ACCRETION DISCS

https://doi.org/10.1017/S0252921100038343 Published online by Cambridge University Press

ACCRETION DISC VISCOSITY

CHRISTOPHER A. TOUT
Institute of Astronomy, The Observatories, Madingley Road, Cambridge CB3 0HA, UK

Abstract. We review the various physical processes that could lead to viscosity in accretion discs. A local magnetic dynamo offers the most plausible mechanism and we discuss a simple model in some detail. The dynamo operates even in partially and very weakly ionized discs without much modification.

1. Introduction

Accretion discs occur where high angular momentum material is falling on to a central object. Material that cannot be accreted directly forms a disc rotating around the object. Accretion discs (of about $1\,R_\odot$ in radius) form in cataclysmic variables. Material overflowing from the low-mass star filling its Roche lobe cannot accrete directly on to the white dwarf, collides with itself and forms an accretion disc around the star. Accretion discs (of about 100AU $\approx 20\,000\,R_\odot$) are inferred to be an integral part of the process of star formation. Once a denser, self-gravitating core has formed within a rotating cloud it cannot directly accrete further material. This material first collapses and then accretes on to a disc perpendicular to the rotation axis. Accretion discs (of about $1\,\mathrm{kpc} \approx 4\,10^{10}\,R_\odot$) are probably the supply route to black holes at the centre of active galactic nuclei.

If material is to fall inwards through the accretion disc its angular momentum must be transferred outwards. If the material in the disc is orbiting in circular Keplerian orbits with angular velocity $\Omega \propto r^{-3/2}$, where r is the radius in the disc, such a transfer is energetically favourable because the lowest energy state for any rotating object of given angular momentum is one of complete corotation. The transfer of angular momentum outwards serves to slow down the inner, rapidly rotating parts of the disc while spinning up the outer, more slowly rotating parts. This can be achieved if a simple shear viscosity acts within the disc fluid. The stress between adja-

A. Evans and J. H. Wood (eds.), Cataclysmic Variables and Related Objects, 97–106.

cent annuli will then be

$$\sigma_{r\phi} = \eta_v \frac{dV}{dr}, \tag{1}$$

where the suffices $r\phi$ indicate the force per unit area in the azimuthal direction owing to the velocity gradient dV/dr in the radial direction. The dynamic viscosity is $\eta_v = \rho\nu$, where ρ is the density of the fluid and ν its kinematic viscosity. Gravitational forces will generally dominate over viscous forces ensuring that material in the disc follows near Keplerian orbits for a non-self-gravitating disc and tidal effects will ensure that these orbits are circular. If ν is known we can write a diffusion equation for the disc surface density (Lynden-Bell & Pringle 1974). The time-scale for viscous processes to influence the disc will be $\tau_\nu = R^2/\nu$, where R is the radius of the disc. For the disc to remain in a steady accreting state this must be much less than the accretion time-scale, which is about 10^9 yr for typical cataclysmic variables. Taking the radius of the disc to be about 10^{10} cm we find $\nu \gg 3 \times 10^3\,\mathrm{cm^2 s^{-1}}$.

2. Molecular viscosity

For normal material, molecular viscosity is several orders of magnitude too small. Even honey has a kinematic viscosity of about $5\,\mathrm{cm^2\,s^{-1}}$ and astrophysical discs are made of much more slippery stuff. On the other hand if a disc becomes degenerate then its electrons have very long mean free paths. In just the same way that degenerate matter is highly conducting it is also very viscous (Paczyński & Jaroszyński 1978).

3. Turbulent viscosity

Turbulent motions within the fluid can transport angular momentum. First consider a radial turbulent cell of length l, average velocity c_t and cross section σ. There will be a velocity difference δV between the ambient material at the two ends of the cell such that $\delta V = Sl$, where S is the shear in the medium. Now in a time $\Delta t = l/c_t$ the cell transports momentum $\Delta p = \rho\sigma l.Sl$ from one end to the other. This provides an effective force

$$F = \frac{\Delta p}{\Delta t} = \rho\nu S\sigma \tag{2}$$

from which we deduce that $\nu \approx lc_t$. We expect $l \leq H$, the disc thickness, and $c_t \leq c_s$, the sound speed, so that

$$\nu \approx \alpha H c_s, \tag{3}$$

https://doi.org/10.1017/S0252921100038343 Published online by Cambridge University Press

with $\alpha \leq 1$ (Shakura & Sunyaev 1973). A calculation of vertical structure within the disc gives $H \approx c_s/\Omega$ so that

$$\nu \approx \frac{\alpha c_s^2}{\Omega} \approx 10^{14}\alpha \ \mathrm{cm^2 s^{-1}}. \tag{4}$$

However, this simple argument breaks down because we have exchanged higher angular momentum material from the outer end of the cell for lower from the inner end. This is neither what we want nor is it energetically favourable. Had we consulted Rayleigh's criterion for stability to axisymmetric perturbations we would have found our disc to be stable to such turbulence. Consider an incompressible uniform fluid rotating in cylindrical shells at $\Omega(r) = V(r)/r$. Interchange two cylinders of fluid each of mass m at r_1 and r_2 with $r_2 > r_1$. If the specific angular momentum $h = rV = r^2\Omega$ is conserved then the energy change on interchange is

$$\Delta E = \frac{1}{2}m\left\{\left(\frac{h_2}{r_1}\right)^2 + \left(\frac{h_1}{r_2}\right)^2 - \left(\frac{h_1}{r_1}\right)^2 - \left(\frac{h_2}{r_2}\right)^2\right\} \tag{5}$$

$$= \frac{1}{2}m\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)\left(h_2^2 - h_1^2\right). \tag{6}$$

If the interchange is energetically favourable and thence unstable $\Delta E < 0$ and hence $h_1^2 > h_2^2$ or specific angular momentum must decrease outwards. Alternatively if

$$\frac{d}{dr}(r^2\Omega)^2 > 0 \tag{7}$$

the disc is stable to axisymmetric perturbations. This is indeed the case with Keplerian discs.

On the other hand, should the disc be unstable to turbulent motions predominantly in an azimuthal direction, the situation is different. Consider a small region of fluid moving at an azimuthal velocity c_t faster than the ambient flow. This region has excess angular momentum and experiences a coriolis force in the outward radial direction. In this way higher angular momentum material can be transported outwards while lower angular momentum material falls inwards.

We can now ask when such turbulence might set in. For fluid of known viscosity we can construct the Reynolds number

$$\Re_e = \frac{LV}{\nu}, \tag{8}$$

where L is a typical length scale over which the fluid velocity, typically V, varies. In the disc $L \approx R$ and $V \approx R\Omega$. If we write $\nu \approx \alpha H^2\Omega$ then

$$\Re_e \approx \frac{1}{\alpha}\left(\frac{R}{H}\right)^2 \gg 1. \tag{9}$$

In the laboratory, turbulence sets in when $\Re_e \geq 10^3$. Lynden-Bell & Pringle (1974) argued that the fluid in the disc might be self-regulating in the sense that the viscosity is such that the fluid is just turbulent and

$$\alpha \approx 10^3 \left(\frac{H}{R}\right)^2 . \qquad (10)$$

If the viscosity were smaller $\Re_e$ would be larger and turbulence would increase. If the viscosity were larger then turbulence would be suppressed. Although a reasonable numerical value is obtained there is no real physical justification for this approach. It has proved popular because it allows α to take much smaller values in thin discs with $H/R \ll 1$ than in thicker discs, a phenomenon that might explain dwarf novae outbursts.

4. Convection

Convective turnover would indeed give rise to turbulence that can cause viscosity. Whether or not a disc is convectively unstable depends on how and where the gravitational energy of the material flowing through the disc is liberated. Although parts of some cataclysmic variable discs may be convective, we can conclude from Livio & Shaviv (1977) that they are generally stable and convection is not the source of viscosity. However in protostellar discs Lin & Papaloizou (1980) have claimed that convection will be important simply because, as the disc contracts to its mid plane, the released gravitational energy sets up a temperature gradient sufficient to drive convection. Convective cells, although essentially vertical, must close on themselves to avoid accumulation of matter. Suppose a cell is able to move a distance l before being disrupted. If disruption takes place when the two extremes of the cell have been sheared around the disc, $l \approx v_c/\Omega$, where v_c is the convective velocity to be calculated using mixing length theory. The viscosity is then $\nu \approx v_c^2/\Omega$.

Kley, Papaloizou & Lin (1993) were able to create two-dimensional axisymmetric fluid simulations in which they found convective cells transporting angular momentum. However they had to introduce an additional artificial viscosity to get the model to work and then found that the cells themselves carry angular momentum inwards because their axisymmetry forces them to close in a radial direction. The situation might be improved if convective cells can close in an azimuthal direction but, since top and bottom must move in opposite directions, each end of the cell will experience oppositely directed coriolis forces and it is difficult to see how such a cell can close on itself at all.

5. Dynamical processes

Paczyński (1978) pointed out that, if it is left to cool, an accretion disc will contract until its own self-gravity becomes important. The density at which this occurs is given by

$$\rho_{\rm disc} > \frac{M_{\rm WD}}{r^3} < 1\,{\rm g\,cm^{-3}}, \tag{11}$$

where the second condition avoids degenerate support. A similar situation to that seen in galactic discs results, with the formation of denser clumps throughout the disc. Torques between these clumps will act so as to enforce corotation which directly transfers angular momentum outwards in Keplerian discs. Laughlin & Bodenheimer (1994) constructed numerical models of a cool disc which becomes unstable and gives rise to viscosity equivalent to $0.01 < \alpha < 0.03$. Cataclysmic variable discs, however, are generally hot and far from self-gravitating.

6. Magnetic fields

Magnetic fields are the most promising source of viscosity. Magnetohydrodynamics in astrophysics is based on Maxwell's equations with two simplifying assumptions. The first that

$$|\nabla \wedge \mathbf{B}| \gg \frac{1}{c^2}\left|\dot{\mathbf{E}}\right|, \tag{12}$$

that electromagnetic waves are unimportant, holds well in all quantifiable situations. This leads to the induction equation

$$\dot{\mathbf{B}} = \nabla \wedge (\mathbf{u} \wedge \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{13}$$

where $\eta = 1/4\pi\sigma$ is the magnetic diffusivity (σ being the electrical conductivity). The second assumption commonly made is that η is small in astrophysical plasmas so that, except where $|\nabla^2 \mathbf{B}|$ is large, field lines are linked to the fluid or fluid can flow freely only along the field lines.

A particular consequence of this is that if there are radial field lines in a disc the shear flow will tend to wrap them around the disc generating azimuthal field. In so doing work must be done on the field and mechanical energy is converted to magnetic energy. This conversion is the basis of a magnetic dynamo. At the same time the field lines will be bent. Curved field lines will attempt to straighten themselves and in so doing enforce corotation of the fluid and hence angular momentum transport outwards in the disc. Lynden-Bell (1969) suggested that magnetic fields are in this way responsible for viscosity in accretion discs. The magnetic torque can be calculated from the Lorentz force

$$\begin{aligned} \mathbf{F} &= \mathbf{j} \wedge \mathbf{B} \\ &= \frac{1}{4\pi}(\nabla \wedge \mathbf{B}) \wedge \mathbf{B} \\ &= \frac{1}{4\pi}[(\mathbf{B}.\nabla)\mathbf{B} - \nabla(\tfrac{1}{2}|\mathbf{B}|^2)]. \end{aligned} \tag{14}$$

The second term provides an additional contribution to isotropic pressure while the first is the magnetic curvature force. By evaluating the mean of the component of this force in the azimuthal direction,

$$\overline{F}_{\mathrm{curv},\phi} = \frac{1}{4\pi}\overline{[(\mathbf{B}.\nabla)\mathbf{B}]_\phi} \approx \frac{B_R B_\phi}{4\pi r}, \tag{15}$$

we obtain an effective viscosity [from equation (1)] of

$$\nu \approx \frac{B_R B_\phi}{4\pi\rho\Omega} \tag{16}$$

or

$$\alpha \approx \frac{B_R B_\phi}{4\pi\rho c_\mathrm{s}^2}. \tag{17}$$

Now in order to have viscosity all we need do is generate radial magnetic field in the disc. Radial turbulent motions can generate radial field from azimuthal but we have already shown that the disc is stable to such turbulence and isotropic turbulence might in itself generate the necessary viscosity. A major break-through was made when Balbus & Hawley (1991) recognised the importance of an instability first recorded by Velikhov (1959).

6.1. THE BALBUS–HAWLEY INSTABILITY

Discs with weak vertical fields are unstable if

$$\frac{d(\Omega)^2}{dr} < 0, \tag{18}$$

if $|\Omega|$ decreases outwards. The instability can be understood by considering a vertical field line. Suppose a perturbation moves material together with the field line in the azimuthal direction of the ambient flow. In the absence of the field this would be a neutral perturbation. In this case the field line is stretched and bent so that the curvature force slows down the displaced material relative to the surrounding fluid. Losing angular momentum it falls radially inwards where it is caught up in yet faster flowing material.

https://doi.org/10.1017/S0252921100038343 Published online by Cambridge University Press

If the shear is stronger than the magnetic tension the material will be dragged yet further from its equilibrium increasing the magnetic tension as it goes. More angular momentum is carried outwards and the material falls in further. Alternatively, if the field is strong enough the tension will win over the shear straightening the field line and stabilizing against the inflow. Growth occurs only on wavelengths $\lambda > \lambda_{\rm crit} \propto B_z$ so that if $\lambda_{\rm crit} > 2H$, the total disc thickness, it will be stable.

We now have an instability that will generate viscosity if there is a weak vertical magnetic field in the disc. Various sources of vertical field can be envisaged. If the whole system is embedded in a region of magnetised space, field might be advected in with the material flowing through the disc. Alternatively field anchored on the central accreting object may thread through the disc and interact with it. This is the case in DQ Her systems or intermediate polars in which the central-object field disrupts the inner parts of the disc. However it is not clear that weaker fields can thread a differentially rotating disc at all. The disc may appear super-conducting to external field which can then be excluded entirely. A third, more promising mechanism is the regeneration of vertical field within the disc itself by some dynamo process.

6.2. PARKER INSTABILITY

Magnetic buoyancy can provide the source of vertical field. Consider a tube of magnetic flux $\mathbf{B}$ embedded in non-magnetic fluid of density $\rho_{\rm e}$ and pressure $p_{\rm e}$. If the tube is in pressure balance with its surroundings its own thermal pressure $p_{\rm i}$ will be supplemented by its magnetic pressure.

$$p_{\rm e} = p_{\rm i} + \frac{B^2}{8\pi} > p_{\rm i}. \tag{19}$$

Thus its internal density $\rho_{\rm i} < \rho_{\rm e}$ and the tube will float to the surface. Once a section of a flux tube begins rising material can flow down the tube towards the mid plane leaving the rising part of the tube less dense still, so that it rises yet faster. This is the Parker instability. Its fastest growing mode has a wavelength some eight times the disc scale height (Horiuchi et al. 1988) and a growth rate $\tau_{\rm P}^{-1}$ two to five times slower than the Alfvén crossing rate. In the disc it is the azimuthal and radial field in the plane of the disc that buoys up generating new vertical field.

6.3. RECONNECTION

Unless the vertical field B_z decays it will build up until the Balbus–Hawley instability is stabilized, when the generation of radial and azimuthal field ceases. The vertical field generated through the two instabilities will not be

https://doi.org/10.1017/S0252921100038343 Published online by Cambridge University Press

uniform but will change direction on a length scale $\lambda_{rec} \approx H$, determined by the wavelengths of the fastest growing modes of the Balbus–Hawley and Parker instabilities and the action of shear in the disc which will tend to reduce λ_{rec} (see Tout & Pringle 1992 for details). Between regions of oppositely directed field reconnection can take place rapidly. Once reconnection begins at one point reconnected loops attempt to straighten pulling material away from the reconnection region. Excess pressure outside the region pushes more field in close enough to continue reconnecting. The reconnection time-scale is then $\tau_{rec} \approx \lambda_{rec}/0.1[V_A]_z$, where the vertical Alfvén speed, $[V_A]_z = B_z/\sqrt{4\pi\rho}$.

6.4. OPERATIONAL MODEL

Tout & Pringle (1992) put together these processes in a description of a magnetic dynamo that leads directly to disc viscosity without the need for any externally imposed turbulence. The equilibrium $\mathbf{B} = \mathbf{0}$ is unstable. Initially both B_R and B_ϕ grow on a time-scale Ω^{-1} but decay only on a time-scale $\max(\tau_P, \tau_{rec}) \gg \Omega^{-1}$; a second equilibrium is reached only when B_z is close to its maximum for instability when the ratios of the Alfvén speeds to the ambient sound speed for the three field components are

$$\frac{[V_A]_z}{c_s} \approx \frac{[V_A]_\phi}{c_s} \approx 0.8, \quad \frac{[V_A]_R}{c_s} \approx 0.1. \tag{20}$$

The equilibrium value of α is about 0.1. This equilibrium is also unstable and Fig. 1 shows how the fields oscillate around their equilibrium values. In each cycle B_R and B_ϕ build up until B_z is sufficient to shut off the Balbus–Hawley instability. While B_R and B_ϕ begin to decay B_z increases further. Eventually B_z decays faster than it is replenished until the Balbus–Hawley instability begins to operate again and B_R and B_ϕ rise once more. Fig. 2 illustrates the variation of α [from equation (17)] for the same system.

6.5. NUMERICAL SIMULATIONS

The above model simplifies all the processes involved to make the solution tractable. Numerically we can try to model more of the details but a number of specific problems beset such attempts. First, because of limited resolution, an artificial magnetic diffusivity is always present so that the magnetic Reynolds number $\Re_m = RV/\eta$ is several orders of magnitude too large, seed magnetic fields must already be large in order to grow and reconnection cannot be modelled properly. Second, only a small region of the disc can be followed and boundary conditions limit the size of large scale growth and enforce certain field structures.

https://doi.org/10.1017/S0252921100038343 Published online by Cambridge University Press

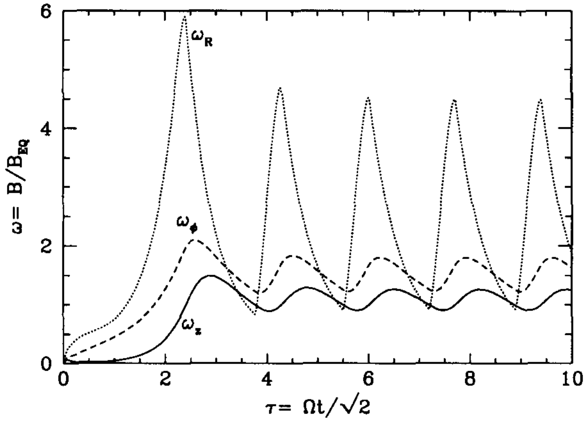

Figure 1. Variation of component field strengths relative to their equilibrium values.

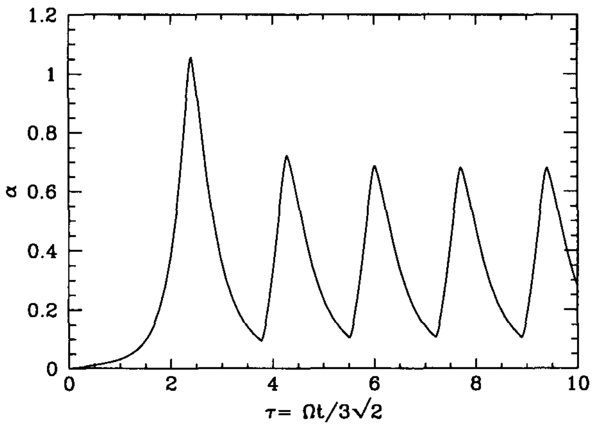

Figure 2. The variation of α with time.

Not withstanding these problems Hawley, Gammie & Balbus (1995) showed that a self excited dynamo can be maintained even with zero gravity. Brandenburg et al. (1995) have included gravity but their surface boundary condition still limits the growth of Parker's instability. They found $\alpha \approx 0.004$ from magnetic stress.

6.6. PARTIAL IONIZATION

We have described processes operating in fully ionized media. Regős (1996) has asked the question of what happens when some material is neutral. Neutral particles feel the magnetic fields only through collisions with ions and two-component magnetohydrodynamic equations are needed. Cataclysmic variable discs in quiescence are partially ionized with $0.1 \leq \rho_i/\rho_n \leq 0.9$. Regős has recalculated the rates for the various instabilities and finds that α does not differ much from the fully ionized case.

In protostellar discs the ionization fraction may be as little as $\rho_i/\rho_n \approx 10^{-10}$. In this case the velocities of neutral particles and ions differ widely and the induction equation becomes a diffusion equation. As a result reconnection is faster and α increases because B_R and B_ϕ can build up to larger values before the Balbus–Hawley instability switches off. Brandenburg et al. find their models in agreement with these conclusions.

7. Conclusions

Magnetic fields are probably the source of viscosity in cataclysmic variable discs and are a viable source in protostellar and AGN discs too. A radial component of magnetic field is needed and this can be generated by the Balbus–Hawley instability which overcomes the Rayleigh stability criterion. The Parker instability or ensuing turbulence can regenerate vertical field by dynamo activity. Theoretically acceptable values of $\alpha \approx 0.1$ are predicted. Limited numerical models give smaller but still reasonable values, $\alpha \approx 0.004$. The gravitational energy from the inward flowing material will not all be dissipated where the viscosity acts. It is released by reconnection of vertical field which may occur in the disc corona or it may escape altogether in a disc wind or jet. The inability of a disc dynamo to find a stable equilibrium may account for disc flickering.

References

Balbus, S. A., Hawley, J. F., 1991, Ap. J., **376**, 214
Brandenburg, A., Nordlund, Å., Stein, R. F., Torkelsson, U., 1995, Ap. J., **446**, 741
Hawley, J. F., Gammie, C. F., Balbus, S. A., 1995, Ap. J., **440**, 742
Horiuchi, T., Matsumoto, R., Hanawa, T., Shibata, K., 1988, PASJ, **40**, 147
Kley, W., Papaloizou, J. C. B., Lin, D. N. C., 1993, Ap. J., **416**, 679
Laughlin G., Bodenheimer, P., 1994, Ap. J., **436**, 335
Lin, D. N. C., Papaloizou, J. C. B., 1980, MNRAS, **191**, 37
Livio, M., Shaviv, G., 1977, A&A, **55**, 95
Lynden-Bell, D., 1969, Nature, **223**, 690
Lynden-Bell, D., Pringle, J. E., 1974, MNRAS, **168**, 603
Paczyński, B., 1978, Acta Astr., **28**, 91
Paczyński, B., Jaroszyński, M., 1978, Acta Astr., **28**, 111
Regős, E., 1996, MNRAS, submitted
Shakura, N. I., Sunyaev, R. A., 1973, A&A, **24**, 337
Tout, C. A., Pringle, J. E., 1992, MNRAS, **259**, 604
Velikhov, E. P., 1959. Sov. Phys. JETP, **36**, 1398