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Nonlinear Modeling of the Solar Tachocline

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Abstract. As a first step toward a more realistic model of the solar tachocline, we report simulations of rotating, stably-stratified turbulence in a thin spherical shell.

1. Introduction

Helioseismic Inversions have revealed a thin layer of radial shear in the solar internal rotation located near the base of the convective envelope (Schou et al. 1998). Although it may extend into the convection zone, this shear layer, termed the *solar tachocline*, appears to lie mainly in the stably-stratified region below (e.g. Charbonneau et al. 1999). Spiegal & Zahn (1992) proposed that the tachocline is maintained against radiative spreading by anisotropic turbulence which is kept nearly horizontal by the stratification and rotation. However, the details of how this may or may not occur remain controversial (Gilman & Fox 1997; Gough & McIntyre 1998). Further insight may be provided by numerical simulations which specifically address turbulence in thin spherical shells under the influence of rotation and stable stratification. This is the approach we have adopted and this poster represents the first results from our model.

2. The Model System

The model equations are derived from the thin-shell limit of the Boussinesq equations in a rotating, spherical geometry and are solved using a modified version of a parallel computer code previously applied to solar convection (Clune et al. 1999; Miesch et al. 2000; Elliott, Miesch & Toomre 2000). The nondimensional system is specified by six parameters: the shell thickness δ (assumed $\ll 1$), the Rossby number R_o, the internal Froude number F_r, the Reynolds number R_e, the Prandtl number P_r, and the diffusive anisotropy A, which allows for an anisotropic viscosity and diffusivity with a reduced vertical component. Without such anisotropy, diffusion would rapidly dissipate all vertical structure in the thin shell, eliminating much of the nonlinear dynamics of interest.

For the simulations reported here, $\delta = 0.04$, $R_e = 100$, $P_r = 1$, and A is chosen such that the vertical viscosity and diffusivity are reduced by a factor of δ^2 relative to their horizontal components. The Rossby and Froude numbers are varied in order to study the influence of rotation and stratification. Two types of simulations have been considered. In the first, the system is allowed to evolve freely beginning from an initial spectrum of random velocity fluctuations. The

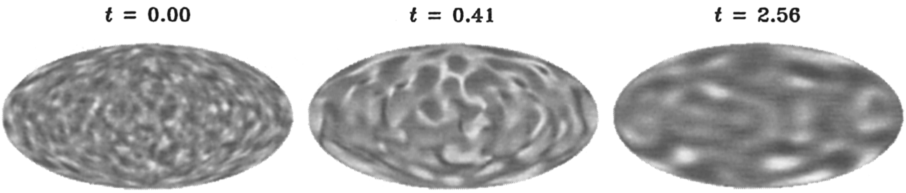

Figure 1. Evolution of the vertical vorticity near the top of the shell (z=0.94) for a decaying simulation with vortex initial conditions ($F_r = 0.1$, $R_o = 0.1$).

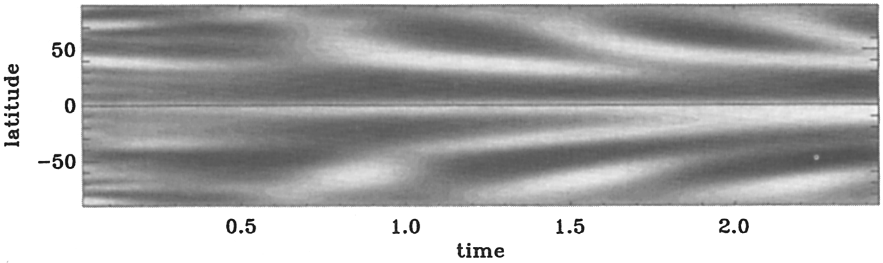

Figure 2. The mean zonal velocity as a function of latitude and time for a layer near the top of the shell (decaying simulation; wave initial conditions; $F_r = 0.1$, $R_o = 0.1$).

second type of simulation includes random, high-wavenumber, external forcing of the vertical vorticity field.

Rotating, stably-stratified turbulence can be loosely regarded as a superposition of two distinct velocity components which are coupled only through nonlinear interactions and Coriolis accelerations. The *vortex* component represents quasi-2D turbulence with vanishing horizontal divergence while the *wave* component represents internal gravity waves with vanishing vertical vorticity. In the freely-evolving (decaying) simulations, both *vortex* and *wave* initial conditions have been considered in which only one of the velocity components is nonzero. For the wave initial conditions, the temperature and pressure fields have also been initialized with relative amplitudes and phases appropriate for internal gravity waves.

3. Simulations of Decaying and Forced Turbulence

The decaying simulations initially exhibit a filamentary structure characterized by narrow vortex sheets and lanes of horizontal convergence (Fig. 1). An *inverse cascade* of kinetic energy from small to large scales is observed which is similar to but less efficient than that which occurs in strictly 2D turbulence. Later behavior is dominated by paired vortices and longitudinally-elongated flow structures which drift toward the equator. These give rise to mean zonal flows which propagate toward the equator on a timescale of 6–8 rotation periods (Fig. 2).

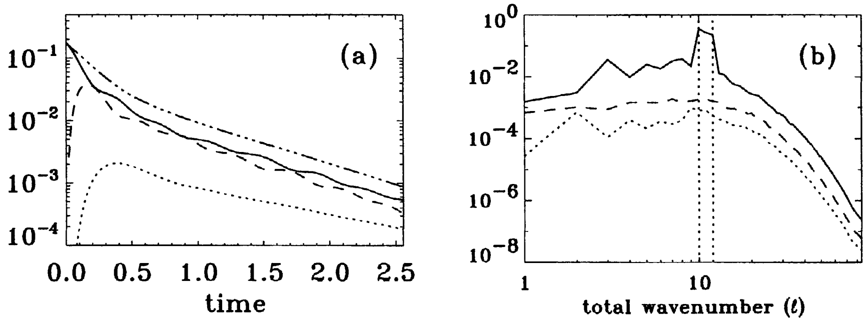

Figure 3. (a) Energy partition for a decaying simulation with vortex initial conditions and (b) time-averaged spectra at the middle of the shell for a simulation with high-wavenumber (ℓ=10–12) vortical forcing. In both cases $F_r = 0.1$, $R_o = 0.1$ and the solid line represents vortex kinetic energy, the dashed line wave kinetic energy, the dotted line potential energy, and the dash-dotted line total kinetic energy.

The wave flow component generally dissipates faster than the vortex flow component due in part to a more efficient downscale energy transfer by nonlinear interactions. In non-rotating simulations, the kinetic energy at early times is dominated by the flow component which makes up the initial conditions. However, in the rapidly rotating case, the two flow components are closely coupled and achieve approximate equipartition within about one rotation period (Fig. 3a). In simulations with external forcing, Coriolis forces and nonlinear interactions transfer energy to large and small scales and between the wave and vortex flow components (Fig. 3b).

Further results will be presented in forthcoming papers. Plans are also in store for higher-resolution simulations with external forcing, imposed shear flows, and magnetic fields.

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