Complex Planetary Systems II: Latest Methods for an Interdisciplinary Approach
Proceedings IAU Symposium No. 382, 2024
A. Lemaitre & A.-S. Libert, eds.
doi:10.1017/S1743921323004763

Investigation of the dynamical evolution of planetary systems with isotropically varying masses

M. Zh. Minglibayev[1,2], **A. N. Prokopenya**[3] **and A. B. Kosherbayeva**[1]

[1]Al-Farabi Kazakh National University, Almaty, Kazakhstan

[2]Fesenkov Astrophysical Institute, Almaty, Kazakhstan
email: Minglibayev@gmail.com

[3]Warsaw University of Life Sciences, Warsaw, Poland

Abstract. In this work, the secular evolution of exoplanetary systems is investigated, when the variability of the masses of celestial bodies is the leading factor of dynamical evolution. The masses of the parent star and the planets change due to the particles leaving the bodies and falling on them. At the same time, bodies masses are assumed to change isotropically at different rates. The law of mass change is considered to be known and given function of time. The relative motions of the planets are investigated by the methods of the canonical perturbation theory in the absence of resonances. It is assumed that the orbits of the planets do not intersect. Evolutionary equations in analogues of Poincaré variables $(\Lambda_i, \lambda_i, \xi_i, \eta_i, p_i, q_i)$ are obtained and used to study the *K2-3* exoplanetary system. All analytical and numerical calculations are performed with the aid of the Wolfram Mathematica.

Keywords. variable mass, canonical variables, perturbation theory, exoplanetary system, Poincaré variables.

1. Introduction

To date, more than 4,000 exoplanetary systems are known (Exoplanet Exploration 2023). The variability of the masses of real celestial bodies significantly affects the possible trajectories of their motion and determines the rich variety of evolutionary orbits. In our work, within the framework of the classical problem of $n+1$ bodies with variable masses, the relative motion of n planets in quasi-elliptical non-intersecting orbits is investigated. It is assumed that the celestial bodies are spherically symmetrical and attract each other according to Newton's law of gravitation.

2. Problem statement

In the considered case of the n planets problem, it is convenient to write the differential equations in the following form (see Minglibayev (2012)):

$$\ddot{\vec{r}}_i + f\frac{(m_0+m_i)}{r_i^3}\vec{r}_i - \frac{\ddot{\gamma}_i}{\gamma_i}\vec{r}_i = grad_{\vec{r}_i} f\sum_{j=1}^{n} m_j\left(\frac{1}{r_{ij}} - \frac{\vec{r}_i\cdot\vec{r}_j}{r_j^3}\right) - \frac{\ddot{\gamma}_i}{\gamma_i}\vec{r}_i, \quad i\neq j, \qquad (1)$$

here $\vec{r}_i$ is the radius-vector of the considered planet relative to the center of mass of the parent star, f is the gravitational constant, m_0, m_i are the masses of the parent star

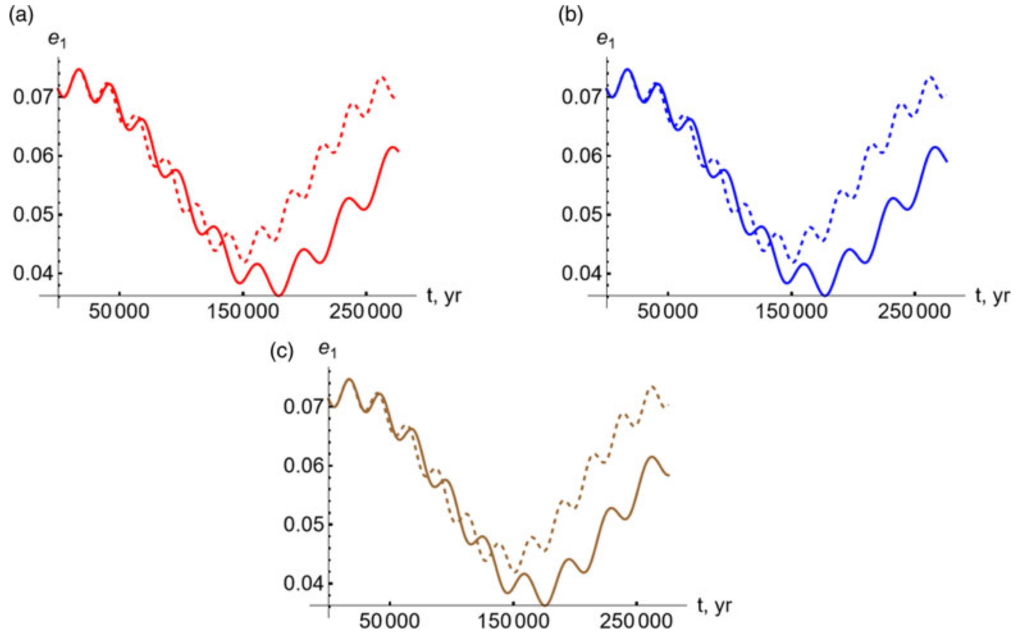

Figure 1. Comparison of the obtained results for the eccentricities in the constant mass case (dashed line) and in the cases of variable mass (continuous line). The Eddington-Jeans laws are considered when the degree of the mass law for the parent star are a) $n_0 = 5/2$; b) $n_0 = 3$ and c) $n_0 = 7/2$.

and planet, respectively, $\vec{r}_{ij} = \vec{r}_j - \vec{r}_i$, $\gamma_i = \frac{m_0(t_0)+m_i(t_0)}{m_0(t)+m_i(t)} = \gamma_i(t)$, t_0 is the initial instant of time.

3. Secular perturbations

In the system of differential equations (2.1), if the right-hand side is zero, we get aperiodic motions along the quasi-conic sections (as in see Minglibayev (2012) for more information), which are used as unperturbed motion. The corresponding canonical theory of perturbations in analogues of Poincaré variables has been developed in Prokopenya et al. (2022) and in Kosherbayeva et al. (2023).

4. Dynamical evolution of the system *K2-3*

The *K2-3* exoplanetary system is considered as an example (Almenara et al. 2015). Numerical solutions of the evolutionary equations written in dimensionless variables in Kosherbayeva et al. (2023) are obtained, and the dependence of the analogues of the osculating elements on time is found. Let's assume that the mass of the parent star changes over time according to the Eddington-Jeans law $\dot{m}_0 = -\alpha_0\ m_0^{n_0}$, where $0.4 < n_0 < 4$ and parameter α_0 is chosen from the condition $\dot{m}_0 = -10^{-5} m_0/yr$ at the initial instance of time. Masses of the planets are assumed to increase linearly with time with the rates $\dot{m}_1 = \frac{m_1}{40000} yr^{-1}$, $\dot{m}_2 = \frac{m_2}{70000} yr^{-1}$ and $\dot{m}_3 = \frac{m_3}{80000} yr^{-1}$. Figure 1 shows the results obtained for the analogue of the eccentricity of the first planet during the interval of 250 000 yr.

5. Conclusion

The resulting system of evolutionary equations includes $4n$ linear non-autonomous differential equations, the coefficients of which are various complicated functions of time.

https://doi.org/10.1017/S1743921323004763 Published online by Cambridge University Press

In this work, the evolutionary equations for $n = 3$ are explicitly obtained for the system *K2-3*. Various tracks of osculating elements' evolution are studied by numerical methods.

6. Acknowledgements

This research was funded by the Committee of Science of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP14869472).

References

Exoplanet Exploration, url: https://exoplanets.nasa.gov/, Last update: October 16, 2023.

Minglibayev, M. Zh., 2012, LAP LAMBERT Academic Publishing, 224 (in Russ.)

Prokopenya, A. N., Minglibayev, M. Zh., Kosherbaeva, A. B., 2022, Programming and Computer Software, 48(2),107–115, DOI: 10.1134/S0361768822020098

Kosherbayeva, A. B., Minglibayev, M. Zh. 2023, Proceedings of the International Astronomical Union, 17, S370: Winds of Stars and Exoplanets, 283–284, DOI:10.1017/S1743921322003611

Almenara, J. M., et al. 2015, A&A, 581, id.L7, 6, DOI:10.1051/0004-6361/201525918