INVERSE INELASTIC SCATTERING THEORY

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For inverse scattering theories , it is well known that in order to guarrantee the existence and uniquness of the solution , drastic restrictions must be imposed on the scattering potential or on the scattering data collected from experiments (see for example Newton 1966) . The existing theories in their original form such as the Geldfan Levitan or the Marchenko equations therefore cannot be used in a straightforward manner and some further steps toward simplifications have to be considered .

In inelastic scattering problems where many coupled channels must be taken into account to construct various elements of the scattering matrix the problem become much more complicated both from thetheoretical and experimental point of view , The present work deal only with the energy fixed case and show that a solution can nevertheless be obtained by using on one hand the conventional technique of the Abelian transformation which has been extensively applied in the JWKB approach to the one channel problem (see for example a general review by Buck 1971) with on the other , some results obtained recently concerning a system of coupled differential equations (Cao 1982 I) .

The case of two states approximation which may be interesting for its possible applications is investigated starting from the results of the direct problem (Cao 1984 II) but its possible extension to the general case of a finite number of coupled channels will be omitted for space limitation .

The mathematical description of the 2 states problem for simplicity , is assumed to be , in matrix notation :

$$(L + U)Y = 0$$

$$Y = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} \qquad L = \begin{pmatrix} L_0 & 0 \\ 0 & L_1 \end{pmatrix} \qquad U = \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11} \end{pmatrix}$$

$$L_i = \frac{d^2}{dr^2} + k_i^2 - \frac{\ell(\ell+1)}{r^2} \qquad i = 0, 1$$

where we continue to keep the notations and conventions of Ref II . The interaction matrix U is generally symmetric so that we may set

$U_{01} = U_{10} = B$, B will be referred to as the coupling function of the system .

Let $\delta_\ell^{\pm}$ be the proper phase schift i.e. the phase schift one would have after separation of the coupled equations With the following definitions :

$$I_\pm(s_\pm, E) = \mathrm{Log}\,\frac{r(s_\pm)}{s_\pm}$$

$$s_\pm^2 = r^2\left\{1 - \frac{1}{2E}\left[\frac{1}{2}(U_{00}+U_{11}) \pm \frac{1}{2}\left[(\Delta k^2 + \Delta U)^2 + 4B^2\right]^{1/2}\right]\right\}$$

$$\Delta k^2 = k_1^2 - k_0^2 \qquad \Delta U = U_{11} - U_{00}$$

E : incident energy

and using the technique of Abelian transformation we obtain :

$$I_\pm - \alpha_\pm = \frac{2}{K\pi}\int_{s_\pm}^{\infty} \frac{1}{(b^2 - s_\pm^2)^{1/2}}\frac{d\delta_\ell^\pm}{db}\,db$$

$$K^2 = \frac{1}{2}(k_0^2 + k_1^2) \qquad b = \frac{1}{K}(\ell + 1/2) \qquad \alpha_\pm = \mathrm{Log}\,\frac{E}{E \mp \Delta k^2}$$

The various elements U_{ij} of the interaction matrix U are therefore related to the quantities $I\pm$ by a system of 2 equations :

$$\begin{aligned} U_{00} + U_{11} &= \frac{1}{2E}\left\{2 - \left(e^{-2I_-} + e^{-2I_+}\right)\right\} \\ (\Delta k^2 + \Delta U)^2 + 4B^2 &= \frac{1}{4E^2}\left[e^{-2I_-} - e^{-2I_+}\right]^2 \end{aligned} \qquad (1)$$

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In the one channel case, the diagonal terms U_{ii} are often referred to as the interaction potential of the ith channel which is described by an equivalent homogeneous equation. They can usually be inferred by other means, for example for sufficiently strong attractive forces, U_{ii} can yield a discrete energy spectrum. Therefore they can be evaluated from the wave function corresponding to channel i. Particularly, it is reasonable to assume that the U_{oo} term which is related to the ground state can be attained by this way. The two equations in (1) are then sufficient in principle to lead to the unknowns U_{11} and B provided that $d\delta^{\pm}_{\ell}/d\ell$ can be determined from experimental data.

In order to simplify the presentation, we shall limit ourselves to the case of "near resonance" (i.e. $\Delta k^2 \simeq 0$) for which $\alpha_{\pm}$ is set equal to zero. Assuming then that the elastic and inelastic partial cross section Q^o_{ℓ}, Q^1_{ℓ} are known, it can be shown that $\delta^{\pm}_{\ell}$ are given by the following equations:

$$\delta^+_{\ell} = P_1 + \delta^-_{\ell} \qquad (2) \qquad P_1 = \sin^{-1}\left[\frac{2k_o}{[(2\ell+1)\pi]^{1/2}}\,\frac{Q_{1\ell}^{1/2}}{\sin 2\varepsilon}\right]$$

$$\sin^2\varepsilon = \frac{d^2}{c^2+d^2} \qquad \cos^2\varepsilon = \frac{c^2}{c^2+d^2} \qquad f_o = \frac{k_o^2}{4\pi(2\ell+1)}\,\frac{Q_{1\ell}+Q_{o\ell}}{\cos^2\varepsilon}$$

$$\left(\cos 2P_1 + tg^2\varepsilon\right)\sin^2\delta^-_{\ell} + \frac{1}{2}\sin 2P_1 \sin 2\delta^-_{\ell} + \left(\sin^2 P_1 - f_o\right) = 0 \qquad (3)$$

The quantities $d\delta^{\pm}_{\ell}/d\ell$ needed in the determination of $I_{\pm}$ are then given by :

$$d\delta^+_{\ell}/d\ell = 2\left(\sin 2\delta^+_{\ell} + tg^2\varepsilon \sin 2\delta^-_{\ell}\right)^{-1}\left[g_o - \frac{g_1}{(1-f_1)^{1/2}}\sin 2\delta^+_{\ell}\right]$$

$$d\delta^-_{\ell}/d\ell = 2\left(\sin 2\delta^+_{\ell} + tg^2\varepsilon \sin 2\delta^-_{\ell}\right)^{-1}\left[g_o + \frac{g_1}{(1-f_1)^{1/2}}\,tg^2\varepsilon \sin 2\delta^+_{\ell}\right]$$

$$g_o = -\frac{k_o^2}{4\pi(2\ell+1)^2}\,\frac{1}{\cos^2\varepsilon}\left[(2\ell+1)\left(\frac{dQ_{1\ell}}{d\ell} + \frac{dQ_{o\ell}}{d\ell}\right) - \left(Q_{1\ell}+Q_{o\ell}\right)\right]$$

$$g_1 = \frac{1}{2}\,\frac{k_o}{[(2\ell+1)\pi]^{1/2}}\,\frac{1}{\sin 2\varepsilon}\,\frac{1}{Q_{1\ell}^{1/2}}\left(\frac{1}{2}\frac{dQ_{1\ell}}{d\ell} - \frac{1}{2\ell+1}Q_{1\ell}\right); \quad f_1 = \frac{k_o^2}{(2\ell+1)\pi}\,\frac{Q_{1\ell}}{\sin^2 2\varepsilon}$$

The quantity ε is related to an arbitrary parameter n (see Ref II) which must be chosen according to the validity condition of the theory :

$$\frac{B}{\Delta k^2 + \Delta U} \ll \frac{1}{\alpha^2(n)}\left(2n + 1/4\right) \qquad (4)$$

Assume first that the element U_{11} is known a priori. The following algorithm is needed to solve the problem :

(i) Let $\{n\}$ be the set of positive values of real number. To each element of this set correspond an element of the set $\{ I_{\pm}(E, n) \}$

(ii) Let $\{n_j\}$ be a subset of $\{n\}$ such that the first equation of system (1) is verified.

(iii) Corresponding to each n_j we can now compute the element of the set $\{B(n_j)\}$ from the second equation of (1) and for each case check the validity condition (4). The appropriate value n_o is then the one for which this condition is satisfied best. The non existence of such n_o mean on the other hand that we must go over to the second order separation of the equations as indicated in Ref I.

The case where both U_{11} and B are unknown is obviously more complicated but nevertheless tractable, the results are summarised below : Define

$$V_{\pm} = \frac{1}{2}\left\{ U_{11} \pm \left[(\Delta k^2 + \Delta U)^2 + 4B^2 \right]^{1/2} \right\} \; ; \quad A_{\pm\ell} = \left(\frac{2}{\mu}\right)^{1/2}\left(\delta_\ell^{\pm} - \delta_\ell^{(c)} \right)$$

in which μ is the reduced mass, $\delta_\ell^{(c)}$ is the conventional JWKB phase schift corresponding to the potential $\frac{1}{2}U_{oc}$. We find then

$$\begin{aligned} V_{\pm}(r) &= \frac{2}{\pi} f(U_{oo}) \int_r^{\infty} \frac{1}{(s^2 - r^2)^{1/2}} \frac{dA_{\pm\ell}}{ds} ds \\ V_+ - V_- &= \frac{1}{2E}\left(e^{-2I_-} - e^{-2I_+} \right) \end{aligned} \qquad (5)$$

where the expression of $f(U_{oo})$ is (Zerarke 1984)

$$f(U_{oo}) = E - \frac{1}{2}U_{oo} - \frac{1}{4} r \frac{dU_{\infty}}{dr}$$

The above algorithm remains valid, the first equation of (5) can be used to evaluate V_+, V_- while the second one serving to define the subset $\{n_j\}$ of the second step (ii).

The non resonance case ($\Delta k^2 \neq 0$, $\alpha_{\pm} \neq 0$) is more complicated in the sense that the quantities $\delta^{\pm}_\ell$ have tobe reinterpreted and the relations (2), (3) must be modified accordingly. It can be shown that the problem can nevertheless be made tractable on the same basis .

To conclude , we may point out that the above results are derived in the frame of a JWKB treatment which imply strict limitations inherent to a short wave lenght approximation . We find however that it is also possible to generalise the theory to the case where such an approximation does not apply by including to the above approach some properties of non linear differential equations .

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