RELATIVISTIC EFFECTS IN EARTH BASED AND COSMIC LONG BASELINE INTERFEROMETRY

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ABSTRACT. With present day technology the technique which provides the greatest precision in astrometric and geodetic measurement is Very Long Baseline Interferometry (VLBI) (Robertson, 1975; Dravskykh, 1981; Gubanov, 1983). The precision of present day astrometrical measurements by VLBI exceeds those of the best modern optical observations by an order and a half of magnitude and is capable of further improvement by the future development of phase stable, wide band, global networks and by the future deployment of VLBI antennas in space. Such precision of observation places the technique of VLBI well within the regime of special and general relativity. The present paper presents an analysis of relativistic effects on VLBI measurements with an accuracy of 0.0001 arc seconds.

1. INTRODUCTION

1.1 Long Baseline Interferometry Observations

In conducting long baseline interferometry observations, a microwave signal from an extra-galactic radio source is independently recorded at each antenna site, together with appropriate timing information derived from synchronized atomic clocks, on magnetic tape. The tape recorded data is subsequently cross correlated at a special purpose correlator facility. Since the natural radio emissions are broad band white noise the recorded signals will not correlate unless the magnetic tapes are relatively delayed by an amount equal and opposite to the differential propagation delay between the antennas before cross correlation is attempted. For most applications to astrometry and geodynamics the interferometer delay, τ, which maximizes the cross correlation and its first

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time derivative, $\dot{\tau}$, constitute the fundamental observables from which all other quantities of interest are inferred.

1.2 Newtonian Interferometer Time Delay

It is well known that in Newtonian physics time is considered to possess absolute properties which are independent of an observer's state of motion or position in a gravitational field. If we consider a source of radiation fixed at position $\vec{r}_o$ and (possibly mobile) receptors at positions $\vec{r}_1(t)$ and $\vec{r}_2(t)$ then the interferometer static geometric time delay $T_o(t)$ is given by

$$T_o(t) = \frac{1}{c}[|\vec{r}_o - \vec{r}_1(t)| - |\vec{r}_o - \vec{r}_2(t)|]$$

where we have introduced the convention that the time delay is considered positive if the signal arrives at antenna #2 before it arrives at antenna #1. If the source of radiation is at a great distance (which is almost always the case in VLBI when using extra-galactic sources) then the effects of parallax on the arrival of the wave fronts at the two antenna sites is negligible and the radiation can be regarded as a plane wave originating from a direction given by the unit vector $\hat{k}$. In this case it is relatively easy to show that the static geometric time delay can be written as

$$T_o(t) = \frac{1}{c}\,\hat{k}\cdot\hat{b}(t)$$

where we have introduced the interferometer baseline vector $\vec{b}(t)$ given by

$$\vec{b}(t) = \vec{r}_2(t) - \vec{r}_1(t) \ .$$

In this paper we shall not consider the effects of the atmosphere and ionosphere nor the effects of the electronic delays in the VLBI data acquisition system on the measured time delay, all of which are of considerable importance in the practical application of the technique.

1.3 Relativistic Interferometer Time Delay

The breakdown of Newtonian physics in its ability to correctly describe the observed interferometer time delay can be attributed to a combination of the effects of both special and general relativity. These effects include:
(1) relativity of simultaneity between relatively moving coordinate frames (this is an expression of the effects of aberration on the microwave radiation)
(2) relative differences in the rate of proper time between clocks at the two antenna sites
(3) curvature and position dependent metrical properties of the trajectory of the microwave radiation generated by gravitational fields of the solar system.

The precision of delay measurement by VLBI depends on the effective

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band width of the cross correlated signals. Using band width synthesis techniques delay measurement precision is of the order of 50 picoseconds or less which exceeds the precision with which the clocks at each antenna site are synchronized and so a model for the relative behaviour of the clocks on each baseline is required for the reduction of VLBI data. In the last decade the frequency and time stability of laboratory clocks has reached the level of a few parts in 10^{16} for integration intervals of the order of one hundred to several thousand seconds (Jimenez, 1979; Hellwig, 1979; Vessot, 1979). The use of super conducting cavity oscillators coupled to cooled hydrogen masers is expected to yield a further factor of ten improvement (Vessot, 1979). Even though the performance of the frequency standards in routine use for VLBI appears to be about two orders of magnitude lower than the best laboratory performances it would appear that future clock performances provide the motivation for examining the relativistic effects on VLBI time delay to the order of parts in 10^{16}.

2. CLOCK SYNCHRONIZATION

VLBI time delay is a measure of the time interval between the arrival of a given wave front at the two antenna sites as reckoned by the differences in the readings of the two clocks marking the two events and so the issue of clock synchronism is central to the technique. The synchronization of a pair of clocks at the same position $\vec{r}$ in three dimensional space is accomplished by simply setting the second clock to "read the same time" as the first. The synchronization of a pair of clocks at separate locations in three dimensional space requires a conventional definition. This problem was first addressed clearly by Einstein who proposed the following defining convention for clock synchronism.

Two clocks at position $\vec{r}$ and $\vec{r}'$ keeping time scales t and t' are said to be synchronized if they both assign the same time of emission t_e to a light signal emitted from position $\vec{r}_e$. The time of emission is to be determined in each case by:
(1) noting the times t, t' of reception of the signal at each location $\vec{r}$, $\vec{r}'$,
(2) subsequently subtracting from these times of reception the intervals Δt, $\Delta t'$ required for the signal to propagate from the point of emission to the respective clock locations.

The two clocks are synchronized if

$$t' - \Delta t' = t - \Delta t \; .$$

This is the basis of clock synchronization by Loran-C transmissions and by the reception of emissions from earth satellites such as Navstar GPS. The technique of clock synchronization by a satellite transponder is similar in principle and differs only in the fact that the source of synchronizing emissions is co-located with one of the clocks. In the case of the satellite transponder a signal is emitted from position $\vec{r}$ at time t and received, via a satellite link, at position r' at time t'.

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The signal is returned via the satellite link to position $\vec{r}$ arriving at time $t + \delta t$. The clocks are synchronized if

$$t' = t + \frac{\delta t}{2}$$

which is identical to the previous synchronizing condition with

$$\Delta t = -\frac{\delta t}{2}$$

$$\Delta t' = 0 .$$

In applying these clock synchronization techniques to clocks on the earth, any motion of the clocks relative to the source of the synchronizing emission subsequent to the signal emission will affect the nature of the achieved synchronization. For a source of synchronizing emission "at rest" relative to the solar system barycentre; if the effects of earth orbital motion and earth rotation are accounted for in calculating the signal propagation delays then the clocks will appear synchronized in a nonrotating barycentric frame. If the effects of the earth's orbital motion are neglected in calculating the signal propagation delays the clocks will appear synchronized in a nonrotating geocentric frame. The failure to account for the effects of earth rotation when synchronizing earth bound clocks will give rise to nontransitive synchronization effects (Ashby and Allan, 1979).

In classical physics the relative rate of ideal clocks is considered to be exactly unity, independent of the clocks motion or position. Relativity theory recognizes that the relative rates of ideal clocks will differ from unity according to the relative motion and relative position of the clocks. Except in special circumstances, free running ideal clocks, whose epochs have been once synchronized, will not remain synchronous unless the rates of at least one of the clocks is adjusted. A time scale in which the rates of otherwise ideal free running clocks are adjusted for purposes of synchronization with a single "master clock" is known as a coordinate time scale. In the presence of local gravitational fields, space-time is considered to be asymptotically flat at large distances from the gravitating masses and it is customary to introduce a coordinate time scale in which local coordinate clock epochs and rates are adjusted so that the coordinate clocks run synchronously with an ideal clock at rest at infinity.

3. PROPER TIME AND COORDINATE TIME

The relationship between propertime and coordinate time is expressed by the space-time metric. In a coordinate system in which the space-time metric tensor has components $g_{\mu\nu}$, the proper time interval ds separating space-time coordinate positions x^μ and $x^\mu + dx^\mu$, both on the world line of the clock, is given by

$$ds = \frac{1}{c}\sqrt{g_{\mu\nu}\, dx^\mu\, dx^\nu} \qquad \mu,\nu = 0,1,2,3$$

where c is the velocity of light. The solar system is characterized by "weak" gravitational fields and the space-time metric tensor can be written in the form $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu} << 1$ and where $\eta_{\mu\nu}$ is the metric tensor of flat space-time with signature (+1, -1, -1, -1). The post-Newtonian approximation to the solar system space-time metric expressed in a barycentric nonrotating Cartesian coordinate system with $x^o = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$, has the form (Will, 1974)

$$g_{oo} = 1 + \frac{2\Phi}{c^2} + \frac{2\beta\Phi^2}{c^4} + 4\,\frac{\Theta}{c^4} - \zeta\,\frac{A}{c^4}$$

$$g_{oj} = -\frac{7}{2}\Delta_1 \frac{{}_j}{c^3} - \frac{1}{2}\Delta_2 \frac{W_j}{c^3}$$

$$g_{ij} = -\delta_{ij}[1 - \frac{2\gamma\Phi}{c^2}]$$

where $\rho = \rho(\vec{r})$ is the local rest mass density and

$$\Phi(\vec{r}) = -G\int_{vol.} \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\,dV'$$

is the Newtonian gravitational potential and where

$$\Theta(\vec{r}) = \int_{vol.} \frac{\rho(\vec{r}')\theta(\vec{r}')}{|\vec{r}-\vec{r}'|}\,dV'$$

$$\theta(\vec{r}) = \beta_1 v^2 + \beta_2\Phi + \frac{1}{2}\beta_3\pi + \frac{3}{2}\beta_4\,\frac{p(\vec{r}')}{\rho(\vec{r}')}$$

$$A(\vec{r}) = \int_{vol.} \frac{\rho(\vec{r}')[(\vec{r}-\vec{r}')\cdot\vec{v}(\vec{r}')]^2}{|\vec{r}-\vec{r}'|^3}\,dV'$$

$$\vec{V}_j(r) = \int_{vol.} \frac{\rho(\vec{r}')v_j(\vec{r}')}{|\vec{r}-\vec{r}'|}\,dV'$$

$$\vec{W}_j(\vec{r}) = \int_{vol.} \frac{\rho(\vec{r}')[(\vec{r}-\vec{r}')\cdot\vec{v}(\vec{r}')][\vec{r}_j-\vec{r}_j']}{|\vec{r}-\vec{r}'|^3}\,dV'$$

The quantities π and p are the internal energy per unit rest mass and the local isotropic stress (pressure) respectively, measured in a comoving Lorentz frame. The numerical value of the dimensionless post-Newtonian parameters β, γ, β_1, β_2, β_3, β_4, Δ_1 and Δ_2 depend on the choice

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of gravitational theory. We shall assume their general relativity values of unity.

For antennas in the vicinity of the earth, including orbital antennas at altitudes of up to 4000 kilometers, the above general expressions for the post-Newtonian space-time metric can be greatly simplified while retaining accuracy of parts in 10^{16}. The barycentric position vector $\vec{r}$ can be regarded as the vector sum, $\vec{r} = \vec{r}_e + \vec{r}_g$, of $\vec{r}_e$, the barycentric position vector of the earth's centre of mass, and $\vec{r}_g$, the geocentric position vector of the antenna relative to the earth's centre of mass. The local mass density field can be subdivided into two components $\rho(\vec{r}) = \rho_e(\vec{r}) + \rho_g(\vec{r})$ where $\rho_g(\vec{r})$ is the mass density field of the earth and $\rho_e(\vec{r})$ is the mass density field of bodies external to the earth. Substituting this expression for the mass density field into the integral for the Newtonian potential gives $\Phi(\vec{r}) = \Phi_e(\vec{r}) + \Phi_g(\vec{r})$ where $\Phi_g(\vec{r})$ is the Newtonian potential of the earth's mass and $\Phi_e(\vec{r})$ is the Newtonian potential of the masses external to the earth. For antennas in the vicinity of the earth, including antennas in orbit with altitudes of up to 4000 kilometers, we have

$$\left(\frac{\dot{\vec{r}}_e}{c}\right)^2 \simeq 10^{-8} \qquad \left(\frac{\dot{\vec{r}}_g}{c}\right)^2 \leqslant 4 \times 10^{-10}$$

$$\frac{\Phi_e}{c^2} \simeq 10^{-8} \qquad \frac{\Phi_g}{c^2} \simeq 10^{-9}$$

and so to an accuracy of parts in 10^{16} we can write

$$g_{oo} = 1 + \frac{2\Phi}{c^2} + \frac{2\beta\Phi_e^2}{c^4}$$

$$g_{oj} = 0$$

$$g_{ij} = -\delta_{ij}\ [1 - \frac{2\gamma\Phi}{c^2}]$$

and

$$\frac{ds}{dt}(\vec{r}) = \left[1 + \frac{2\Phi(\vec{r})}{c^2} + 2\beta\,\frac{\Phi_e^2(\vec{r})}{c^4} - \frac{1}{c^2}(1 - 2\gamma\frac{\Phi(\vec{r})}{c^2})(\frac{d\vec{r}}{dt})^2\right]^{1/2} .$$

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Now for antennas in the vicinity of earth

$$\Phi_e(\vec{r}) = \Phi_e(\vec{r}_e+\vec{r}_g) = \Phi_e(\vec{r}_e) + \nabla\Phi_e(\vec{r}_e)\cdot\vec{r}_g + 0(10^{-17})$$

where

$$\nabla\Phi_e(\vec{r}_e) = -\vec{a}_e$$

and $\vec{a}_e$ is the barycentric acceleration of the earth's centre of mass. Following Thomas (1975), the use of the identity

$$-\vec{a}_e\cdot\vec{r}_g = \vec{v}_e\cdot\vec{v}_g - \frac{d}{dt}(\vec{v}_e\cdot\vec{r}_g)$$

allows us to obtain the following expression

$$\frac{ds}{dt} = 1 - \frac{1}{c^2}\frac{d}{dt}(\vec{v}_e\cdot\vec{r}_g) + \psi(\vec{r}_e) + \theta(\vec{r}_g) + \varepsilon(\vec{r}_e) + 0(10^{-17})$$

where

$$\psi(\vec{r}_e) = \frac{1}{c^2}[\Phi_e(\vec{r}_e) - \frac{1}{2}v_e{}^2]$$

$$\theta(\vec{r}_g) = \frac{1}{c^2}[\Phi_g(\vec{r}_g) - \frac{1}{2}v_g{}^2]$$

$$\varepsilon(\vec{r}_e) = \frac{1}{c^4}[\beta\Phi_e^2(\vec{r}_e) + \gamma\Phi_e(\vec{r}_e)v_e{}^2 - \frac{1}{8}c^2\,\psi^2(\vec{r}_e)]$$

It is useful to express $\psi(\vec{r}_e)$, $\theta(\vec{r}_g)$, and $\varepsilon(\vec{r}_e)$ in terms of their mean values $<\psi(\vec{r}_e)>$, $<\theta(\vec{r}_g)>$, and $<\theta(\vec{r}_e)>$ and variations $\Delta\psi(\vec{r}_e)$, $\Delta\theta(\vec{r}_g)$, and $\Delta\varepsilon(\vec{r}_g)$ about their means. In which case we have

$$\frac{ds}{dt} = 1 - \frac{1}{c^2}\frac{d}{dt}(\vec{v}_e\cdot\vec{r}_g) + <\psi(\vec{r}_e)> + <\theta(\vec{r}_g)> + <\varepsilon(\vec{r}_e)>$$

$$+ \Delta\psi(\vec{r}_e) + \Delta\theta(\vec{r}_g) + \Delta\varepsilon(\vec{r}_e)$$

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In particular for the earth in a Keplerian orbit about the sun with major axis a, eccentricity e, and mean anomaly f,

$$\Phi_e(\vec{r}_e) = -\frac{GM_\odot}{r_e}$$

where $M_\odot$ is the mass of the sun. Element orbital mechanics gives

$$\Phi_e(\vec{r}_e) + 1/2\, v_e^2 = -\frac{GM_\odot}{2a}$$

$$\frac{1}{r_e} = \frac{1 + e \cos f}{a(1-e^2)}$$

from which it follows that

$$\langle\psi(\vec{r}_e)\rangle = -\frac{GM_\odot}{2ac^2}\,\frac{3+e^2}{1-e^2}$$

$$\langle\varepsilon(\vec{r}_e)\rangle = \frac{G^2M_\odot^2}{a^2c^4}\left[\frac{2-(1-F)e^2}{(1-e^2)^2} - \frac{9+(6+16F)e^2+e^4}{32(1-e^2)^2}\right]$$

where $$F = \langle\cos^2 f\rangle\ .$$

These results are similar to those of Brumberg (1972) when the formulas presented here are reduced in accuracy to five parts in 10^{10} in which case the expressions take the form

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$$\frac{ds}{dt} = 1 - \frac{1}{c^2}[\phi + \frac{\dot{\vec{r}}^2}{2}]$$

For antennas in space for which, 6400 km $\leq r_g \leq$ 22400 km, it is necessary to take

$$\phi = \frac{GM_\odot}{|\vec{r}_e+\vec{r}_g|} + \frac{GM_\oplus}{|\vec{r}_g|}$$

where $M_\oplus$ is the mass of the earth; and for antennas in space for which, 22400 km $\leq r_g \leq 7\text{x}10^6$ km it is necessary only to take

$$\phi = \frac{GM_\odot}{|\vec{r}_e+\vec{r}_g|}$$

To an accuracy of five parts in 10^{10} one can also take

$$\dot{\vec{r}}^2 = v_e^2 + \vec{v}_e \cdot \vec{v}_g \ .$$

If the formulas presented here are further reduced in accuracy to five parts in 10^9 then it is only necessary to take

$$\phi = \frac{GM_\odot}{|\vec{r}_e+\vec{r}_g|}$$

and to take

$$\dot{\vec{r}}^2 = v_e^2 \ .$$

4. RELATIVISTIC EXPRESSION OF INTERFEROMETER TIME DELAY

The proper time delay τ, actually measured by a long baseline interferometer differs from the first order quantity referred to earlier in this paper as the Newtonian static geometric delay $T_o(t)$ as a consequence of a number of contributing effects. These effects include:
(1) the kinematical consequences of the motion of the antennas relative to the source of radiation during the interval by which the signal propagates from one antenna to the other. The expression for the static geometric delay is valid in a Newtonian sense only if the interferometer geometry is static or the velocity of light is infinite. In reality the finite velocity of light implies that the motion of the antennas alters the effective interferometer baseline and, consequently, the observed time delay.

(2) the relativistic effects of the curvature of space-time which alters the Newtonian coordinate time interval separating the space-time events of the arrival of the signal at each antenna site.
(3) the relativistic effects on the rates of proper time of the synchronized clocks. The relative rate of a clock's proper time and coordinate time is given by the expression for the line element which must be integrated along the clock's world line from the coordinate time of clock synchronization until the coordinate time of time delay measurement to correct for the differences between coordinate time and proper time.

The relativistic coordinate time interval $T(^{s}_{1},t^{s}_{2})$ separating the coordinate time of arrival t^{s}_{2} of the signal at antenna # 2 and the coordinate time of arrival t^{s}_{1} of the signal at antenna # 1 can be written in heliocentric space-time coordinates in the form (Brumberg, 1972; Finkelstein, 1983)

$$T(t^{s}_{1},t^{s}_{2}) = t^{s}_{1} - t^{s}_{2}$$

$$= \frac{1}{c}[|\vec{R}_{o}(t^{s}_{o}) - \vec{R}_{1}(t^{s}_{1})| - |\vec{R}_{o}(t^{s}_{o}) - \vec{R}_{2}(t^{s}_{2})|]$$

$$+ \frac{(1+\gamma)GM_{\odot}}{c^{3}}\left[\ell n\left(\frac{R_{1}-\vec{R}_{1}\cdot\hat{k}_{1}}{R_{o}-\vec{R}_{o}\cdot\hat{k}_{1}}\right) - \ell n\left(\frac{R_{2}-\vec{R}_{2}\cdot\hat{k}_{2}}{R_{o}-\vec{R}_{o}\cdot\hat{k}_{2}}\right)\right]$$

$$+ \sum_{j=1}^{N} \frac{(1+\gamma)Gm_{j}}{c^{3}}\left[\ell n\left(\frac{|\vec{R}_{1}-\vec{R}_{j}| - (\vec{R}_{1}-\vec{R}_{j})\cdot\hat{k}_{1}}{|\vec{R}_{o}-\vec{R}_{j}| - (\vec{R}_{o}-\vec{R}_{j})\cdot\hat{k}_{1}}\right)\right.$$

$$\left. - \ell n\left(\frac{|\vec{R}_{2}-\vec{R}_{j}| - (\vec{R}_{2}-\vec{R}_{j})\cdot\hat{k}_{2}}{|\vec{R}_{o}-\vec{R}_{j}| - (\vec{R}_{o}-\vec{R}_{j})\cdot\hat{k}_{2}}\right)\right]$$

In the above expression $R_{o}(t^{s}_{o})$ is the heliocentric position of the source of radiation and t^{s}_{o} is the coordinate time of the signal emission from the source. $\vec{R}_{2}(t^{s}_{2})$ and $\vec{R}_{1}(t^{s}_{1})$ are the heliocentric positions of the antennas at the coordinate times t^{s}_{2} and t^{s}_{1} corresponding to the events of signal reception at antennas #2 and #1 respectively; while the unit vectors $\hat{k}_{2}$ and $\hat{k}_{1}$ are the apparent directions of the source of radiation as viewed from antenna sites #1 and #2 respectively. $M_{\odot}$ is the mass of the sun and m_{j}, j=1,2...N, are the masses of other gravitating bodies whose heliocentric positions at the time of observation are given by $\vec{R}_{j}$, j=1,2...N, respectively. For extra-galactic radio sources $\vec{R}_{o}>>\vec{R}_{i}$, i=1,2, and the parallax which distinguishes the unit vector $\hat{k}_{2}$ from the unit vector $\hat{k}_{1}$ vanishes. As a result, for extra-galactic sources, the above expression for coordinate time delay reduces to

$$
\begin{aligned}
T(t_1^s, t_2^s) &= t_1^s - t_2^s \\
&= \frac{1}{c}\,[\vec{R}_2(t_2^s) - \vec{R}_1(t_1^s)] \\
&+ \frac{(1+\gamma)GM_\odot}{c^3}\left[\ln\left(\frac{R_1+\vec{R}_1\cdot\hat{k}}{R_2+\vec{R}_2\cdot\hat{k}}\right)\right] \\
&+ \sum_{j=1}^{N}\frac{(1+\gamma)GM_j}{c^3}\left[\ln\left(\frac{|\vec{R}_1-\vec{R}_j| + (\vec{R}_1-\vec{R}_j)\cdot\hat{k}}{|\vec{R}_2-\vec{R}_j| + (\vec{R}_2-\vec{R}_j)\cdot\hat{k}}\right)\right].
\end{aligned}
$$

In each of these results the leading term in the expression for the coordinate time delay is the Newtonian static geometric delay modified by the kinematic effects of the antenna motions. The remaining terms represent the effects of space-time curvature on the coordinate time delay arising from the gravitational fields of the bodies of the solar system.

The heliocentric coordinate time delay, $t_1^s - t_2^s$, is given implicitly by the above expressions in the form $t_1^s - t_2^s = T(t_1^s, t_2^s)$ and in this form does not refer to an explicit instant of coordinate time t. For purposes of comparing theoretical models with observations and for least squares parameter estimation it is convenient to have a theoretical model for coordinate time delay which appears as an explicit function with coordinate time t as an independent variable. This is achieved by reducing the expression for coordinate time delay to some standard epoch t where $t_2^s \leqslant t \leqslant t_1^s$. In Finkelstein (1983) the position of the standard epoch t within the interval $t_2^s \leqslant t \leqslant t_1^s$ is selectable by a choice of his parameter γ, (not to be confused with the post-Newtonian expansion parameter $\dot{\gamma}$). The standard procedure used to achieve this result is to expand the heliocentric positions of the antenna sites in a Taylor series about the standard epoch t and solve the resulting equation T(t) by an iterative numerical method (Fanselow, 1983) or; to substitute the expression for the static geometric delay into the Taylor series expansion which is then truncated after one (Thomas, 1975) or more terms (Robertson, 1975; Finkelstein, 1983).

For purposes of illustration we can consider the case of an extra-galactic source of radiation where we chose to reduce the coordinate time delay to the standard epoch t_2^s. Following Robertson (1975) we can set

$$
\vec{R}_1(t_1^s) = \vec{R}_1(t_2^s) + \dot{\vec{R}}_1(t_2^s)\cdot T(t_1^s, t_2^s) + \frac{1}{2}\,\ddot{\vec{R}}_1(t_2^s)\cdot T^2(t_1^s, t_2^s) + \cdots
$$

and then substitute

$$T(t_1^s, t_2^s) \cong \frac{1}{c}[\vec{R}_2(t_2^s) - \vec{R}_1(t_2^s)]$$

into the Taylor series expansion. For post-Newtonian accuracy the errors should be confined to terms of order $1/c^3$ or higher and so it is necessary to carry the Taylor series expansion out to, at least, second order. The expansion of Thomas (1975) is therefore valid only to Newtonian accuracy.

4.1 Conversion from Coordinate Time to Proper Time

The above relativistic expression for the interferometer coordinate time delay $T(t_1^s, t_2^s)$ must be converted from coordinate time to proper time by the use of the theoretical formulas developed in section 3. If the clocks are synchronized at some coordinate time t_c then the proper time intervals, $s_2(t_1^s)$, $s_2(t_2^s)$ indicated on each of the two clocks at the coordinate times t_1^s, t_2^s of signal reception are

$$s_1(t_1^s) - s_1(t_c) = \int_{t_c}^{t_1^s} \frac{ds_1}{dt}\, dt = t_1^s - t_c + \int_{t_c}^{t_1^s} (\frac{ds_1}{dt} - 1)\, dt$$

and

$$s_2(t_2^s) - s_2(t_c) = \int_{t_c}^{t_2^s} \frac{ds_2}{dt}\, dt = t_2^s - t_c + \int_{t_c}^{t_2^s} (\frac{ds_2}{dt} - 1)\, dt .$$

The interferometer proper time delay τ is given by $\tau = s_1(t_1^s) - s_2(t_2^s)$ which can be written as $\tau = [s_1(t_1^s)-s_1(t_c)]-[s_2(t_2^s)-s_2(t_c)]$ since clock synchronization gives $s_1(t_c) = s_2(t_c)$. These relationships yield a formula for interferometer proper time delay given by

$$\tau = t_1^2 + \int_{t_c}^{t_1^s} (\frac{ds_1}{dt} - 1) dt - t_2^s - \int_{t_c}^{t_1^s} (\frac{ds_2}{dt} - 1)\, dt$$

$$= T(t_1^s, t_2^s) + \int_{t_c}^{t_1^s} (\frac{ds_1}{dt} - 1)\, dt - \int_{t_c}^{t_2^s} (\frac{ds_2}{dt} -1)\, dt .$$

To an accuracy of five parts in 10^{10} this can be written

$$\tau = T(t_1^s, t_2^s) + (t_1^s - t_c)(\frac{ds_1}{dt} - 1) - (t_2^s - t_c)(\frac{ds_2}{dt} - 1) .$$

In the above treatment of the problem it is assumed that the coordinate clocks are synchronized while at rest in a heliocentric frame. The problem can equally well be solved using coordinate clocks which are synchronized while at rest in a non-rotating geocentric frame. Alternatively the solution presented here can be Lorentz transformed into a non-rotating geocentric frame by the formulas in Finkelstein (1983). Transforming from the heliocentric frame to the geocentric frame gives rise to an additional term of the form $1/c^2\, \vec{v}_e \cdot \vec{b}$ and where $\vec{v}_e$ is the heliocentric velocity of the earth's center of mass (Thomas, 1975; Soffel, 1985). This term is a manifestation of the relativity of simultaneity between relatively moving reference frames.

Differences between the formulas of all the authors cited, including those of Pavlov (1984) and Pavlov (1985) which are expressed in the non-inertial reference frame of the rotating earth, are a consequence of the choice of reference system. However while the different formulas reflect the different choices of coordinate systems the interferometer proper time delay is a coordinate independent quantity and in all cases the consistent use of the chosen coordinate system and the formulas which apply will yield coordinate independent results.

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DISCUSSION

Alley : what kind of clocks are used in soviet cosmical experiments ?

Kreinovich : I do not know the exact technical details. Maser clocks are used in orbit, corrected by periodic signals of more precise Earth-based masers.

Pavlov : why do you use Newtonian expressions for fringe frequency in relativistic case ?

Kreinovich : we use the formula $F = \Delta\tau/\Delta$ because it is the way that one computes F in real VLBI. In the case another formulation would be used we would of course also apply it.

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