SOME DIFFERENCES BETWEEN GEOMETRICAL AND DYNAMICAL FIGURES OF THE MOON

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Abstract. The geometrical shape of the Moon is determined from measurements of absolute heights of the lunar surface, while its dynamical shape is described by means of the Moon's gravity field parameters. All these data are derived from observations of the lunar artificial satellites ('Luna-10', 'Orbiters 1-4') and astronomical measurements.

In the paper differences of the lunar geometrical and dynamical figures are analysed. It is shown, that the homogeneous model of the Moon is not capable of explaining these differences. It is found, that the lunar centre of gravity situated about 0.9 km to the north, and 1.1 km nearer to the Earth, than the centre of its geometrical figure.

The geometrical shape of the Moon is determined from measurements of absolute heights of the lunar surface, while its dynamical shape is described by means of the Moon's gravity field parameters. However there are no reasons to consider, that the smoothed surface of the Moon and its equipotential surface coincide with one another [1]. Therefore it is useful to determine differences between the lunar geometrical and dynamical figures and to compare them with some theoretical lunar models.

Let $\mathbf{R}=\{X, Y, Z\}$ be the position vector of the lunar surface point, and let $\mathbf{r}=\{x, y, z\}$ be the position vector of the equipotential surface point. The absolute heights of the lunar and equipotential surfaces are respectively

$$\begin{aligned} H &= 1738.0\,(\sqrt{X^2 + Y^2 + Z^2} - 1), \\ h &= 1738.0\,(\sqrt{x^2 + y^2 + z^2} - 1). \end{aligned} \tag{1}$$

For the first case the reference surface is a sphere, the centre of which coincides with the origin of the X, Y, Z system, and for the second case – a baricentric sphere.

The dynamical figure of the Moon can be represented by means of spherical harmonics

$$h = \sum_{n=0}^{\infty} \sum_{m=0}^{n} (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda)\, P_{nm}(\sin \beta), \tag{2}$$

where C_{nm}, S_{nm} are harmonic coefficients, that are determined from an analysis of the motion of artificial satellites of the Moon [2, 3]. Analogically we can write for the geometrical figure of the Moon

$$H = \sum_{n=0}^{\infty} \sum_{m=0}^{n} (J_{nm} \cos m\lambda + J'_{nm} \sin m\lambda)\, P_{nm}(\sin \beta). \tag{3}$$

The harmonic coefficients J_{nm} and J'_{nm} can be determined by use of absolute heights of the selenodetic reference points. As at present the reference points on the far side of the Moon are lacking the coefficients J_{nm} and J'_{nm} are determined from various lunar

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models using symmetry and homogeneity hypotheses [4]. However these procedures are open to severe criticism.

We must determine more strictly the approximate absolute heights of the lunar far side from existing relations between heights of the lunar and equipotential surfaces. For example these relations may be expressed by the formula

$$H = \alpha + \beta h , \tag{4}$$

where α and β are parameters determined from measurements of the lunar and equipotential surfaces of the visible side.

The following relations of harmonic coefficients are known from theory

$$\begin{aligned} C_{nm} &= kJ_{nm} , \\ S_{nm} &= kJ'_{nm} . \end{aligned} \tag{5}$$

The parameter k depends on the density distribution along the lunar radius.

All data obtained up to the present show that the homogeneous model of the Moon is not capable of explaining existing differences between the lunar geometrical and dynamical figures. It is very probably, that the real Moon is slightly nonhomogeneous in all three directions (radius, longitude, latitude). Moreover, the geometrical centre of the Moon does not coincide with its centre of gravity.

The nonhomogeneity of the Moon demands an additional analysis.

Comparison of the parameters of the lunar geometrical and dynamical figures permits to obtain the position of the lunar centre of gravity in its body.

For this purpose, the most probable sphere fitting the lunar visible surface can be found from a solution of the equations

$$aX + bY + cZ + d = H , \tag{6}$$

where a, b, c are coordinates of the centre, d is the radius correction.

The most probable sphere is found on the assumption, that a certain correlation exists between the absolute heights of the lunar and equipotential surfaces. Equation (6) should then be written

$$AX + BY + CZ + \beta h + \alpha = H . \tag{7}$$

Here A, B, C are new coordinates of the centre, α is the new radius correction, β is the proportionality coefficient of the heights H and h.

The differences $A\text{-}a = \Delta X$, $B\text{-}b = \Delta Y$, $C\text{-}c = \Delta Z$ show that the centres of the reference spheres of absolute heights h and H do not coincide one with another. So far as the reference sphere of absolute heights h is a barycentric sphere, the vector $M = \{\Delta X, \Delta Y, \Delta Z\}$ can be interpreted as a position vector of the lunar centre of gravity in the X, Y, Z system.

This method was tested using data obtained from the 'Luna-10' and 'Orbiters 1–4' observations. Absolute heights of the visible lunar surface were determined from the lunar hypsometric chart [1], prepared at the Kiev Observatory. The results are represented in Table I.

TABLE I

Coordinates	Position of the lunar centre of gravity in km	
	From 'Luna-10' data	From 'Orbiters 1-4' data
ΔX	-0.33 ± 0.13	-0.20 ± 0.14
ΔY	$+0.48 \pm 0.14$	$+0.36 \pm 0.14$
ΔZ	$+1.45 \pm 0.56$	$+1.05 \pm 0.35$

The weighted mean results (weights 1 and 4, depending on the number of used artificial satellites of the Moon) is

$$\Delta X = -\ 0.26\ km$$

$$\Delta Y = +\ 0.38\ km$$

$$\Delta Z = +\ 1.13\ km\,.$$

Previously during the construction of the Kiev hypsometric lunar chart the following data were used $\Delta X' = +0.26$ km, $\Delta Y' = +0.52$ km, $\Delta Z' = 0.0$ km [1]. Therefore the final results are

$$\Delta X = \quad 0.00\ km$$

$$\Delta Y = +\ 0.90\ km$$

$$\Delta Z = +\ 1.13\ km\,.$$

The obtained data show that the lunar centre of gravity is situated approximately to the north, and nearer to the Earth, than the centre of the lunar geometrical figure. These results do not contradict the data obtained from other investigations [5].

References

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