are liberally quoted, there is a somewhat surprising lack of references to other important textbooks; e.g. uniformisation is discussed cursorily on p. 523, but the standard book of Nevanlinna is not mentioned. Nor does one find references to the well-known textbooks of Ahlfors, Sario, Hille, Lehner, Thron. Pfluger and Behnke-Sommer, however, are quoted several times.

Complex analysis has assumed such huge dimensions that a textbook can never be complete. A personal impression of mine is that the theory of univalent functions - with coefficient estimates along Jenkins' line - and some rudiments of Schiffer's variational methods should have found at least a tiny refuge in such a large survey.

As for the style, the book is written for the greater part in the carefully thought-out way of the fathers and grandfathers of complex analysis; some exceptions occur in the new appendix where modern English corruptions mar the beauty of the German presentation (e.g. "geliftete Kurve" on p. 685!). However I would be at a loss to find a better translation with the proper technical connotation.

The index of the former editions has not always been rewritten conforming to the new arrangement of the text; e.g. on p. 364 one looks in vain for non-Euclidean motions as announced by the index.

Summing up: The new version of Hurwitz-Courant-Röhrl will continue to be a standard textbook of the first class. One cannot escape the impression, however, that the wealth of material seems to have overwhelmed the authors to such an extent that the organisation has suffered sometimes. Naturally, in this age of gentle educational approach, the aforementioned repetitions can also be interpreted as highly sophisticated pedagogical devices enabling the student to digest piecemeal the more complicated aspects of the theory!

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Mathematical Theory of Optics, by R. K. Luneburg. University of California Press, Berkeley, 1964. xxx + 448 pages. $12.50.

The volume under review is a corrected version of a famous set of lectures delivered by the late Rudolf Luneburg at Brown University in 1944. Mimeographed copies were prepared, but had a very limited distribution as they were not generally available to the scientific public. It is no exaggeration to state that these notes have had a profound impact upon the progress of classical optics in spite of their inaccessibility, and for this reason their publication in book form is a major event. Luneburg was that rara avis, the productive scientist who did not publish his results! As a manifestation of this attitude, one finds that the volume has an intensely personal flavor reminiscent of Lord Rayleigh's

Theory of Sound. Unfortunately the author has not indicated the very substantial corpus of his new results with the result that the neophyte will not appreciate the vast amount of original research that Luneburg integrated into the text. In fact, the strength as well as the weakness lies in the almost complete absence of references to the literature! The book naturally divides into three independent sections; each section will be reviewed separately.

The first section (Chapter 1) is entitled "Wave and Geometrical Optics", but could more properly be called "Initial Value Problems Associated with Maxwell's Equations". After stating Maxwell's equations and discussing periodic fields, the author then transforms Maxwell's equations into integral form and proceeds to derive various theorems on the propagation of discontinuities using the theory of characteristics. The transport equations relating the electromagnetic field vector along geometrical rays are then outlined. Luneburg's conception of the transport equations is one of his major achievements, and the techniques he pioneered have been utilized by many workers in wave propagation; e.g., Friederich and Keller's formulation of geometrical acoustics. One particularly happy feature to an applied mathematician lies in the detailed analysis of the construction of wave-fronts with the aid of light rays including Huyghen's construction. There are other miscellaneous topics, finally ending with a hint about the possibility of expanding Maxwell's equations in inverse powers of the frequency. The asymptotic development of steady state electromagnetic fields was later developed by Luneburg in unpublished work done at New York University and forms the subject of a monograph by Kline and Kay[1] only recently published.

Even a cursory inspection by the reader of the second section (Chapters 2-6) will dispel the common fallacy that geometrical optics is exclusively concerned with the application of ray tracing formulas to lens design. Fortunately, Luneburg chose to present Hamilton's theory of geometric optics in full detail. The power and elegance of this approach has never been fully appreciated by applied mathematicians and the usual accounts always seem to omit just enough of the theory to obscure the modus operandi. Not so with Luneburg; by beginning with Fermat's principle and marshalling the apparatus of the calculus of variations he carefully exposes the underlying foundations of the point characteristic, angular characteristic, and mixed characteristic functions. Integral invariants are introduced and the theorem of Malus receives a thorough treatment. Application of the theory to special problems occupies over one hundred pages. One finds here the original treatment of the Luneburg lens which has had widespread application in microwave technology and is still the subject of mathematical investigations.

(1) M. Kline and I. Kay, Electromagnetic Theory and Geometrical Optics (Interscience Publishers, New York, 1965).

The next two chapters are devoted to first order optics and aberration theory (both topics constituting the hard core of lens design theory) set in the context of Hamilton's theory. In reading these sections bear in mind that Luneburg was a practicing lens designer and that almost everything presented has some application. The first order optics section contains some unusual difference equations but is on the whole fairly straightforward in spite of being rather specialized. The theory of aberrations is confined to third order aberrations in systems of rotational symmetry in view of the obvious practical utility. Aberration theory is one area in desperate need of help, with respect to both the formal aspects as well as convergence conditions. For example, the usual approach to aberration theory involves a power series expansion of one of the characteristic functions. This expansion is formal and the question of its convergence has never been investigated. The class of functions which admit power series is limited and it is conceivable that practical systems exist for which power series expansions are not adequate, such as a Fresnel lens. On the other hand orthogonal function expansions, such as in Zernike polynomials, admit a larger class of functions but suffer other disadvantages. Numerical methods are in their infancy and certainly the least squares approach to curve fitting the aberration function is questionable. The classification of aberrations by group theoretic methods has yet to be performed; in fact one could ask endless questions. The reviewer hopes that mathematicians will respond at least in part to these pleas especially since the problems are definitely not simple mathematical problems. Anyone entertaining the thought of doing serious work in this area should also consult the comprehensive treatise of Herzberger[(2)].

The last section (Chapter 6) on the diffraction theory of optical instruments is virtually a research monograph in itself; well over eighty percent is new material. It is generally recognized that the Kirchhoff diffraction theory (actually not a theory, but an ansatz) upon which practically all optical diffraction work is based, is an approximate, non self-consistent theory which for reasons only now being understood - works! The critical point is that the Kirchhoff theory is independent of the particular boundary conditions at the edge of the aperture. Even if we could obtain the rigorous solution of the electromagnetic diffraction problem, it is doubtful if it would be very useful because of its complexity. Alternate approaches based upon somewhat less than the rigorous electromagnetic solution yet retaining some measure of the complete problem are required. The most successful of these approaches is due to Luneburg. His analysis is based upon the second Rayleigh Green's function (again no references!) so that his theory is self-consistent. Luneburg seeks to determine the solution of Maxwell's equations which have a prescribed behavior at infinity (satisfy a radiation condition) and are valid over the half space $Z > 0$. The actual

(2) M. Herzberger, Modern Geometrical Optics (Interscience Publishers, New York, 1958).

boundary conditions are replaced by their geometrical optics approximations as in the Kirchhoff theory and a solution of Maxwell's equations is sought which has the same value at infinity as the geometrical optics solution. One unique feature of the theory is the existence of a complex scalar function (remember the theory is vectorial) which when operated upon yields the diffracted electric and magnetic field vectors. In the important case of unpolarized light, the energy density can be written in terms of the absolute square of an integral over the complex scalar functions. The usefulness of this theory is slowly becoming evident to optical diffraction practitioners.

After a qualitative review of the diffraction patterns for different types of aberrations, the author proceeds to a discussion of resolution criteria for coherent and incoherent sources, rediscovering the Sparrow resolution criterion. Luneburg then formulates and partially solves a series of optimizing problems involving apodization. The Luneburg apodization problems and their variants now have an extensive literature. The last few pages are devoted to resolution of objects of periodic structure. He derives the transfer function and shows that an optical system acts as a low pass filter of spatial frequencies. Unfortunately the limited distribution of the old notes has prevented current workers in optics from appreciating the remarkable fact that Luneburg was in possession of the full theory of optical system analysis as early as 1943.

The volume closes with a series of appendices and supplementary notes, some by M. Herzberger. The notes are definitely worth studying as much of the material is not generally available. A foreword by Emil Wolf (not Emile Wolf as on the title page!) and a short bibliographical note round out the book.

In summary we can do no better than quote Wolf: "I consider it to be one of the most important publications on optical theory that has appeared within the last few decades".

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Topologie II, by Wolfgang Franz. Vol. 2, Algebraische Topologie Samml. Göschen 1182/82a. W. de Gruyter, Berlin, 1965. 153 pages. Price D.M. 5.80.

This is a sequel to the Allgemeine Topologie of the same author, and, as in the case of the previous volume, the style is clear and concise, and the treatment remarkably comprehensive. In fact as before it seems quite surprising that so much information can be included in so small a space.

The book starts off with a general description of the scope of