BULL. AUSTRAL. MATH. SOC. 20C25, 20D40, 20F25
VOL. 7 (1972), 461-462.

The multiplicator of various products of groups

William Haebich

Most results relating a group to its multiplicator are "non structural" in the sense that they relate various numerical invariants of group and multiplicator. This means that the calculation of a specific multiplicator often involves lengthy, specialised arguments. Standard product constructions like the direct and wreath products provide a way of describing certain group structures. When G is a given product of the groups A_λ , $\lambda \in I$, its multiplicator $M(G)$ is necessarily dependent on the A_λ . We are led to ask whether $M(G)$ is also a recognisable product of the A_λ , or perhaps the $M(A_\lambda)$. Until recently the only known relationship of this sort was a classical result due to Schur [1]: if G is the direct product, $A_1 \times A_2$, then $M(G)$ is isomorphic to $M(A_1) \times M(A_2) \times (A_1 \otimes A_2)$. In this thesis an attempt is made to furnish some useful "structural" theorems on the multiplicator by establishing similar results for various other products.

All products dealt with are the type where G is generated by the A_λ , or (multiple) isomorphic copies of each A_λ , $\lambda \in I$. $M(G)$ can be characterised as $R \cap F'/[R, F]$ where F/R is a presentation for G . From this point of view, the most direct way of approaching $M(G)$ is to try to express $R \cap F'/[R, F]$ in terms of the $R \cap F'_\lambda/[(R \cap F_\lambda), F_\lambda]$ where F_λ is the group of elements in F which map onto A_λ . $(F_\lambda/R \cap F_\lambda$ is a presentation for A_λ .$)$ In fact this is the method applied by Schur to the direct

Received 9 August 1972. Thesis submitted to the Australian National University, July 1972. Degree approved, September, 1972. Supervisors: Dr J. Wiegold, Professor Hanna Neumann and Dr P.J. Cossey.

https://doi.org/10.1017/S000497270004538X Published online by Cambridge University Press

product. The difficulty in extending it to more complicated products lies in not knowing how the F_λ will generate F. To get around this problem we use a technique that is essentially the reverse of the direct approach. We start with presentation F_λ/R_λ for each A_λ and use them to *construct* an appropriate presentation for G. With one exception, it is best just to take the free product $F = \prod^*_{\lambda\in I} F_\lambda$ and to find the kernel of the natural epimorphism from F onto G.

The most complete results are obtained for regular products and verbal wreath products. In both instances the multiplicator of the product is the direct product of the multiplicators of its factors together with another group. More generally it is shown that if G is a splitting extension of A by B, then $M(B)$ is a direct factor of $M(G)$. (This has been established by Tahara [2] using homological methods.) These main results are pushed further under various restrictions. For example nilpotent products are examined in detail as a special case of regular products. Finally some limited information is obtained about the multiplicator of a central product.

References

[1] J. Schur, "Untersuchungen über die Darstellungen der endlichen Gruppen durch gebrochene lineare Substitutionen", *J. reine angew. Math.* 132 (1907), 85-137.

[2] Ken-ichi Tahara, "On the second cohomology groups of semi-direct products", (unpublished).