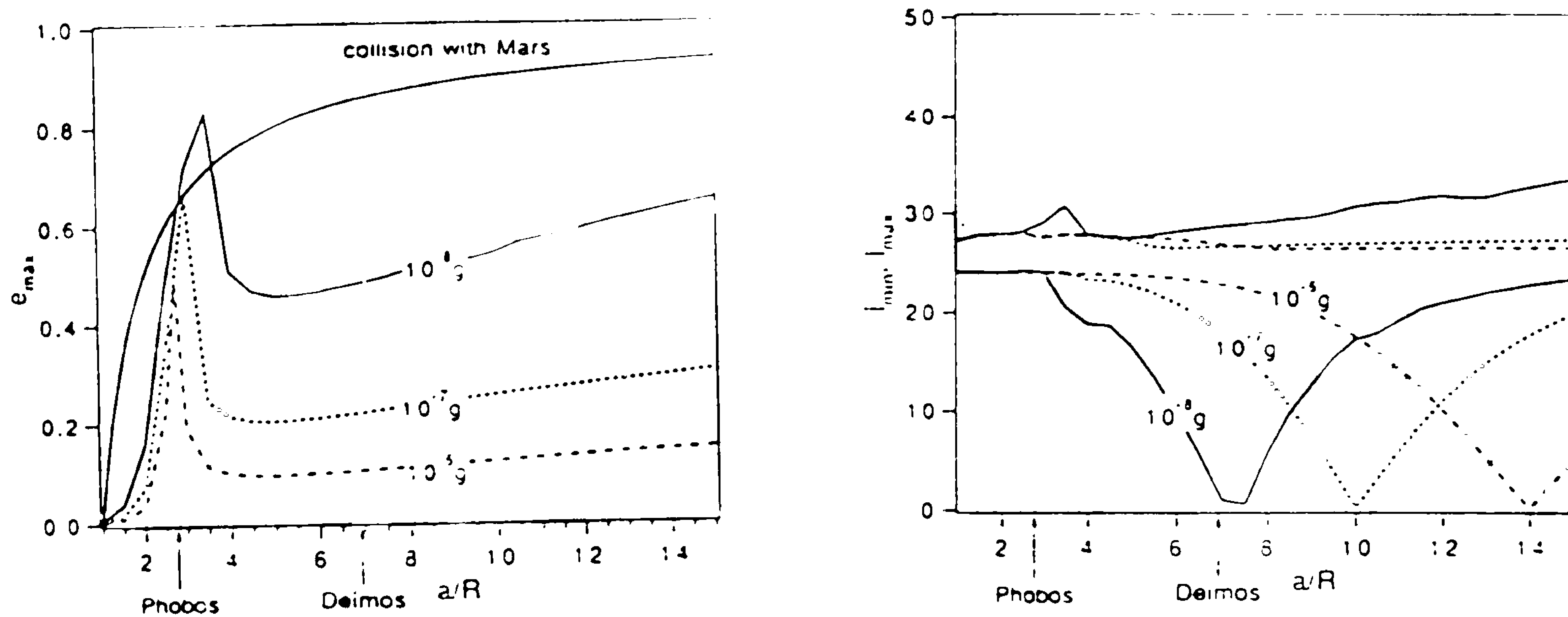


Figure 1. Maximum eccentricities and minimum/maximum inclinations of orbits for different particle masses ( $10^{-8}$  g (solid line),  $10^{-7}$  g (dotted line), and  $10^{-6}$  g (dashed line)), as a function of initial semi-major axis of the particle.

## 2. Effect of solar radiation pressure and Mars oblateness

Figure 1 shows the maximum eccentricity  $e_{max}$  and minimum/maximum values of inclinations  $i_{min}/i_{max}$  of dust particle orbits as a function of the orbit's initial semi-major axis. The initial state of test particles that are assumed to be made of silicates and to be spherical, is that they move along circular orbits in the equatorial plane of Mars (Mars' obliquity :  $i \sim 26^\circ$ ) as the initial state. We can estimate the typical mass of a ring particle using Figure 1. If the particle's maximum eccentricity becomes larger than the critical eccentricity of its orbit (see Figure 1), the particle collides with Mars within a relatively short time scale, and its contribution to the number density of dust in the rings becomes small. On the other hand, there are many more small particles if we assume that the ring particles were produced by impacts. Therefore, we can expect the typical mass of a ring particle to be that of the smallest particle that avoids collisions with Mars. In both of the initial orbits of Phobos and Deimos, particles with mass of less than  $10^{-9}$  g collide with Mars. From Figure 1, we can expect that the typical particle's mass is about  $10^{-7}$  g for Phobos' dust rings and  $10^{-8}$  g for Deimos' dust rings.

The maximum orbital eccentricity peaks strongly at semi-major axes near  $3R$  ( $R$  is Mars' radius). This is a case when the motion of  $\omega_S$  becomes slow ( $\omega_S$  is the longitude of the particle's orbital periapsis measured from the direction of the Sun). The secular change of orbital eccentricity of the particle perturbed by solar radiation pressure depends strongly on the cycle of  $\omega_S$  (Burns et al. 1979). And the planetary motion rotates  $\omega_S$  clockwise while the planetary oblateness acts to rotate  $\omega_S$  counterclockwise. When these two effects nearly cancel, the perturbation of orbital eccentricity accumulates for a long time. The coupled perturbation equations for the system are (e.g. Hamilton 1993).

$$\frac{de}{dt} = \frac{3a^{\frac{1}{2}}G^{\frac{1}{2}}M_{\odot}\beta}{2M_P^{\frac{1}{2}}r^2} (1-e^2)^{\frac{1}{2}} \sin \omega_S \quad (1)$$

$$\frac{d\omega_S}{dt} = \frac{3GM_P J_2 R^2}{2a^5 n} (1-e^2)^{-2} + \frac{3a^{\frac{1}{2}}M_{\odot}\beta}{2G^{\frac{1}{2}}M_P^{\frac{1}{2}}r^2} (1-e^2)^{\frac{1}{2}} \cos \omega_S - n_P. \quad (2)$$

where  $G$ ,  $M_{\odot}$ ,  $M_P$ ,  $r$  and  $J_2$  are the gravitational constant, the mass of the Sun, Mars' mass, the heliocentric distance of Mars' orbit, and the second harmonic coefficient of Mars' gravitational field. The quantities  $n$ ,  $n_P$ , and  $\beta$  are, respectively, the mean motion of particles around Mars, the mean motion of Mars around the Sun, and a ratio of solar radiation pressure force to the solar gravity on the particle. The value of  $d\omega_S/dt$  becomes small at the orbit where the first term of equation (2) is approximately canceled by  $n_P$ . Therefore, the characteristic semi-major axis  $a_1$  where the steep increase of orbital eccentricity occurs, can be derived as

$$\frac{3GM_P J_2 R^2}{2a^5 n} \sim n_P, \quad \Rightarrow \quad \frac{a_1}{R} = \left(\frac{3}{2} J_2\right)^{2/7} \left(\frac{M_P}{M_{\odot}}\right)^{1/7} \left(\frac{r}{R}\right)^{3/7} \quad (3)$$

The value of  $a_1/R$  for Mars is 2.62 which is very close to the orbit of Phobos ( $2.76R$ ). Therefore, particles ejected from Phobos go into an orbit of higher eccentricity than Deimos particles (see, Figure 1).

On the other hand, the secular variation of orbital inclination for Deimos particles depends strongly on the time evolution of their orbital node  $\Omega$  (where the orbital inclination and node are measured from the ecliptic plane). As shown in Figure 2, solar radiation pressure acts to rotate the orbital node of particles clockwise. When the orbital node changes, the planetary oblateness perturbation acts to push it back to the planetary equatorial plane, and at the same time, the particle's orbital inclination decreases. Moreover, when the orbital node passes the equatorial plane, the oblateness perturbation force makes the orbital inclination increase. Consequently, the orbital inclination becomes minimum when the orbital node passes the equatorial plane. From the perturbation equation,

$$\frac{d\Omega}{dt} \sim \frac{3GM_P J_2 R^2}{4a^5 n} (1 - e^2)^{-2} \left[ \frac{\cos 2i}{\sin i} \sin 2i_0 + 2 \cos i \cos 2i_0 \right] - \frac{3aGM_{\odot}\beta^2}{\pi n_P r^4} \quad (4)$$

where  $i_0$  is the initial value of the orbital inclination (corresponding to the obliquity of Mars). And assuming  $d\Omega/dt \sim 0$  when  $i = (i_{min} + i_0)/2$  and  $i_{min} \sim 0$ , we can derive another characteristic semi-major axis  $a_2$  where the inclination decreases to zero.

$$\frac{a_2}{R} \sim \left(\frac{\pi J_2}{2} \cos \frac{i_0}{2}\right)^{2/9} \left(\frac{M_P}{M_{\odot}}\right)^{1/3} \left(\frac{r}{R}\right)^{2/9} \beta^{-4/9} \quad (5)$$

This characteristic semi-major axis depends on the size (or mass) of particle through the dependence on  $\beta$ . Substituting the values for Mars,  $a_2 = 7R$  ( $10^{-8}$  g),  $10R$  ( $10^{-7}$  g) and  $14R$  ( $10^{-6}$  g) can be derived. The characteristic semi-major axis for  $10^{-8}$  g particles is very close to that of Deimos orbit ( $6.92R$ ). Therefore, a large secular variation of orbital inclination for Deimos dust particles is observed.

These two types of characteristic semi-major axes of initial particle orbits (Eqs. (3) and (5)) both depend on the values of  $J_2$ ,  $M_P$  and  $r$ , and Eqs. (3) and (4) implicitly show that these resonance orbits are closer to the surface of the planet for inner planets than that for outer planets. Since the Martian  $J_2$  is small, significant orbital evolution can occur for grains close to Mars.

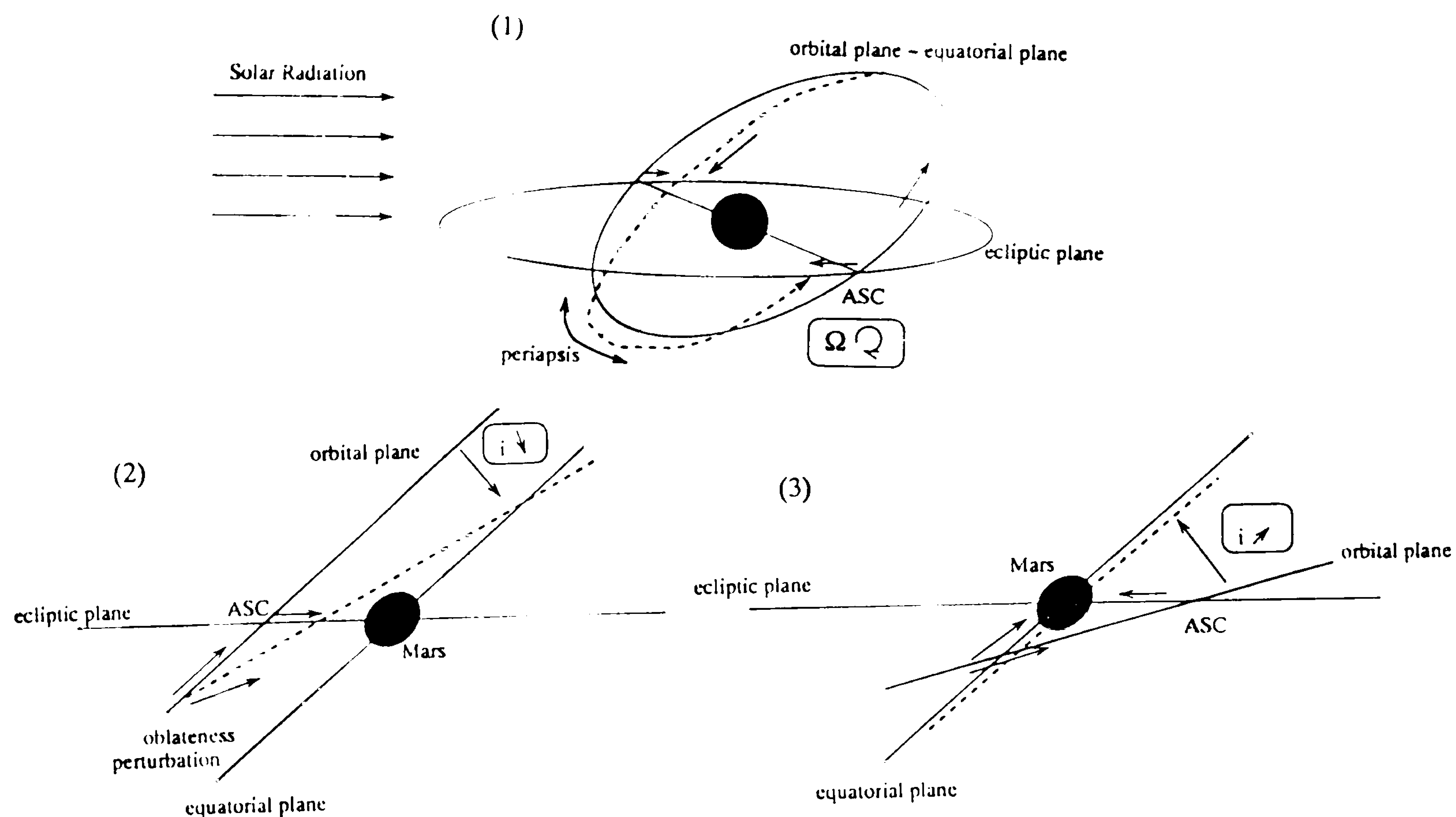


Figure 2. Schematics of the perturbation process for orbital inclination of Deimos particles. Solar radiation pressure rotates the particle's orbital node clockwise (1). The planetary oblateness perturbation acts to push back the orbital node and also changes the orbital inclination (2)-(3).

Other kinds of perturbation forces in addition to planetary oblateness may cause the same kind of orbital resonances when they are combined with the major perturbation force (i.e. solar radiation pressure) around Mars. Future studies of the dynamics of circumplanetary particles will be required to elucidate the complexities involved.

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