A NEW LIFE-TABLE FOR ENGLAND AND WALES.

(Constructed by an extended method.)

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Data.

When the publication of the series of Reports for each separate County, dealing with the facts ascertained at the Census of 1901, was completed last March, it was possible for anyone, who would take the trouble, to arrive at the facts relating to the whole of England and Wales, that is, to ascertain not only the finally corrected total population number, but the numbers classified into age and sex groups.

From the separate Annual Reports of the Registrar-General for the years 1891–1900, and for some preceding years, it was equally possible to compile the numbers of Deaths and of Births requisite for the construction of a new Life-Table.

While therefore it must be understood that the succeeding Tables have no sort of official sanction or guarantee, but are given on the sole responsibility of a private individual, it must also be understood that they are based on officially published figures.

Methods.

The mean population numbers, or "years of life," were calculated from the data of the Censuses of 1891 and 1901 by the method described by Mr A. C. Waters in the *Journal of the Royal Statistical Society*, Vol. lxiv., Part ii., June 1901, p. 203.

In order to save space the actual numbers of "years of life" and of deaths in the respective age-groups are not given.

If reference be made to a series of papers contributed by the writer to the *Journal of Hygiene*, Vol. II., Nos. 1, 2 and 3, but little additional description will be required of the methods employed in the construction of the following Tables.

(1) The log p_x values for ages 5 to 64 inclusive were obtained by the "graphic" process described in this *Journal* in Nos. 2 and 3 of Vol. II., the curve used being that described as "Section 1" in No. 3, page 206.

(2) Although it has been shown that the same process might be employed for obtaining the succeeding log p_x values from age 65 onwards by drawing the curve in different sections and on different scales, the writer has found that it is easier and more expeditious, as well as more accurate, to obtain the log p_x values from age 65 onwards by a simple process of analytical interpolation. Space does not permit of going into full detail or giving actual illustrative calculations, but at the references already given it will be found how to calculate readily the values of log p'_x for the following values of x; $77\frac{1}{2}$, $82\frac{1}{2}$, $87\frac{1}{2}$, $92\frac{1}{2}$, and $97\frac{1}{2}$.

On setting down this series of values in a column and differencing them it is easy to obtain from the line of 5-yearly differences opposite to $u_{77\frac{1}{2}}$, (as u_0), the differences corresponding to intervals of one year, by the formulæ:

$$\delta^4 u_0 = \cdot 0016\Delta^4 u_0$$

$$\delta^3 u_0 = \cdot 008\Delta^3 u_0 - 6\delta^4 u_0$$

$$\delta^2 u_0 = \cdot 04\Delta^2 u_0 - 4\delta^3 u_0 - 8\delta^4 u_0$$

$$\delta u_0 = \cdot 2\Delta u_0 - 2\delta^2 u_0 - 2\delta^3 u_0 - \delta^4 u_0.$$

The series can then be carried upwards as far as u_{65} and downwards as far as may be required.

The values thus obtained are log p_x values, that is, they represent the chance of living from age x to age $x+1$, and they are ready to be used for the calculation of the l_x column of the Life-Table without any further alteration.

It has been found in each case that the two curves have joined each other fairly accurately at ages 64, 65.

In comparing different methods of constructing "extended" Life-Tables, attention must be concentrated upon the fact that the differences only apply to the calculation of the log p_x values.

Whatever method may be chosen, the labour of compiling the data and of calculating the l_x, P_x, Q_x, and E_x columns remains the same.

While admitting to the fullest extent the merits of the graphic method of distributing population and death numbers which is so ably advocated and illustrated by his friend Dr A. Newsholme, the writer is of opinion that from the points of view of economy of time and labour, as well as of accuracy, the *combined* method, herein briefly alluded to, is to be preferred.

An abstract of a Life-Table, constructed by an extended method, for ENGLAND AND WALES, *based on the census enumerations of Population in* 1891 *and* 1901, *and on the recorded Deaths in the ten years* 1891—1900, *shown in comparison with a Life-Table for the preceding decennium* 1881—90, *which has been recalculated by the same method.*

[Columns (*a*) give the figures for 1891—1900, and columns (*b*) the differences of these from the corresponding values for 1881—90.]

TABLE I. *Males.*

Age x	Probability of surviving from age x to age $x+1$ p_x		Number of survivors at each age out of 100,000 born l_x		Mean expectation of life or after-lifetime at each exact age x E_x	
	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
0	·82861	− ·01064	100,000		44·17*	+0·81
1	·94696	+ ·00415	82,861	−1064	52·23	+1·63
2	·97922	+ ·00300	78,467	− 658	54·13	+1·49
3	·98685	+ ·00221	76,836	− 408	54·26	+1·35
4	·99033	+ ·00169	75,826	− 232	53·98	+1·25
5	·99298	+ ·00114	75,093	− 101	53·50	+1·17
10	·99770	+ ·00048	73,524	+ 253	49·60	+0·95
15	·99678	+ ·00043	72,631	+ 449	45·18	+0·84
20	·99541	+ ·00071	71,233	+ 586	41·01	+0·76
25	·99453	+ ·00084	69,502	+ 871	36·97	+0·62
30	·99312	+ ·00101	67,457	+1165	33·02	+0·48
35	·99083	+ ·00098	64,895	+1459	29·22	+0·33
40	·98835	+ ·00096	61,686	+1697	25·60	+0·19
45	·98503	+ ·00063	57,807	+1834	22·15	+0·10
50	·98044	+ ·00027	53,147	+1814	18·86	+0·05
55	·97391	+ ·00004	47,547	+1654	15·78	+0·05
60	·96465	+ ·00062	40,933	+1483	12·91	+0·03
65	·95008	+ ·00053	33,281	+1336	10·30	−0·01
70	·92719	− ·00088	24,643	+ 992	8·02	−0·02
75	·89368	− ·00164	15,778	+ 519	6·13	+0·04
80	·84748	+ ·00080	8,147	+ 223	4·64	+0·13
85	·78743	+ ·00957	3,103	+ 150	3·49	+0·21
90	·71356	+ ·02602	780	+ 112	2·64	+0·28
95	·62758	+ ·05095	113	+ 39	2·01	+0·29

* If the value of p_0 had been the same as it was for 1881—90 the value of E_0 would have been 44·73, or increased by 0·56.

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TABLE II. *Females.*

Age x	Probability of surviving from age x to age $x+1$ p_x		Number of survivors at each age out of 100,000 born l_x		Mean expectation of life or after-lifetime at each exact age x E_x	
	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
0	·85963	− ·00945	100,000		47·82*	+1·04
1	·95061	+ ·00326	85,963	− 945	54·57	+1·80
2	·97900	+ ·00286	81,717	− 615	56·38	+1·71
3	·98669	+ ·00180	80,074	− 368	56·52	+1·57
4	·99047	+ ·00147	79,008	− 218	56·28	+1·50
5	·99312	+ ·00122	78,255	− 99	55·82	+1·43
10	·99763	+ ·00050	76,613	+ 261	51·97	+1·21
15	·99683	+ ·00062	75,661	+ 438	47·59	+1·11
20	·99566	+ ·00099	74,268	+ 715	43·44	+0·96
25	·99511	+ ·00117	72,591	+1102	39·38	+0·75
30	·99383	+ ·00122	70,668	+1493	35·38	+0·54
35	·99197	+ ·00112	68,272	+1847	31·53	+0·36
40	·99044	+ ·00091	65,388	+2106	27·81	+0·21
45	·98833	+ ·00071	62,063	+2234	24·16	+0·12
50	·98474	+ ·00043	58,143	+2259	20·62	+0·06
55	·97938	+ ·00027	53,297	+2156	17·26	+0·03
60	·97123	+ ·00032	47,297	+1968	14·12	+0·02
65	·95814	+ ·00008	39,897	+1715	11·26	±0·00
70	·93746	− ·00030	30,961	+1320	8·77	±0·00
75	·90676	− ·00043	21,084	+ 855	6·70	+0·02
80	·86377	+ ·00041	11,810	+ 466	5·06	+0·06
85	·80688	+ ·00309	4,999	+ 235	3·79	+0·08
90	·73558	+ ·00820	1,436	+ 110	2·85	+0·11
95	·65098	+ ·01584	245	+ 36	2·15	+0·11

* If the value of p_0 had been the same as it was for 1881—90 the value of E_0 would have been 48·34, an excess of 0·52.

For the sake of experiment the p_x curve from age 5 to age 24 for England and Wales (males) for 1891–1900, has been calculated throughout by the writer, both by complete analytical interpolations in the series of log p'_x values, as already described in the *Journal of Hygiene*, and also by the method used for the new London Life-Table, and the results in each case almost exactly correspond with the series of values obtained by the graphic process which has been used for these Life-Tables.

Probably by a skilful use of Dr Newsholme's method results would be obtained almost the same.

Although the values of log p'_{10} and of log p'_{15} have been *calculated*, it has been found that there was no real need for this, as the values found by the empirical rules were practically identical.

If any reader should care to take the trouble to compare the newly calculated E_x values for England and Wales 1881–90 (which can be

arrived at from Tables I. and II. by means of the differences given) with the corresponding values given in the officially published Life-Table, it will be found that, proceeding upwards, they correspond with only differences of $\pm 0{\cdot}01$ until E_{35} is reached.

The differences found to exist from E_{30} to E_0 depend upon a different, and, as the writer believes, a more rational, system of interpolation having been employed for arriving at the values of p_5 to p_{34}.

Before these extended Life-Tables had been constructed the writer had already calculated the E_x and $E_{x \text{ to } x+n}$ values by the modified short method, and the results agree as closely as in the now considerable number of instances which have been previously tried, so closely, that but for the sake of obtaining accurate l_x values, the calculation of the extended Life-Tables might have been almost dispensed with.

Comments.

Space will only permit a brief allusion to the most important points.

(1) The mean rate of infant mortality for 1891–1900 having been considerably in excess of that for 1881–90, both as regards males and females, there is necessarily a considerable *inverse* difference between the respective p_0 values.

With this exception, and with the exception that for males the p_x values for ages 68 to 79 inclusive, and for females, those for ages 67 to 78 inclusive, which are *less*, all the other p_x values are *greater* than the corresponding ones for 1881–90.

(2) As regards the l_x columns it follows that as the p_0 values are lower, the l_1 values must also be less, and that the succeeding l_x values will also be less until the loss in the p_0 values is counterbalanced by the gain in the values of p_1.....

The adverse balance is wiped off and transferred to the other side both for males and females, when l_7 is reached.

(3) As regards the E_x values, with the exception in the case of males that for ages 65 to 71 inclusive they fall fractionally below, and in the case of females that for ages 65 to 70 inclusive they coincide with the corresponding values of 1881–90, there is an increase all throughout, but the lowered values of p_0 have taken off both for males and females about half a year from the E_0 values.

(4) The comparatively small increase of average "Life-capital" shown in Table V. is due to altered proportional age-distribution of population.

TABLE III.

Showing how the total expectation of Life at Birth, (E_0), *is on the average distributed over the main age-periods of life.*

Age-periods	Males			Females		
	1881—90	1891—1900	Differences	1881—90	1891—1900	Differences
0—5	4·02	3·99	−0·03	4·16	4·14	−0·02
5—15	7·34	7·36	+0·02	7·65	7·67	+0·02
15—65	28·71	29·39	+0·68	30·67	31·52	+0·85
65 and upwards	3·29	3·43	+0·14	4·30	4·49	+0·19
Totals	43·36	44·17	+0·81	46·78	47·82	+1·04

TABLE IV.

Showing the average future lifetime or mean expectation of Life of all the individuals included within the age-groups indicated ($E_{x \text{ to } x+n}$).

Age-groups	Males			Females		
	1881—90	1891—1900	Differences	1881—90	1891—1900	Differences
0—5	51·36	52·74	+1·38	53·58	55·16	+1·58
5—10	50·63	51·69	+1·06	52·72	54·03	+1·31
10—15	46·47	47·39	+0·92	48·62	49·78	+1·16
15—25	40·32	41·06	+0·74	42·54	43·48	+0·94
25—35	32·62	33·09	+0·47	34·91	35·45	+0·54
35—45	25·50	25·70	+0·20	27·66	27·88	+0·22
45—55	18·94	19·00	+0·06	20·67	20·74	+0·07
55—65	13·09	13·12	+0·03	14·30	14·31	+0·01
65—75	8·35	8·34	−0·01	9·08	9·08	±0·00
75—85	4·93	5·03	+0·10	5·40	5·45	+0·05
85 and upwards	2·78	3·00	+0·22	3·12	3·21	+0·09

TABLE V.

Showing the average "Life-capital" of the census populations of England and Wales in 1891 *and* 1901, *the figures having been obtained by applying the life-values of the preceding decennium, as given in Table IV., to the numbers enumerated in the respective age-groups at the following census.*

	1891	1901	Differences
Males	36·09	36·15	+0·06
Females	37·44	37·61	+0·17
Parsons	36·78	36·90	+0·12

ADDENDA.

(1) The accuracy of the numbers of population compiled from the separate County Census Reports has been verified by their correspondence with the numbers given in the recently published *Summary Tables* of the census of 1901.

(2) If reference be made to the current number of *The Journal of the Royal Statistical Society*, a comparison may be made between the results given in the preceding Tables and those obtained by the "modified short method," and other Tables giving death-rates and mean age and sex distribution of population may be found.

(3) The writer desires to make acknowledgement of help received in the construction of the above Tables from Mr Maddison, Mining Surveyor of Haydock, in drawing and measuring the p_x curves, and from Mr Chas. Dickinson, Sanitary Inspector of Haydock, in making some of the routine calculations and in checking others.