Appendix 8
Symmetry properties of the Cartesian reaction parameters

We consider $A + B \to A + B$, where all particles have spin 1/2 but A and B need not be identical. The additional symmetries when $A \equiv B$ are given separately. Many of the results given were derived by Thomas (Thomas, 1969). Results for $0 + 1/2 \to 0 + 1/2$ are obtained by simply suppressing the α, α' labels everywhere. For $1/2 + 1/2 \to 0 + 0$ the labels α' and β' are suppressed everywhere.

A8.1 The CM reaction parameters

To begin with there are 256 parameters.

(*a*) *Parity.* Use of parity in both H amplitudes gives (see (5.6.4))

$$(\alpha\beta|\alpha'\beta') = \xi_\alpha^{\mathscr{P}} \xi_\beta^{\mathscr{P}} \xi_{\alpha'}^{\mathscr{P}} \xi_{\beta'}^{\mathscr{P}} (\alpha\beta|\alpha'\beta') \tag{A8.1}$$

where

$$\xi_0^{\mathscr{P}} = \xi_Y^{\mathscr{P}} = 1 \qquad \xi_X^{\mathscr{P}} = \xi_Z^{\mathscr{P}} = -1.$$

This implies that the parameter is zero when the number of X labels plus the number of Z labels in it is an odd number. This eliminates one half of the coefficients, leaving 128.

Use of parity in just one amplitude leads to

$$(\alpha\beta|\alpha'\beta') = \xi_\alpha \xi_\beta \xi_{\alpha'}^* \xi_{\beta'}^* (\alpha_{\mathscr{P}} \beta_{\mathscr{P}} | \alpha'_{\mathscr{P}} \beta'_{\mathscr{P}}) \tag{A8.2}$$

where

$$\xi_0 = \xi_Y = 1 \qquad \xi_Z = -\xi_X = i$$

and for any label α, $\alpha_{\mathscr{P}}$ means

$$0 \longleftrightarrow Y \qquad X \longleftrightarrow Z.$$

An example: under $\mathscr{P}$, $(00|Y0) = (YY|0Y)$.

This eliminates 64 coefficients, leaving 64.

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(*b*) *Time reversal.*[1] Use of time reversal in both H amplitudes gives

$$(\alpha\beta|\alpha'\beta') = \xi_\alpha^{\mathcal{T}}\,\xi_\beta^{\mathcal{T}}\,\xi_{\alpha'}^{\mathcal{T}}\,\xi_{\beta'}^{\mathcal{T}}(\alpha'\beta'|\alpha\beta) \tag{A8.3}$$

where

$$\xi_0^{\mathcal{T}} = \xi_Y^{\mathcal{T}} = \xi_Z^{\mathcal{T}} = 1 \qquad \xi_X^{\mathcal{T}} = -1.$$

This eliminates 12 coefficients, and the use of parity combined with time-reversal a further 12, leaving 40. Use of time reversal in one H only gives

$$(\alpha\beta|\alpha'\beta') = \eta_\alpha\eta_\beta\eta_{\alpha'}^*\eta_{\beta'}^* \sum_{\nu,\nu',\gamma,\gamma'} C_{\gamma'\gamma}^{\alpha'_{\mathcal{T}}\alpha_{\mathcal{T}}} C_{\nu'\nu}^{\beta'_{\mathcal{T}}\beta_{\mathcal{T}}}(\gamma\nu|\gamma'\nu') \tag{A8.4}$$

where, for any label α, $\alpha_{\mathcal{T}}$ means

$$X \longleftrightarrow Y \qquad 0 \longleftrightarrow Z.$$

Also,

$$\eta_0 = \eta_Z = 1 \qquad \eta_X = -\eta_Y = i$$

and

$$C_{\gamma'\gamma}^{\alpha'\alpha} = -\tfrac{1}{16}\zeta_\alpha\zeta_\gamma \,\mathrm{Tr}\,(\sigma_{\gamma'}\sigma_{\alpha'}\sigma_\gamma\sigma_\alpha)$$

with

$$\zeta_0 = \zeta_X = \zeta_Z = 1 \qquad \zeta_Y = -1.$$

This leads to four new conditions:

$$\begin{aligned}(XX|ZZ) &= (XX|XX) - (YY|00) - 1\\ (XZ|XZ) &= (XX|XX) + (0Y|0Y) - 1\\ (ZX|XZ) &= -(XX|XX) + (Y0|Y0) + 1\\ (Z0|Z0) &= (0Z|0Z) + (X0|X0) - (0X|0X).\end{aligned} \tag{A8.5}$$

The first three of these were given by Thomas for NN scattering.[2]

We are now left with 36 linearly independent reaction parameters, just what is expected since there are six independent helicity amplitudes in the reaction. The expressions for the 36 observables parameters in terms of the helicity amplitudes are given in Appendix 10.

[1] Clearly the results of this section do not hold for $1/2 + 1/2 \to 0 + 0$.

[2] Thomas found these by 'brute force' from studying the relations between observable parameters and helicity amplitudes – he knew that three extra conditions had to exist.

(*c*) *Identical particles.* When $A \equiv B$ as in nucleon–nucleon scattering we get

$$(\alpha\beta|\alpha'\beta') = \xi_\alpha^{\mathscr{S}} \xi_\beta^{\mathscr{S}} \xi_{\alpha'}^{\mathscr{S}} \xi_{\beta'}^{\mathscr{S}} (\beta\alpha|\beta'\alpha') \tag{A8.6}$$

where

$$\xi_X^{\mathscr{S}} = \xi_Y^{\mathscr{S}} = -1 \qquad \xi_O^{\mathscr{S}} = \xi_Z^{\mathscr{S}} = 1.$$

This eliminates 11 of the parameters, leaving the customary 25.

A8.2 The Argonne Lab reaction parameters

The label 'ARG' is only appended where confusion is possible.

(*a*) *Parity.* An Argonne Lab parameter vanishes if the number of S labels plus the number of L labels in it is an odd number.

Also one has

$$(\alpha\beta|\alpha'\beta')_{\text{Lab}}^{\text{ARG}} = \xi_\alpha \xi_\beta \xi_{\alpha'}^* \xi_{\beta'}^* (\alpha_{\mathscr{P}}\beta_{\mathscr{P}}|\alpha'_{\mathscr{P}}\beta'_{\mathscr{P}})_{\text{Lab}}^{\text{ARG}} \tag{A8.7}$$

where ξ_α and the parity operation $\mathscr{P}$ are defined in (A8.2). Of course the $\mathscr{P}$ operation now reads

$$0 \longleftrightarrow N \qquad S \longleftrightarrow L$$

and

$$\xi_0 = \xi_N = 1 \qquad \xi_L = -\xi_S = i.$$

Some examples are the following

$$C_{NN} = A_{NN} \qquad D_{NN}^{(A)} = D_{NN}^{(B)} \qquad K_{NN}^{(A)} = K_{NN}^{(B)}$$
$$(LS|NN)_{\text{Lab}}^{\text{ARG}} = (SL|00)_{\text{Lab}}^{\text{ARG}} = A_{SL}.$$

(*b*) *Time reversal.* One finds

$$(\alpha,\beta|\alpha',\beta')_{\text{Lab}}^{\text{ARG}} = (\alpha'_{\mathscr{T}},\beta'_{\mathscr{T}}|\alpha_{\mathscr{T}},\beta_{\mathscr{T}})_{\text{Lab}}^{\text{ARG}} \tag{A8.8}$$

where for particle A (i.e. for α, α')

$$0_{\mathscr{T}} = 0 \qquad N_{\mathscr{T}} = N$$
$$S_{\mathscr{T}} = -\cos\alpha_C\, S + \sin\alpha_C\, L \qquad L_{\mathscr{T}} = \sin\alpha_C\, S + \cos\alpha_C\, L$$

while for particle B (i.e. for β, β')

$$0_{\mathscr{T}} = 0 \qquad N_{\mathscr{T}} = N$$
$$S_{\mathscr{T}} = \cos\theta_{\text{R}}\, S + \sin\theta_{\text{R}}\, L \qquad L_{\mathscr{T}} = \sin\theta_{\text{R}}\, S - \cos\theta_{\text{R}}\, L.$$

Here α_C is of course the Wick helicity rotation angle for $C =$ final particle A; $\alpha_C = \theta_{\text{L}}$, the Lab scattering angle, for $NN \to NN$, see subsection 2.2.4 and θ_{R} is the Lab recoil angle.

Some examples are the following:

$$A^{(A)}_{\mathrm{ARG}} \equiv (N0|00)^{\mathrm{ARG}}_{\mathrm{Lab}} = (00|N0)^{\mathrm{ARG}}_{\mathrm{Lab}} \equiv \mathscr{P}^{(A)}_{\mathrm{ARG}}$$

that is, the analysing power for particle A = the polarizing power for particle A.

Also

$$\begin{aligned} A_{SS} &\equiv (SS|00)^{\mathrm{ARG}}_{\mathrm{Lab}} \\ &= (00|-\cos\alpha_C\, S + \sin\alpha_C\, L, \cos\theta_{\mathrm{R}}\, S + \sin\theta_{\mathrm{R}}\, L)^{\mathrm{ARG}}_{\mathrm{Lab}} \\ &= -\cos\alpha_C \cos\theta_{\mathrm{R}}\, C_{SS} - \cos\alpha_C \sin\theta_{\mathrm{R}}\, C_{SL} \\ &\quad + \sin\alpha_C \cos\theta_{\mathrm{R}}\, C_{LS} + \sin\alpha_C \sin\theta_{\mathrm{R}}\, C_{LL} \end{aligned}$$

and

$$\frac{D^{(A)}_{LS} + D^{(A)}_{SL}}{D^{(A)}_{LL} - D^{(A)}_{SS}} = \tan\alpha_C \qquad \frac{D^{(B)}_{LS} + D^{(B)}_{SL}}{D^{(B)}_{SS} - D^{(B)}_{LL}} = \tan\theta_{\mathrm{R}} \qquad \text{etc.}$$

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