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LETTER TO THE EDITOR

Comments on the Paper "The Monozygotic Twinning Rate: Is It Really Constant?"

In a paper in this journal Allen and Hrubec (1988) refer to the author of this note. In order to bring the personal communication to a broader audience some results and some references are given here.

Allen and Hrubec assume a constant MZ ratio and suggest the model

$$\mathrm{SSR} = \alpha + \beta \mathrm{OSR} + e$$

where

SSR = the rate of twins of the same sex
OSR = the rate of twins of opposite sex
α = a parameter = the constant MZ rate
β = a parameter measuring the proportion $\frac{\text{DZ twins of the same sex}}{\text{DZ twins of opposite sex}}$

Furthermore, they suggest ordinary least squares (OLS). The random error in the twinning rate depends on the total number of maternities. In order to obtain efficient estimates, weighted least squares (WLS) should be used. However, if only the twinning rates are known, OLS has to be used. Unfortunately, the independent variable OSR is observed with error. This will cause biased and inconsistent estimates.

Following, eg, Wetherill [3], we consider the linear model

$$Y = \alpha + \beta X + \varepsilon$$

If X is observed without error, OLS gives the estimator

$$\hat{\beta} = \frac{\mathrm{Cov}(Y, X)}{\mathrm{Var}(X)}$$

and $E(\hat{\beta}) = \beta$.

If X is observed with error, we observe $x = X + \delta$. Assume that δ is $N(0, \sigma_\delta^2)$ and that δ is independent of Y, X and ε. Then we estimate β in the model

$$Y = \alpha + \beta(x - \delta) + \varepsilon$$

or, if $\eta = \beta\delta + \varepsilon$,

$$Y = \alpha + \beta x + \eta$$

Now, the estimator is

$$\tilde{\beta} = \frac{\mathrm{Cov}(Y, x)}{\mathrm{Var}(x)} = \frac{\mathrm{Cov}(Y, X) + \mathrm{Cov}(Y, \delta)}{\mathrm{Var}(X) + \mathrm{Var}(\delta)} = \frac{\mathrm{Cov}(Y, X)}{\mathrm{Var}(X) + \mathrm{Var}(\delta)} =$$
$$= \hat{\beta} \cdot \frac{\mathrm{Var}(X)}{\mathrm{Var}(X) + \mathrm{Var}(\delta)} = \hat{\beta} - \hat{\beta}\frac{\mathrm{Var}(\delta)}{\mathrm{Var}(X) + \mathrm{Var}(\delta)} < \hat{\beta}$$

Furthermore,

$$E(\tilde{\beta}) = \beta - \beta \cdot \frac{\mathrm{Var}(X)}{\mathrm{Var}(X) + \mathrm{Var}(\delta)}$$

We observe that the estimator is biased and that the bias does not tend to zero when n tends to infinity. Hence, the estimator is inconsistent. The econometric literature has paid attention to this problem and some suggestions how to evade this problem have been given [2].

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REFERENCES

1. Allen G, Hrubec Z (1987): The monozygotic twinning rate: is it really constant? Acta Genet Med Gemellol 36:389-396.
2. Kmenta J (1971): Elements of Econometrics. New York: Macmillan Publ Co Inc, XIII + 655 pp.
3. Whetherill GB (1986): Regression Analysis with Applications. London: Chapman and Hall, XI + 311 pp.