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SUPERCONVERGENCE OF NUMERICAL SOLUTIONS TO SECOND KIND INTEGRAL EQUATIONS

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This thesis examines certain numerical methods for the solution of second kind Fredholm integral equations of the form

(1) $$u_0(x) - (Ku_0)(x) = f(x) \ , \quad x \in [0, 1] \ ;$$

where K is the operator defined by

$$(Ku)(x) = \int_0^1 k(x, \xi)u(\xi)d\xi \ .$$

Let u_n be the Galerkin solution to (1) using an n-dimensional space of piecewise polynomials of degree r as the basis space. It is known that $\|u_n-u_0\|_\infty \le O\left(n^{-r-1}\right)$. Chapter 2 shows that if u_n is used to calculate the iterated Galerkin solution, $u_n^* = f + Ku_n$, then (under suitable regularity conditions on k and f) the order of convergence is doubled to $\|u_n^*-u_0\|_\infty \le O\left(n^{-2r-2}\right)$. That is, u_n^* is globally superconvergent. If k fails to satisfy the regularity conditions because of a discontinuity along the diagonal $x = \xi$, then u_n^* still exhibits this $O\left(n^{-2r-2}\right)$ superconvergence at the grid points, but not globally. Chapter 3 shows for smooth k and f that global superconvergence is preserved when the integrations required to form the Galerkin equations are performed numerically. The proofs of Chapters 2 and 3 use the duality argument from

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the finite element literature.

In practice the kernel function k is rarely smooth. Chapters 4 and 5 consider product integration solutions to (1) when the kernel is of convolution type with a weak singularity. The high rates of convergence observed for the product integration solution when u_0 is smooth have been explained previously. However the singularity in the kernel introduces certain typical singularities into u_0 which reduce the rate of convergence. Chapter 4 uses a modified duality argument and a characterization of the singularity of the solution in terms of Nikol'skii spaces to prove these reduced orders of convergence.

Chapter 5 reports numerical experiments which indicate that the order of convergence can be restored by using an appropriate non-uniform grid. Such grids may be generated automatically by an adaptive method. This method uses the characterisation of the product integration solution as an iterated collocation solution.