# Theoretical Models of Classical Pulsating Stars

# Any Recent Progress in the Theory of Pulsating Stars?

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**Abstract.** To answer the topical question we survey synoptically the recent literature in pulsation theory. We restrict the topics to those which are not dealt with otherwise in this volume. We address research on roAp stars, EC 14026 variables, strange modes, luminous blue variables, pulsation-rotation coupling, and pulsations in compact objects seen from the classical, as well as the relativistic, viewpoint.

## 1. Introduction

Well in accordance with the information era in which we live, variable-star research prospers, producing huge amounts of high-quality data. For the various families of pulsators, a large number of member stars are monitored regularly. Concerning theory, this calls for new – probably statistical – avenues to be taken to eventually improve our understanding of stellar physics and evolution. We are finally no longer confined to detailed studies of single objects only, but have access to statistically significant ensembles. Therefore, conceptual advances are to be expected once the ensemble aspect is given proper consideration in theoretical studies.

The last few years, however, have mostly seen the continuation of traditional approaches to stellar pulsation theory. The contributions from these approaches show that important basic aspects are still to be clarified, and even discovered. We have not yet reached a plateau in our understanding with only minor quantitative bunny-hill problems left to be solved.

The section 2 of this contribution deals with classical, Newtonian stars for which formal tool-making aspects and excitation results are discussed. Section 3 is devoted to compact objects for which we have seen a considerable body of papers appearing recently. Much of the excitement is generated by additional efforts in theory in connection with the upcoming gravitational-wave detectors, and the recent launch of the Chandra spacecraft.

## 2. Newtonian Stars

### 2.1. Formal Developments

Lee and Saio (1986, 1987) introduced a series expansion of the latitudinal ($\theta$) dependence of pulsational perturbations in rotating stars. The (usually rather low) number of terms in this series representation always left some doubt about the quality of the results, as it was not obvious if the dominant contributions were

included since the basis functions – associated Legendre polynomials – generally do not converge quickly. This phase has finally been overcome with the introduction of a direct integration method for Laplace's tidal equation in the $\theta$ direction (Bildsten, Ushomirsky, & Cutler 1996, Lee & Saio 1997). Solving this additional eigenvalue problem provides further sets of eigenvalues associated with modified g and r modes and oscillatory convective modes, respectively. An important observation is that the g modes' amplitudes become increasingly more confined to low latitudes as the ratio of the rotational to the oscillation frequency increases. Oscillatory convective modes, on the other hand, have only low amplitudes close to the equator. Lee & Saio (1997) concluded positively that the instability results for resonant couplings of envelope modes with oscillatory convective modes in upper main-sequence stars (Lee & Saio 1986) remain valid when using the new scheme, rather than the crude two-term approximation used in the past.

For tidally disturbed stars in close binary systems, Savonije & Papaloizou (1997) went a step further, fully accounting for the Coriolis force and casting the perturbation problem into a *two-dimensional* boundary-value problem which was solved with a complex-valued, finite-difference approach in both radial and latitudinal directions. The effect of the time-dependent external tidal potential was included as a fixed-$\ell$ (quadrupole) perturbation with adjustable orbital frequency. Based on this scheme, Witte & Savonije (1999) studied the tidal exchange of energy and angular momentum in a rotating, massive, slightly evolved main-sequence star in an eccentric binary system. For various uniform rotation rates and series of orbital periods, the resonances with r and g modes were computed. These resonances appear to efficiently alter the orbital evolution, as well as synchronize the massive star's rotation.

## 2.2.   Excitation of Pulsations

The most celebrated pulsating variables of the recent past are clearly the EC 14026 or variable sdB (sdBV) stars. Their popularity arose from the rare incidence that the theoretical prediction appeared before the publication of observational evidence of such objects (Charpinet et al. 1996, Kilkenny et al. 1997). The observational data for this class continues to grow rapidly (e.g. Koen et al. 1998a) and an instability domain is beginning to emerge from these data. Theoretically, the instability is clearly induced by the Z-bump in the opacity. The sdBVs have rather large surface gravities and comparably high densities in their envelopes. A consideration of the 'opacity mountain', plotted over the density-temperature plane, shows that the sharpness of the Z-bump decreases as the density increases in the critical temperature interval. Therefore, there is no further surprise to realize that canonical heavy-element abundances are not sufficient to provide enough driving to overcome radiative damping. Nevertheless, the radiation field seems to be appropriate for radiative levitation to induce a spatial distribution of heavier ions to sufficiently steepen the Z-bump so that eventually the model stars become pulsationally overstable in the observed temperature – $\log g$ domain (e.g. Fontaine et al. 1998). Concerning the particularities of the Z-bump, PG 1605+072 is of interest (Koen et al. 1998b). It shows about a dozen, and possibly even more, oscillation modes with periods about in the range of 200 to 540 s. These periods are a factor two to three longer than what is found in the other sdBV stars. Spectroscopic deduction of

the stellar parameters indicates that PG 1605+072 has a surface gravity of up to 0.5 dex lower than the rest of the class. This might indeed hint at an enhanced excitation of p modes under lower-density conditions.

The success with the sdBV class fostered evolutionary and pulsational studies of post-horizontal branch stars evolving towards the white dwarf cooling tracks across the subdwarf domain. Therefore, the prediction of gravity-mode instabilities induced by the $\epsilon$-mechanism in the thin H-burning shell of low-mass stars having settled close to the white dwarf cooling track is not surprising (Charpinet, Fontaine, & Brassard 1997). Potential instabilities were found for low-$\ell$ and low radial-order g modes with periods between 40 and 120 s. The instability region extends from about 4.64 to 4.88 in log $T_{\mathrm{eff}}$. The corresponding objects in the sky would probably be classified as DAO stars. Currently, however, there is no observational evidence for the existence of such variables.

Based on the coincidence of the thermal time scale on top of a prospective driving region and the pulsation period, it is clear that the H I/He I ionization zone must play the dominant role in the excitation of the high-order p modes of roAp stars. Dziembowski & Goode (1996) also argued in this direction, but they were not successful in actually finding overstable modes. After postulating a chromosphere, Gautschy, Saio, & Harzenmoser (1998) could raise the outer edge of the p-mode cavity sufficiently high to eventually identify overstable roAp-like acoustic modes. Homogeneous stellar envelopes allowed, however, for rapidly oscillating Ap stars, as well as longer-period $\delta$ Scuti-like modes to be excited. Only after additionally introducing chemically stratified envelopes (hypothetically caused by element sedimentation) could they restrict the excited modes to the short-period domain. The Gautschy et al. (1998) picture still has weak points, such as the absence of any observational evidence for chromospheres in roAp stars, and too many simultaneously excited modes of various spherical degrees.

The nature, and in particular the physics, of strange modes is still debated. Their origin is becoming considerably clearer now. Buchler, Yecko, & Kolláth (1997) even found strange modes in Cepheid models. Their strange modes can be analyzed in the adiabatic limit, very much like in the massive main-sequence star models of Glatzel & Kiriakidis (1993) or Saio et al. (1998). Finding strange modes in the adiabatic limit removes much of their strangeness. It is seen that they owe their existence to a sharp ridge in the acoustic cavity where waves with appropriate oscillation frequency are effectively reflected, giving birth to an additional oscillation spectrum of the stellar envelope. Also, the large growth rates of these modes – let us call them adiabatic strange modes – are easily explainable. They are confined so much to the outermost stellar layers that the mode inertia is very small. From quasi-adiabatic treatments we know that the imaginary part of the eigenfrequency scales inversely with the mode inertia. Therefore, even for a rather low driving efficiency, the growth rate can become very large. These adiabatic strange modes are, however, only part of the whole story. As pointed out in Saio et al. (1998), there are stars with 'nonadiabatic strange modes' that do not show up in the adiabatic limit. In this latter case the cavity splitting is believed to develop only in the fully nonadiabatic case. In other words, for these modes, the interaction between the thermal and the mechanical reservoir of the oscillator is essential. The nonadiabatic strange modes are of particular interest, since they contain all the 'strange' mode physics,

such as instability bands in the nonadiabatic reversible limit. The crossings of adiabatic strange modes with regular modes, on the other hand, unfold into avoided crossings only. The detailed physics of the unfolding is still unexplained. A first step towards understanding the instability of nonadiabatic strange modes was made by Saio et al. (1998). In a simple model system, the radiation-pressure gradient is identified to drive strange-mode instabilities very much like the gas-pressure gradient does for dynamical instabilities. Numerical experience shows indeed that nonadiabatic strange modes pop up whenever the model stars show strongly radiation-dominated layers.

Partly in connection with strange modes in massive stars a well-documented controversy between Glatzel & Kiriakidis (1998) and Stothers (Stothers & Chin 1993, Stothers 1999) developed around the concept of dynamical instability. Stothers (1999) attributed nonlinear oscillatory instabilities of massive model-star envelopes to the nonadiabatic manifestation of an adiabatically diagnosed $\langle \Gamma \rangle < 4/3$ instability. The brackets denote a suitably defined spatial average; this suitability was also a point of controversy. Glatzel & Kiriakidis (1993) criticized dynamical-instability claims by Stothers & Chin (1993) and argued for the necessity of a fully nonadiabatic treatment to understand LBV variability and eruptions in these stars. Even if the nonadiabatic, nonlinear simulations of dynamically unstable models (computed with the adiabaticity constraint) are strongly pulsationally unstable, it is formally unclear how to connect the pulsational with the dynamical instability. In a system in which the time scale of energy exchange and the time scale for sound propagation through the system become comparable, the concept of dynamical instability loses its foundation. Adopting, however, a more pragmatic point of view, we can certainly say that whenever we come across a stellar model with a dynamically unstable adiabatic fundamental mode, it is worth a closer look as it is obviously hardly bound anymore, even in a thermodynamically more appropriate framework.

Based on the Chandrasekhar–Milne expansion of the mechanical structure equations for a rotating star, Lee (1998) investigated the influence of rotational deformation on the stability of axisymmetric acoustic and gravity modes in B-type main sequence stars. Acoustic nonradial modes were found to be stabilized at high rotation frequencies, i.e. for rotation speeds approaching break-up speed. Quasi-radial modes, however, remained overstable even in the very rapid rotators. The results were, therefore, comparable with the conclusions by Lee & Baraffe (1995), who studied the same phenomenon for non-axisymmetric perturbations. Lee's (1998) explorative nonadiabatic investigation of low-frequency g modes, representative for the pulsation modes in slowly pulsating B stars (SPBs), led him to conclude that they are unlikely to be damped out by the star's rotation, despite the fact that the importance of rotation, measured by the ratio of the rotational to the oscillation frequency, is higher for the g modes than for the acoustic modes. Quasi-adiabatic eigensolutions by Ushomirsky & Bildsten (1998) showed, on the other hand, that g modes appropriate for SPBs could be damped by rotation. Furthermore, oscillation modes which were found to be stable in non-rotating models could be spun up to overstability. Ushomirsky & Bildsten (1998) argued that it is the period measured in the co-rotating frame of reference which is the important quantity to fit the thermal time scale of the envelope above the driving region. Lee's (1998) view is different; he argues that the mean radius of the equipotential surfaces, introduced in

his analyses, shifts the excitation region to larger effective radii as the rotation speed of a star increases. Thereby, the effective period which can match the thermal time scale envelope overlying the excitation region drops. This effect was found to be more pronounced for the p modes in $\beta$ Cephei stars than for the g modes in SPBs. In contrast to the Ushomirsky & Bildsten (1998) analysis, which relies on the direct solution of the tidal equation, Lee (1998) used a two-term expansion in the $\theta$-dependence of the perturbations which might not be fully adequate for the problem. In other words, the quasi-adiabatic analyses versus the Legendre-polynomial expansions in latitude have not yet converged to a common prediction. In any case, it appears to be clear that rotating models are necessary to fully understand the observed populations of blue main-sequence pulsators, particularly in stellar clusters. It might well be that rotation removes some of the instabilities found in non-rotating models. Rapid rotators might also be unstable at lower effective temperatures than non-rotating stars. As stellar rotation constricts the amplitudes of g modes to lower stellar latitudes, the lack of rapid rotators observed among SPB stars (e.g. Balona & Koen 1994) might possibly be attributable to an observational selection effect in the end.

The behavior of r modes in differentially rotating stellar envelopes was studied by Wolff (1998) in connection with solar oscillations. The geostrophic mode with vanishing oscillation frequency measured in the co-rotating frame picks up a finite oscillation frequency in the differential rotation case. The r-mode spectrum consists then of a slow and a fast branch. Depending on the magnitude of rotational shear and the generalized spherical degrees of rapid and slow r modes, their characters can converge. They even can merge in frequency space (as a function of shear parameterization) and thereafter cease to exist.

## 3. Compact Objects

As white dwarfs cool, the central density eventually rises sufficiently for the stellar matter to pass through a first-order phase transition to crystallize. The existence of a crystalline interior has consequences for the interpretation of the white dwarfs' luminosity function and therefore their dating. As DA white dwarfs can show up as g-mode pulsators whose instability region – measured in the $\rho$-$T$ plane – extends into the domain where crystallization could have started, they are prospective candidates to search for observable consequences of a solid interior. Indeed, the DAV star BPM 37093 is the prime candidate for such investigations. Bradley (1996) contemplated that crystallization should manifest itself in the rate of period change which differs from a star with a Coulomb-fluid interior, first because of the release of latent heat at crystallization and later on due to Debye cooling. The monitoring time necessary to detect such an effect would be very long – of the order of ten to twenty years. The effect dominating the period change might, however, be the growing of the crystallized sphere in the star's interior (Winget et al. 1997). The latter authors suggested, based on a simplified treatment of the linear eigenvalue problem, the measurement of the period spacings of identified g modes in stars like BPM 37093 to deduce the magnitude of crystallization. They found that the mean period spacing, measured relative to an uncrystallized star with otherwise identical properties, increases by up to about 30%, depending on the mass fraction of the crystallized

interior. As the mean period spacing of g modes also depends on other stellar parameters, such as thickness of H and He layers and position on the HR plane, complementary mode properties have to be found to measure crystallization conclusively.

A more favorable environment to observe effects of solid stellar matter was discussed by Duncan (1998). He suggested a search for seismic toroidal modes excited during star-quakes in neutron stars. If soft gamma repeaters (SGR) are highly magnetized neutron stars experiencing fracture events in their solid crusts, global seismic oscillations can be excited very much like on Earth. As the crust is rather superficial, seismic modes might attain significant amplitudes on the surface to make them observable. Duncan (1998) therefore proposed a search for periodicities in SGR signals from some ten Hz upwards.

Considerable efforts have gone into studying relativistic oscillations of compact objects. A huge body of data exists showing time-variable phenomena in the frequency domain from Hz to kHz which are, in principle, attributable to oscillations on neutron stars. Much of the present enthusiasm originated from the prospect to observe such signatures in the forthcoming gravitational wave detectors (Owen et al. 1998).

The Chandrasekhar–Friedman–Schutz (CFS) instability is known to destabilize spheroidal modes in inviscid rotating relativistic bodies. Under the influence of viscous forces, only f modes with $\ell = m = 2$ survive in objects rotating close to break-up speed (e.g. Lindblom 1995). Therefore, the CFS instability was considered to be of little importance in nature. This situation changed when Andersson (1998) found that toroidal modes in rotating stars are strongly overstable to the CFS gravitational-radiation reaction. The instability grows proportional to $(\Omega\sqrt{R^3/M})^{10}$ for sectoral dipole modes; $\Omega$ stands for the rotation rate of a star with mass $M$ and radius $R$. Friedman & Morsink (1998) proved that Andersson's numerical finding applies to any relativistic rotating body. For most of the r modes, the CFS instability persists at any value of $\Omega$. Hence, the severe constraint of very rapidly rotating relativistic bodies associated with spheroidal modes disappeared. Nevertheless, to estimate which modes could survive in realistic neutron stars, the effects of rather uncertain viscosity effects needed to be studied. First attempts revealed that r modes might survive in the temperature window between $10^9$ and $10^{10}$ K. Below the lower boundary, shear viscosity damps the r modes, and above about $10^{10}$ K bulk viscosity overcompensates the CFS instability. In this scenario, young hot, rapidly spinning neutron stars are expected to pass through the instability window within the first few years after their birth, losing up to $0.01 M_\odot c^2$ of energy, thereby slowing their rotation rate to five to ten percent of the break-up rotation speed. This mechanism could explain why some young pulsars are observed to rotate slowly compared with what is expected from angular-momentum conservation during collapse. Furthermore, the r-mode instability would not permit milli-second pulsars to form via accretion-induced collapse of white dwarfs. The white dwarfs would get too hot in the process and lose much of their angular momentum through the CFS instability. Instead, milli-second pulsars must be formed by means of accretion onto a neutron star, keeping the recipient object always at sufficiently low temperature to suppress the r-mode instability. Observationally, gravitational wave radiation generated by the CFS instability is now expected

to be detectable by enhanced LIGO interferometers to distances as far as Virgo cluster (Owen et al. 1998).

The r modes referred to in the last paragraph are only one sub-class of inertial modes that owes their existence to rotation. Others, for which the spheroidal contribution to the eigensolutions is of comparable strength as the toroidal one were investigated for CFS instability by Yoshida & Lee (1999). They find that the 'inertial modes' in these stars are also CFS unstable, but less so than the r modes. Nevertheless, the most overstable inertial modes appear to survive even in the dissipative case.

Thermally or compositionally stratified neutron stars also possess g modes with sufficiently long periods to satisfy the condition for CFS instability. Lai (1999) investigated this case and concluded that g modes become overstable by means of the CFS instability if the rotation rate is comparable or larger than the considered g-mode frequency. Viscosity seems to extinguish the instability, except possibly around temperatures of $10^9$ K. The g-mode instability is, in any case, orders of magnitude weaker than the inertial-mode instability and might, therefore, not be of importance in nature.

Gravity-mode oscillations in neutron stars have been a focus of numerous studies in the past (see e.g. Gautschy & Saio 1995). Lately, Bildsten & Cumming (1998) added a new prospective g-mode cavity to the theoretical picture. They proposed an additional class of g modes in matter-accreting neutron stars. These modes owe their existence to the compositional inhomogeneity developing at the base of the hydrogen and helium layer that builds up on the surface and transmutes into iron-group elements by unstable H/He burning and by electron captures. Very much like at the outer edge of the convective cores of massive main-sequence stars, a sharp gradient in the molecular weight builds up which acts as a cavity for the previously mentioned g modes and an interface mode. In the case of the upper main-sequence stars this kind of gravity mode was baptized core g modes. In the case of neutron stars the cavity lies rather close to the surface, actually at the base of the superficial suprafluid ocean. We refer to modes being essentially trapped in this composition interface as interface g modes. In addition, if the neutron star's background is not isentropic, the non-vanishing Brunt–Väisälä frequency permits the 'normal' thermal buoyancy g modes to show up. The resulting frequency spectrum can then become rather intricate as the frequency domains of the interface and thermal g modes can overlap. In the adiabatic mode treatment of Bildsten & Cumming (1998) the resulting mode interaction between the two families unfolds into avoided crossings.

The complicated nuclear physics and thermodynamics in neutron stars can produce a multitude of narrow g-mode cavities by density jumps alone. Much of the attention given to the interface g modes at the bottom of the superficial suprafluid originates from the frequencies of these modes which are compatible with observed QPOs in the frequency domain of a few times 10 Hz. The adiabatic properties of the interface modes in slowly rotating neutron stars have been well discussed. The case of rapid rotation – measured relative to the g-mode frequency – has still to be tackled in more detail. In particular, however, the excitation mechanism of the interface g modes and f-type interface mode have to be addressed. As the interface modes are well trapped in the compositional transition region, they will hardly see anything of the overlying H/He burning. A conceivable driving agent could, however, be an electron-capture instability

of the transition layer becoming oscillatory by nonadiabaticity so close to the neutron star's surface.

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## Discussion

*Margarida Cunha*: How is it possible that your model accounts for inhomogeneous chemical composition only in the outermost layers of the star (associated with the H-ionization layer) and not all the way down to the base of the 2nd convective layer (associated with the $He^{++}$-ionization layer)? If the two convective layers are connected by overshooting, one should have homogeneous composition across the two convective layers.

*Alfred Gautschy*: The ad hoc idea was to allow sedimentation only in regions where magnetic pressure exceeds the gas pressure. In our models this implied a depletion down to about $10^4$K. Since we do not have any overshooting or comparable transport processes between the ionization zones, we do not have any consequences as deep as suggested. As mentioned, this is an ad hoc model. A more realistic treatment is clearly desirable. At the moment, our problem is the "*how*"!

*Tim Bedding*: If a roAp star has a temperature inversion in the atmosphere, wouldn't you expect to see this in the spectrum (e.g. emission in H$\alpha$ core)?

*Alfred Gautschy*: In principle, yes. The $\Delta T$ is possibly so low, however, that the spectral signature is very weak. Maybe different excitation levels of a suitable atom and their relative phase relations might help one day.