

mixing boxes, on page 22, illustrating operations on a set S that are commutative or associative are not particularly convincing and would probably be better omitted.

The set of natural numbers is defined as an algebraic system $(\mathbb{Z}; +, \cdot, <)$ with two operations and a relation. The main defect in this approach, from the point of view of beginners, is that the reader is subjected immediately to a formidable list of axioms. Some properties of the natural numbers, including prime factorisation, are developed.

The number system is then extended through the following sets:

- (1) the positive rationals (by ordered pairs of natural numbers),
- (2) the non-negative real numbers (using Cauchy sequences of positive rational numbers),
- (3) the real numbers (by ordered pairs of non-negative real numbers).

In the introduction of (2), the familiar modulus notation $|a-b|$ is not available because of the approach through the positive rational numbers rather than through the set of all rational numbers. The collection of symbols $|a, b| < c$ is used to stand for the assertion

$$a < b + c \text{ and } b < a + c,$$

and is read “ a and b are within c of each other”. Students may find difficulty in mastering this situation.

There are sections on least upper bounds and greatest lower bounds and the decimal representation of real numbers. Four appendixes, A, B, C and D, contain, respectively, work on cardinal numbers, an introduction to the complex numbers, a statement of the alternative introduction by Peano's axioms to the natural numbers, and a brief note on continued fractions.

The book is well written with good collections of stimulating examples in the exercises. The printing and layout are excellent.

J. HUNTER

TODD, J., *A Survey of Numerical Analysis* (McGraw-Hill, New York, 1962), 589 pp., 97s.

This excellent book arose from a series of lectures and discussions organised by the National Bureau of Standards to attract experienced mathematicians into the field of numerical analysis. The editor has succeeded in preserving a continuity of material throughout the volume which is unusual in a work where practically every chapter has been written by a different author. Although the book is in the main most suitable for post-graduate students, the reviewer feels that its principal function may well be to make the subject of numerical analysis more respectable in the eyes of many mature applied mathematicians.

After an introduction in which the editor outlines the branches of mathematics which have contributed significantly to the development of numerical analysis, there is an adequate chapter on classical numerical analysis including interpolation, numerical differentiation and integration, and the numerical solution of ordinary differential equations of simple type. This is followed by the constructive theory of functions in which the existence of finite polynomials of best approximation to given continuous functions is shown.

The aim of the next chapter is to introduce the reader to the actual use of automatic computers. A hypothetical computer is constructed and programmes written for it. Then follows a section on computer logic, a topic as yet unconnected with numerical analysis, the subject which has done most to motivate the design of high speed computers.

The chapter on matrix computations is concerned with the solution of linear equations and the inversion of matrices together with the determination of eigenvalues

and eigenvectors. In general, direct methods are described for small matrices and iterative methods for large sparse matrices. Eigenvalues of finite matrices are given separate consideration and many useful theorems are given for locating and giving bounds for particular eigenvalues. Sandwiched between the chapters on matrices is a section on numerical methods for finding solutions of non-linear equations.

The numerical solution of ordinary differential equations is concentrated entirely on initial value problems involving either first or second order equations. Runge-Kutta and finite difference methods including stability conditions for the latter are discussed in detail. Little mention is made of boundary value and eigenvalue problems. An extensive numerical treatment of elliptic partial differential equations follows. Only iterative methods are discussed. These include the successive-overrelaxation and alternating-direction implicit methods, with their guaranteed rates of convergence for certain problems. A rather scanty discussion is then given of parabolic equations in one and two space variables. In fact the main emphasis is placed on methods for solving time dependent problems which lead to new techniques for solving elliptic equations. There is no mention of hyperbolic equations.

The chapter on numerical methods for linear integral equations is devoted mainly to the discussion of Fredholm integral equations. A variety of methods of solution is given including linear algebra, iteration, and analogy methods. The numerical methods of solution are explained in general without proofs.

The application of results in functional analysis to problems in numerical analysis is then discussed. In particular, it is shown that many of the iteration procedures used extensively in approximate methods are applications of the central-fixed-point theorem in functional analysis. The iteration methods described include conjugate-direction methods, the method of steepest descent and Newton's method for solving a linear or non-linear equation.

The remaining chapters contain a complex variable method for estimating errors in approximate formulae for integration, interpolation, etc., orthonormalising codes in numerical analysis, discrete problems mainly in group theory which might be profitably attacked by machine methods, algebraic number theory computations and linear estimation in the theory of applied statistics.

The book is excellently printed and singularly free of errors. Each chapter has an extensive list of references up to and including articles published in 1960.

A. R. MITCHELL

ZURMÜHL, R., *Matrizen und ihre technischen Anwendung*, 3rd edition (Springer-Verlag, Berlin, 1961), 459 pp., DM 36.

At a time when it is almost becoming fashionable to write mathematical books in an obscure and arid style, it is a pleasure to read a book, such as the one under review, in which the exposition is outstandingly clear. For the readers for whom the work is intended, namely, engineers and technologists, the contents of the book and their mode of presentation could hardly be bettered. All of what would be contained in any standard course on the elementary theory of matrices (linear equations, quadratic forms, latent roots etc.) and some additional matters (such as the Jordan canonical form and matrix equations) are treated with Teutonic thoroughness in the first 270 pages; an elementary knowledge of determinants is presupposed. The remainder of the book is devoted to numerical methods (for solving sets of linear equations and finding latent roots) and to applications drawn from various fields of technology. In this part of the book there are many references to work carried out in the 1950's.

The book, which is beautifully printed, is likely to remain a standard work on this aspect of matrix theory for a long time to come and can be confidently recommended to students and teachers alike.

D. MARTIN