Appendix B

Sparticle decay widths

In this appendix, we list formulae for the partial widths for the $1 \to 2$ and $1 \to 3$ tree-level decays of sparticles that are relevant to SUSY searches at colliders.

B.1 Gluino decay widths

B.1.1 Two-body decays

The decay width for $\tilde{g} \to \bar{q}\tilde{q}_i$ ($i = \mathrm{L}$ or R), is given by

$$\Gamma(\tilde{g} \to \bar{q}\tilde{q}_i) = \frac{\alpha_s}{8} m_{\tilde{g}} \lambda^{1/2}(1, \frac{m_q^2}{m_{\tilde{g}}^2}, \frac{m_{\tilde{q}_i}^2}{m_{\tilde{g}}^2}) \left(1 + \frac{m_q^2}{m_{\tilde{g}}^2} - \frac{m_{\tilde{q}_i}^2}{m_{\tilde{g}}^2}\right). \tag{B.1a}$$

If intra-generation squark mixing and quark mass effects are included, the formula is slightly more complicated:

$$\Gamma(\tilde{g} \to \bar{q}\tilde{q}_{1,2}) = \frac{\alpha_s}{8} m_{\tilde{g}} \lambda^{1/2}(1, \frac{m_q^2}{m_{\tilde{g}}^2}, \frac{m_{\tilde{q}_{1,2}}^2}{m_{\tilde{g}}^2}) \times \left(1 + \frac{m_q^2}{m_{\tilde{g}}^2} - \frac{m_{\tilde{q}_{1,2}}^2}{m_{\tilde{g}}^2} \pm 2(-1)^{\theta_{\tilde{g}}} \sin 2\theta_q \frac{m_q}{m_{\tilde{g}}}\right), \tag{B.1b}$$

where the upper (lower) sign is for the decay to $\tilde{q}_1$ ($\tilde{q}_2$).

In models with a very light gravitino, the gluino decay to gluon and a gravitino may be relevant:

$$\Gamma(\tilde{g} \to \tilde{G}g) = \frac{m_{\tilde{g}}^5}{48\pi(m_{3/2}M_\mathrm{P})^2}, \tag{B.2}$$

where M_P is the reduced Planck mass, $M_\mathrm{P} \simeq 2.4 \times 10^{18}$ GeV, and we have ignored the gravitino mass in the phase space.

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B.1.2 Three-body decays to light quarks

If two-body gluino decays are kinematically suppressed, then three-body decays can be important. The three-body decay $\tilde{g} \to \widetilde{W}_i u\bar{d}$ width is given by

$$\Gamma(\tilde{g} \to \widetilde{W}_i u\bar{d}) = \frac{\alpha_s}{16\pi^2}\Big[|A^d_{\widetilde{W}_i}|^2\psi(m_{\tilde{g}}, m_{\tilde{u}_L}, m_{\widetilde{W}_i}) + |A^u_{\widetilde{W}_i}|^2\psi(m_{\tilde{g}}, m_{\tilde{d}_L}, m_{\widetilde{W}_i}) \\ -2(-1)^{\theta_{\tilde{g}}}\mathrm{Re}(A^u_{\widetilde{W}_i}A^d_{\widetilde{W}_i})\phi(m_{\tilde{g}}, m_{\tilde{u}_L}, m_{\tilde{d}_L}, m_{\widetilde{W}_i})\Big], \tag{B.3}$$

where

$$\psi(m_{\tilde{g}}, m_{\tilde{q}}, m) = \int \mathrm{d}q \frac{q^2(m^2_{\tilde{g}} - 2m_{\tilde{g}}q - m^2)^2}{(m^2_{\tilde{g}} - 2m_{\tilde{g}}q - m^2_{\tilde{q}})^2(m^2_{\tilde{g}} - 2m_{\tilde{g}}q)}$$

and

$$\phi(m_{\tilde{g}}, m_{\tilde{q}_1}, m_{\tilde{q}_2}, m) = \frac{m}{2}\int \frac{\mathrm{d}q}{m^2_{\tilde{g}} - m^2_{\tilde{q}_1} - 2m_{\tilde{g}}q}\Bigg[\frac{-q(m^2_{\tilde{g}} - m^2 - 2m_{\tilde{g}}q)}{m_{\tilde{g}}(m_{\tilde{g}} - 2q)} \\ - \frac{2m_{\tilde{g}}q - m^2_{\tilde{q}_2} + m^2}{2m_{\tilde{g}}}\log\frac{m^2_{\tilde{q}_2}(m_{\tilde{g}} - 2q) - m_{\tilde{g}}m^2}{(m_{\tilde{g}} - 2q)(m^2_{\tilde{q}_2} - 2m_{\tilde{g}}q - m^2)}\Bigg],$$

and where the range of integration on ψ and ϕ ranges from 0 to $(m^2_{\tilde{g}} - m^2)/2m_{\tilde{g}}$.

The decay rate for $\tilde{g} \to \widetilde{Z}_i q\bar{q}$ is given by

$$\Gamma(\tilde{g} \to \widetilde{Z}_i q\bar{q}) = \frac{\alpha_s}{8\pi^2}\Big[|A^q_{\widetilde{Z}_i}|^2\big(\psi(m_{\tilde{g}}, m_{\tilde{q}_L}, m_{\widetilde{Z}_i}) \\ -(-1)^{\theta_i+\theta_{\tilde{g}}-1}\phi(m_{\tilde{g}}, m_{\tilde{q}_L}, m_{\tilde{q}_L}, m_{\widetilde{Z}_i})\big) + |B^q_{\widetilde{Z}_i}|^2\big(\psi(m_{\tilde{g}}, m_{\tilde{q}_R}, m_{\widetilde{Z}_i}) \\ - (-1)^{\theta_i+\theta_{\tilde{g}}-1}\phi(m_{\tilde{g}}, m_{\tilde{q}_R}, m_{\tilde{q}_R}, m_{\widetilde{Z}_i})\big)\Big]. \tag{B.4}$$

The formulae for gluino decay to third generation particles are more complicated. They involve Yukawa coupling contributions, squark mixing effects and all final state fermion masses are non-negligible.

B.1.3 $\tilde{g} \to \widetilde{Z}_i t\bar{t}$ *and* $\tilde{g} \to \widetilde{Z}_i b\bar{b}$

The partial width for $\tilde{g} \to t\bar{t}\widetilde{Z}_i$ can be written as

$$\Gamma(\tilde{g} \to t\bar{t}\widetilde{Z}_i) = \frac{\alpha_s}{8\pi^4 m_{\tilde{g}}}\left[\Gamma_{\tilde{t}_1} + \Gamma_{\tilde{t}_2} + \Gamma_{\tilde{t}_1\tilde{t}_2}\right], \tag{B.5}$$

with,

$$\Gamma_{\tilde{t}_1} = \Gamma_{LL}(\tilde{t}_1)\cos^2\theta_t + \Gamma_{RR}(\tilde{t}_1)\sin^2\theta_t \\ - \sin\theta_t\cos\theta_t\left\{\Gamma_{L_1R_1} + \Gamma_{L_1R_2} + \Gamma_{L_2R_1} + \Gamma_{L_2R_2}\right\}(\tilde{t}_1), \tag{B.6a}$$

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$$\Gamma_{\tilde{t}_2} = \Gamma_{\rm LL}(\tilde{t}_2)\sin^2\theta_t + \Gamma_{\rm RR}(\tilde{t}_2)\cos^2\theta_t + \sin\theta_t\cos\theta_t\left\{\Gamma_{\rm L_1R_1} + \Gamma_{\rm L_1R_2} + \Gamma_{\rm L_2R_1} + \Gamma_{\rm L_2R_2}\right\}(\tilde{t}_2), \tag{B.6b}$$

and

$$\Gamma_{\tilde{t}_1\tilde{t}_2} = (\Gamma_{\rm LL}(\tilde{t}_1,\tilde{t}_2) + \Gamma_{\rm RR}(\tilde{t}_1,\tilde{t}_2))\sin\theta_t\cos\theta_t + \Gamma_{\rm LR}(\tilde{t}_1,\tilde{t}_2)\cos^2\theta_t + \Gamma_{\rm RL}(\tilde{t}_1,\tilde{t}_2)\sin^2\theta_t. \tag{B.6c}$$

The Γ_{ij} contributions to the partial width are all written in terms of one-dimensional integrals. The various $\Gamma_{ij}(\tilde{t}_1)$ that enter the expression (B.6a) for $\Gamma_{\tilde{t}_1}$ are:

$$\begin{aligned}\Gamma_{\rm LL}(\tilde{t}_1) &= (\alpha_1^2+\beta_1^2)\psi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) - 4m_t m_{\widetilde{Z}_i}(-1)^{\theta_i}\alpha_1\beta_1\chi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) \\ &+ (-1)^{\theta_{\tilde{g}}} m_{\tilde{g}}\Bigg[(-1)^{\theta_i} m_{\widetilde{Z}_i}\left(\frac{\alpha_1^2}{m_{\tilde{g}} m_{\widetilde{Z}_i}}\phi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) + \beta_1^2 m_t^2 \rho(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i})\right) \\ &- \alpha_1\beta_1 m_t\left(\xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) - m_{\widetilde{Z}_i}^2\rho(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i})\right)\Bigg],\end{aligned} \tag{B.7}$$

where

$$\alpha_1 = \tilde{A}^t_{\widetilde{Z}_i}\cos\theta_t - f_t v_1^{(i)}\sin\theta_t \quad \text{and} \tag{B.8a}$$

$$\beta_1 = f_t v_1^{(i)}\cos\theta_t + \tilde{B}^t_{\widetilde{Z}_i}\sin\theta_t, \tag{B.8b}$$

and

$$\tilde{A}^t_{\widetilde{Z}_i} = \frac{g}{\sqrt{2}}v_3^{(i)} + \frac{g'}{3\sqrt{2}}v_4^{(i)}, \tag{B.9a}$$

$$\tilde{A}^b_{\widetilde{Z}_i} = -\frac{g}{\sqrt{2}}v_3^{(i)} + \frac{g'}{3\sqrt{2}}v_4^{(i)}, \tag{B.9b}$$

$$\tilde{B}^t_{\widetilde{Z}_i} = \frac{4}{3}\frac{g'}{\sqrt{2}}v_4^{(i)}, \quad \text{and} \tag{B.9c}$$

$$\tilde{B}^b_{\widetilde{Z}_i} = -\frac{2}{3}\frac{g'}{\sqrt{2}}v_4^{(i)}. \tag{B.9d}$$

Also,

$$\Gamma_{\rm RR}(\tilde{t}_1) = \Gamma_{\rm LL}(\tilde{t}_1), \tag{B.10}$$

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but with $\alpha_1 \to \alpha_2$ and $\beta_1 \to \beta_2$, where

$$\alpha_2 = \tilde{B}^t_{\widetilde{Z}_i} \sin\theta_t + f_t v_1^{(i)} \cos\theta_t \quad \text{and} \tag{B.11a}$$

$$\beta_2 = -f_t v_1^{(i)} \sin\theta_t + \tilde{A}^t_{\widetilde{Z}_i} \cos\theta_t. \tag{B.11b}$$

Furthermore,

$$\Gamma_{\mathrm{L_1R_1}}(\tilde{t}_1) = 2m_{\tilde{g}} m_t (-1)^{\theta_{\tilde{g}}} \left[(\alpha_1\alpha_2 + \beta_1\beta_2)(-1)^{\theta_i} m_t m_{\widetilde{Z}_i} \zeta(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) - (\alpha_2\beta_1 + \alpha_1\beta_2) X(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) \right], \tag{B.12}$$

with

$$\Gamma_{\mathrm{L_2R_2}}(\tilde{t}_1) = \Gamma_{\mathrm{L_1R_1}}(\tilde{t}_1). \tag{B.13}$$

Finally,

$$\Gamma_{\mathrm{L_1R_2}}(\tilde{t}_1) = \beta_1\beta_2 Y(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) + \alpha_1\alpha_2 m_t^2 \xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) - m_t m_{\widetilde{Z}_i} (-1)^{\theta_i} (\alpha_1\beta_2 + \alpha_2\beta_1) \chi'(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) \tag{B.14}$$

with

$$\Gamma_{\mathrm{L_2R_1}}(\tilde{t}_1) = \Gamma_{\mathrm{L_1R_2}}(\tilde{t}_1). \tag{B.15}$$

Turning to $\Gamma_{\tilde{t}_2}$, we have

$$\Gamma_{\mathrm{LL}}(\tilde{t}_2) = \Gamma_{\mathrm{LL}}(\tilde{t}_1), \tag{B.16}$$

but with the replacements

$$m_{\tilde{t}_1} \to m_{\tilde{t}_2}, \tag{B.17a}$$

$$\alpha_1 \to \tilde{A}^t_{\widetilde{Z}_i} \sin\theta_t + f_t v_1^{(i)} \cos\theta_t, \tag{B.17b}$$

$$\beta_1 \to f_t v_1^{(i)} \sin\theta_t - \tilde{B}^t_{\widetilde{Z}_i} \cos\theta_t, \tag{B.17c}$$

and

$$\Gamma_{\mathrm{RR}}(\tilde{t}_2) = \Gamma_{\mathrm{RR}}(\tilde{t}_1), \tag{B.18}$$

with

$$m_{\tilde{t}_1} \to m_{\tilde{t}_2}, \tag{B.19a}$$

$$\alpha_2 \to -\tilde{B}^t_{\widetilde{Z}_i} \cos\theta_t + f_t v_1^{(i)} \sin\theta_t, \quad \text{and} \tag{B.19b}$$

$$\beta_2 \to f_t v_1^{(i)} \cos\theta_t + \tilde{A}^t_{\widetilde{Z}_i} \sin\theta_t. \tag{B.19c}$$

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Also,

$$\Gamma_{L_1R_1}(\tilde{t}_2) = \Gamma_{L_2R_2}(\tilde{t}_2), \quad \text{and} \tag{B.20a}$$

$$\Gamma_{L_1R_2}(\tilde{t}_2) = \Gamma_{L_2R_1}(\tilde{t}_2), \tag{B.20b}$$

where these expressions can be obtained from the previous $\Gamma_{L_iR_j}$ formulae by replacing $m_{\tilde{t}_1} \to m_{\tilde{t}_2}$ and using the revised α_1, α_2, β_1 and β_2 values.

The expression for $\Gamma_{\tilde{t}_1\tilde{t}_2}$ contains,

$$\begin{aligned}\Gamma_{LL}(\tilde{t}_1, \tilde{t}_2) = {} & 2(\alpha_1\alpha_2 + \beta_1\beta_2)\tilde{\psi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\ & -(-1)^{\theta_i} 4 m_t m_{\widetilde{Z}_i}(\alpha_1\beta_2 + \alpha_2\beta_1)\tilde{\chi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\ & -(-1)^{\theta_{\tilde{g}}} m_{\tilde{g}}\{2m_{\widetilde{Z}_i}(-1)^{\theta_i - 1}[\frac{\alpha_1\alpha_2}{m_{\tilde{g}} m_{\widetilde{Z}_i}}\tilde{\phi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\ & +\beta_1\beta_2 m_t^2 \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i})] \\ & +(\alpha_1\beta_2 + \alpha_2\beta_1) m_t[\xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\ & -m^2_{\widetilde{Z}_i}\tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i})]\},\end{aligned} \tag{B.21}$$

where

$$\alpha_1 = \tilde{A}^t_{\widetilde{Z}_i}\cos\theta_t - f_t v_1^{(i)}\sin\theta_t, \tag{B.22a}$$

$$\alpha_2 = \tilde{A}^t_{\widetilde{Z}_i}\sin\theta_t + f_t v_1^{(i)}\cos\theta_t, \tag{B.22b}$$

$$\beta_1 = f_t v_1^{(i)}\cos\theta_t + \tilde{B}^t_{\widetilde{Z}_i}\sin\theta_t, \quad \text{and} \tag{B.22c}$$

$$\beta_2 = f_t v_1^{(i)}\sin\theta_t - \tilde{B}^t_{\widetilde{Z}_i}\cos\theta_t. \tag{B.22d}$$

Also,

$$\begin{aligned}\Gamma_{RR}(\tilde{t}_1, \tilde{t}_2) = {} & -2(\alpha_1\alpha_2 + \beta_1\beta_2)\tilde{\psi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\ & -(-1)^{\theta_i} 4 m_t m_{\widetilde{Z}_i}(\alpha_1\beta_2 + \alpha_2\beta_1)\tilde{\chi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\ & +(-1)^{\theta_{\tilde{g}}} m_{\tilde{g}}\{2m_{\widetilde{Z}_i}(-1)^{\theta_i - 1}[\frac{\alpha_1\alpha_2}{m_{\tilde{g}} m_{\widetilde{Z}_i}}\tilde{\phi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\ & +\beta_1\beta_2 m_t^2 \tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i})] \\ & -(\alpha_1\beta_2 + \alpha_2\beta_1) m_t[\xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\ & -m^2_{\widetilde{Z}_i}\tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i})]\},\end{aligned} \tag{B.23}$$

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where

$$\alpha_1 = -\tilde{B}^t_{\widetilde{Z}_i} \sin\theta_t - f_t v_1^{(i)} \cos\theta_t, \tag{B.24a}$$
$$\alpha_2 = \tilde{B}^t_{\widetilde{Z}_i} \cos\theta_t - f_t v_1^{(i)} \sin\theta_t, \tag{B.24b}$$
$$\beta_1 = -f_t v_1^{(i)} \sin\theta_t + \tilde{A}^t_{\widetilde{Z}_i} \cos\theta_t, \quad \text{and} \tag{B.24c}$$
$$\beta_2 = f_t v_1^{(i)} \cos\theta_t + \tilde{A}^t_{\widetilde{Z}_i} \sin\theta_t. \tag{B.24d}$$

Next,

$$\begin{aligned}
&\Gamma_{\rm LR}(\tilde{t}_1, \tilde{t}_2) \\
&\quad = 4m_{\tilde{g}} m_t (-1)^{\theta_{\tilde{g}}} \{(-1)^{\theta_i}(-\alpha_1\alpha_2 + \beta_1\beta_2) m_t m_{\widetilde{Z}_i} \zeta(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\
&\qquad +(\alpha_2\beta_1 - \alpha_1\beta_2) X(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i})\} + 2\beta_1\beta_2 Y(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\
&\qquad +2m_t m_{\widetilde{Z}_i} (-1)^{\theta_i} (\beta_1\alpha_2 - \alpha_1\beta_2) \chi'(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}) \\
&\qquad -2\alpha_1\alpha_2 m_t^2 \xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}_i}),
\end{aligned} \tag{B.25}$$

where

$$\alpha_1 = \tilde{A}^t_{\widetilde{Z}_i} \cos\theta_t - f_t v_1^{(i)} \sin\theta_t, \tag{B.26a}$$
$$\alpha_2 = \tilde{B}^t_{\widetilde{Z}_i} \cos\theta_t - f_t v_1^{(i)} \sin\theta_t, \tag{B.26b}$$
$$\beta_1 = f_t v_1^{(i)} \cos\theta_t + \tilde{B}^t_{\widetilde{Z}_i} \sin\theta_t, \quad \text{and} \tag{B.26c}$$
$$\beta_2 = f_t v_1^{(i)} \cos\theta_t + \tilde{A}^t_{\widetilde{Z}_i} \sin\theta_t. \tag{B.26d}$$

Finally,

$$\Gamma_{\rm RL}(\tilde{t}_1, \tilde{t}_2) = -\Gamma_{\rm LR}(\tilde{t}_1, \tilde{t}_2) \tag{B.27}$$

but using

$$\alpha_1 = \tilde{A}^t_{\widetilde{Z}_i} \sin\theta_t + f_t v_1^{(i)} \cos\theta_t, \tag{B.28a}$$
$$\alpha_2 = -\tilde{B}^t_{\widetilde{Z}_i} \sin\theta_t - f_t v_1^{(i)} \cos\theta_t, \tag{B.28b}$$
$$\beta_1 = f_t v_1^{(i)} \sin\theta_t - \tilde{B}^t_{\widetilde{Z}_i} \cos\theta_t, \quad \text{and} \tag{B.28c}$$
$$\beta_2 = -f_t v_1^{(i)} \sin\theta_t + \tilde{A}^t_{\widetilde{Z}_i} \cos\theta_t, \tag{B.28d}$$

and interchanging $m_{\tilde{t}_1} \leftrightarrow m_{\tilde{t}_2}$ in the arguments of the functions ζ, X, Y; and χ' (the first three of which are automatically symmetric).

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The functions appearing above are defined as,

$$\tilde{\psi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}}) = \pi^2 m_{\tilde{g}} \int \mathrm{d}E_t\, p_t E_t \frac{\lambda^{1/2}(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t, m_{\tilde{Z}}^2, m_t^2)}{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t} \times \frac{m_{\tilde{g}}^2 - m_{\tilde{Z}}^2 - 2m_{\tilde{g}}E_t}{(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_1}^2)(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_2}^2)}, \quad \text{(B.29a)}$$

$$\tilde{\phi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}}) = \frac{1}{2}\pi^2 m_{\tilde{g}} m_{\tilde{Z}} \int \frac{\mathrm{d}E_t}{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_1}^2} \times \left[-[E_{\bar{t}}(\max) - E_{\bar{t}}(\min)] - \frac{m_{\tilde{Z}}^2 - m_t^2 + 2m_{\tilde{g}}E_t - m_{\tilde{t}_2}^2}{2m_{\tilde{g}}} \log Z(m_{\tilde{t}_2}) \right], \quad \text{(B.29b)}$$

$$\tilde{\chi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}}) = \pi^2 m_{\tilde{g}} \int \mathrm{d}E_t\, p_t E_t \frac{\lambda^{1/2}(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t, m_{\tilde{Z}}^2, m_t^2)}{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t} \times \frac{1}{(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_1}^2)(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_2}^2)}, \quad \text{(B.29c)}$$

$$\xi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}}) = \frac{1}{2}\pi^2 \int \frac{\mathrm{d}E_t}{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_1}^2} \times \left[[E_{\bar{t}}(\max) - E_{\bar{t}}(\min)] - \frac{m_{\tilde{g}}^2 - m_t^2 - 2m_{\tilde{g}}E_t + m_{\tilde{t}_2}^2}{2m_{\tilde{g}}} \log Z(m_{\tilde{t}_2}) \right], \quad \text{(B.29d)}$$

$$\tilde{\rho}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}}) = -\frac{\pi^2}{2m_{\tilde{g}}} \int \frac{\mathrm{d}E_t}{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_1}^2} \log Z(m_{\tilde{t}_2}), \quad \text{(B.29e)}$$

$$\zeta(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}}) = \pi^2 \int \frac{\mathrm{d}E_t [E_{\bar{t}}(\max) - E_{\bar{t}}(\min)]}{(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_1}^2)(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_2}^2)}, \quad \text{(B.29f)}$$

$$X(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{Z}}) = \frac{\pi^2}{2} \int \mathrm{d}E_t\, p_t \frac{m_{\tilde{g}}^2 - m_{\tilde{Z}}^2 - 2m_{\tilde{g}}E_t}{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t} \times \frac{\lambda^{1/2}(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t, m_{\tilde{Z}}^2, m_t^2)}{(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_1}^2)(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_2}^2)}, \quad \text{(B.29g)}$$

$$Y(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}}) = \frac{\pi^2}{2} \int \frac{\mathrm{d}E_t}{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_1}^2}$$
$$\times \big[[E_{\bar{t}}(\max) - E_{\bar{t}}(\min)](m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t)$$
$$+ \frac{1}{2m_{\tilde{g}}}(m_{\tilde{g}}^2 m_{\widetilde{Z}}^2 - m_{\tilde{g}}^2 m_{\tilde{t}_2}^2 + m_t^4 + 2m_{\tilde{g}}E_t m_{\tilde{t}_2}^2 - m_{\tilde{t}_2}^2 m_t^2) \log Z(m_{\tilde{t}_2}) \big], \quad \text{(B.29h)}$$

$$\chi'(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{Z}}) = -\frac{\pi^2}{2} \int \frac{\mathrm{d}E_t E_t}{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t - m_{\tilde{t}_2}^2} \log Z(m_{\tilde{t}_1}). \quad \text{(B.29i)}$$

The functions with three arguments ψ, χ, ϕ and ρ that appear in various expressions for $\Gamma_{ij}(\tilde{t}_1)$ and $\Gamma_{ij}(\tilde{t}_2)$ are simply the corresponding functions $\tilde{\psi}$, $\tilde{\chi}$, $\tilde{\phi}$ and $\tilde{\rho}$, but with the two top squark mass arguments being the same, i.e.

$$\psi(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}) = \tilde{\psi}(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_{\widetilde{Z}_i}), \quad \text{etc.}$$

The limits of integration on E_t range from m_t to $(m_{\tilde{g}}^2 - 2m_t m_{\widetilde{Z}} - m_{\widetilde{Z}}^2)/2m_{\tilde{g}}$, and

$$Z(m) = \frac{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_{\bar{t}}(\max) - m^2}{m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_{\bar{t}}(\min) - m^2} \quad \text{(B.30)}$$

and

$$E_{\bar{t}}\left(\begin{matrix}\max\\ \min\end{matrix}\right) = \frac{\zeta(m_{\tilde{g}} - E_t) \pm [p_t^2\zeta^2 - 4p_t^2 m_t^2(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t)]^{1/2}}{2(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t)}, \quad \text{(B.31)}$$

where $\zeta = 2m_t^2 + m_{\tilde{g}}^2 - m_{\widetilde{Z}}^2 - 2m_{\tilde{g}}E_t$.

The partial width $\Gamma(\tilde{g} \to b\bar{b}\widetilde{Z}_i)$ can be obtained from the formula for $\Gamma(\tilde{g} \to t\bar{t}\widetilde{Z}_i)$ by making the following substitutions:

$$m_{\tilde{t}_i} \to m_{\tilde{b}_i}, \quad \text{(B.32a)}$$
$$\tilde{A}_{\widetilde{Z}_i}^t \to \tilde{A}_{\widetilde{Z}_i}^b, \quad \text{(B.32b)}$$
$$\tilde{B}_{\widetilde{Z}_i}^t \to \tilde{B}_{\widetilde{Z}_i}^b, \quad \text{(B.32c)}$$
$$f_t \to f_b, \quad \text{(B.32d)}$$
$$v_1^{(i)} \to v_2^{(i)}, \quad \text{(B.32e)}$$
$$\theta_t \to \theta_b, \quad \text{(B.32f)}$$
$$m_t \to m_b, \quad \text{(B.32g)}$$

where,

$$\tilde{A}_{\widetilde{Z}_i}^b = -\frac{g}{\sqrt{2}}v_3^{(i)} + \frac{g'}{3\sqrt{2}}v_4^{(i)}, \quad \text{(B.33a)}$$
$$\tilde{B}_{\widetilde{Z}_i}^b = -\frac{2}{3}\frac{g'}{\sqrt{2}}v_4^{(i)}. \quad \text{(B.33b)}$$

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B.1.4 $\tilde{g} \to \widetilde{W}_i t\bar{b}$ decays

These decays proceed through the exchange of each of the four top and bottom squark mass eigenstates. The formula given below includes effects from t and b Yukawa couplings as well as from intra-generation squark mixing, but with m_b ignored in the squared matrix element (though not in the phase space).

The partial width for the decay $\tilde{g} \to t\bar{b}\widetilde{W}_i^-$ can be written as

$$\Gamma(\tilde{g} \to t\bar{b}\widetilde{W}_i^-) = \frac{\alpha_s}{16\pi^2 m_{\tilde{g}}}\left(\Gamma_{\tilde{t}_1} + \Gamma_{\tilde{t}_2} + \Gamma_{\tilde{t}_1\tilde{t}_2} + \Gamma_{\tilde{b}_1} + \Gamma_{\tilde{b}_2} + \sum_{k,l=1}^{2}\Gamma_{\tilde{t}_k\tilde{b}_l}\right). \tag{B.34}$$

Note that in the limit $m_b \to 0$ the two sbottom exchange diagrams do not interfere with each other. The individual terms in Eq. (B.34) are given by:

$$\Gamma_{\tilde{t}_k} = \left[\left(\alpha^{\tilde{t}_k}_{\widetilde{W}_i}\right)^2 + \left(\beta^{\tilde{t}_k}_{\widetilde{W}_i}\right)^2\right]\left[G_1(m_{\tilde{g}}, m_{\tilde{t}_k}, m_{\widetilde{W}_i}) - (-1)^k \sin(2\theta_t) G_8(m_{\tilde{g}}, m_{\tilde{t}_k}, m_{\tilde{t}_k}, m_{\widetilde{W}_i})\right], \tag{B.35a}$$

$$\Gamma_{\tilde{t}_1\tilde{t}_2} = -2\left(\alpha^{\tilde{t}_1}_{\widetilde{W}_i}\alpha^{\tilde{t}_2}_{\widetilde{W}_i} + \beta^{\tilde{t}_1}_{\widetilde{W}_i}\beta^{\tilde{t}_2}_{\widetilde{W}_i}\right)\cos(2\theta_t) G_8(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{W}_i}), \tag{B.35b}$$

$$\Gamma_{\tilde{b}_k} = \left[\left(\alpha^{\tilde{b}_k}_{\widetilde{W}_i}\right)^2 + \left(\beta^{\tilde{b}_k}_{\widetilde{W}_i}\right)^2\right] G_2(m_{\tilde{g}}, m_{\tilde{b}_k}, m_{\widetilde{W}_i}) - \alpha^{\tilde{b}_k}_{\widetilde{W}_i}\beta^{\tilde{b}_k}_{\widetilde{W}_i} G_3(m_{\tilde{g}}, m_{\tilde{b}_k}, m_{\widetilde{W}_i}), \tag{B.35c}$$

$$\begin{aligned}\Gamma_{\tilde{t}_1\tilde{b}_1} = &\left(\cos\theta_t \sin\theta_b \alpha^{\tilde{b}_1}_{\widetilde{W}_i}\beta^{\tilde{t}_1}_{\widetilde{W}_i} + \sin\theta_t \cos\theta_b \beta^{\tilde{b}_1}_{\widetilde{W}_i}\alpha^{\tilde{t}_1}_{\widetilde{W}_i}\right) G_6(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\widetilde{W}_i})\\ &- \left(\cos\theta_t \cos\theta_b \alpha^{\tilde{b}_1}_{\widetilde{W}_i}\alpha^{\tilde{t}_1}_{\widetilde{W}_i} + \sin\theta_t \sin\theta_b \beta^{\tilde{b}_1}_{\widetilde{W}_i}\beta^{\tilde{t}_1}_{\widetilde{W}_i}\right) G_4(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\widetilde{W}_i})\\ &+ \left(\cos\theta_t \cos\theta_b \beta^{\tilde{b}_1}_{\widetilde{W}_i}\alpha^{\tilde{t}_1}_{\widetilde{W}_i} + \sin\theta_t \sin\theta_b \alpha^{\tilde{b}_1}_{\widetilde{W}_i}\beta^{\tilde{t}_1}_{\widetilde{W}_i}\right) G_5(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\widetilde{W}_i})\\ &- \left(\cos\theta_t \sin\theta_b \beta^{\tilde{b}_1}_{\widetilde{W}_i}\beta^{\tilde{t}_1}_{\widetilde{W}_i} + \sin\theta_t \cos\theta_b \alpha^{\tilde{b}_1}_{\widetilde{W}_i}\alpha^{\tilde{t}_1}_{\widetilde{W}_i}\right) G_7(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\widetilde{W}_i}).\end{aligned} \tag{B.35d}$$

The couplings $\alpha^{\tilde{t}_j}_{\widetilde{W}_i}$ and $\beta^{\tilde{t}_j}_{\widetilde{W}_i}$ are given by,

$$\alpha^{\tilde{t}_1}_{\widetilde{W}_1} = -g \sin\gamma_R \cos\theta_t + f_t \cos\gamma_R \sin\theta_t, \tag{B.36a}$$

$$\beta^{\tilde{t}_1}_{\widetilde{W}_1} = -f_b \cos\gamma_L \cos\theta_t, \tag{B.36b}$$

$$\alpha^{\tilde{b}_1}_{\widetilde{W}_1} = -g \sin\gamma_L \cos\theta_b + f_b \cos\gamma_L \sin\theta_b, \tag{B.36c}$$

$$\beta^{\tilde{b}_1}_{\widetilde{W}_1} = -f_t \cos\gamma_R \cos\theta_b. \tag{B.36d}$$

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The corresponding couplings for heavy sfermions $\tilde{f}_2$ $(f = t,\ b)$ can be obtained from those above by replacing $\cos\theta_f \to \sin\theta_f$ and $\sin\theta_f \to -\cos\theta_f$. The couplings for heavy charginos $\widetilde{W}_2$ can be obtained from those above by replacing $\cos\gamma_{L,R} \to -\theta_{x,y}\sin\gamma_{L,R}$ and $\sin\gamma_{L,R} \to \theta_{x,y}\cos\gamma_{L,R}$. Finally, the other stop–sbottom interference terms can be obtained from (B.35d) by substituting appropriate couplings, squark masses, and squark mixing angle factors.

The eight functions that enter Eq. (B.35a)–(B.35d) are given by,

$$G_1(m_{\tilde{g}}, m_{\tilde{t}}, m_{\widetilde{W}}) = m_{\tilde{g}} \int \frac{\mathrm{d}E_t\, p_t E_t \left(m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\widetilde{W}}^2\right)^2}{\left(m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\tilde{t}}^2\right)^2 \left(m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}}\right)}, \tag{B.37a}$$

$$G_2(m_{\tilde{g}}, m_{\tilde{b}}, m_{\widetilde{W}}) = m_{\tilde{g}} \int \mathrm{d}E_{\bar{b}} E_{\bar{b}}^2 \lambda^{1/2}(m_{\tilde{g}}^2 + m_b^2 - 2E_{\bar{b}} m_{\tilde{g}}, m_{\widetilde{W}}^2, m_t^2) \times \frac{m_{\tilde{g}}^2 + m_b^2 - m_t^2 - 2E_{\bar{b}} m_{\tilde{g}} - m_{\widetilde{W}}^2}{\left(m_{\tilde{g}}^2 + m_b^2 - 2E_{\bar{b}} m_{\tilde{g}} - m_{\tilde{b}}^2\right)^2 \left(m_{\tilde{g}}^2 + m_b^2 - 2E_{\bar{b}} m_{\tilde{g}}\right)}, \tag{B.37b}$$

$$G_3(m_{\tilde{g}}, m_{\tilde{b}}, m_{\widetilde{W}}) = (-1)^{\theta_{\widetilde{W}}} \int \mathrm{d}E_{\bar{b}} E_{\bar{b}}^2 \lambda^{1/2}(m_{\tilde{g}}^2 + m_b^2 - 2E_{\bar{b}} m_{\tilde{g}}, m_{\widetilde{W}}^2, m_t^2) \times \frac{4 m_{\tilde{g}} m_{\widetilde{W}} m_t}{\left(m_{\tilde{g}}^2 + m_b^2 - 2E_{\bar{b}} m_{\tilde{g}} - m_{\tilde{b}}^2\right)^2 \left(m_{\tilde{g}}^2 + m_b^2 - 2E_{\bar{b}} m_{\tilde{g}}\right)}, \tag{B.37c}$$

$$G_4(m_{\tilde{g}}, m_{\tilde{t}}, m_{\tilde{b}}, m_{\widetilde{W}}) = (-1)^{\theta_{\tilde{g}} + \theta_{\widetilde{W}}} m_{\tilde{g}} m_{\widetilde{W}} \int \frac{\mathrm{d}E_t}{m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\tilde{t}}^2} \times \left[E_{\bar{b}}(\max) - E_{\bar{b}}(\min) - \frac{m_{\tilde{b}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\widetilde{W}}^2}{2m_{\tilde{g}}} \log X \right], \tag{B.37d}$$

$$G_5(m_{\tilde{g}}, m_{\tilde{t}}, m_{\tilde{b}}, m_{\widetilde{W}}) = (-1)^{\theta_{\tilde{g}}} \frac{m_t}{2} \int d E_t \frac{m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\widetilde{W}}^2}{m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\tilde{t}}^2} \log X, \tag{B.37e}$$

$$\begin{aligned} &G_6(m_{\tilde{g}}, m_{\tilde{t}}, m_{\tilde{b}}, m_{\widetilde{W}}) \\ &= \frac{1}{2} \int \frac{\mathrm{d}E_t}{m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\tilde{t}}^2} \left\{ \left[m_{\tilde{g}} \left(m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\widetilde{W}}^2\right) \right.\right. \\ &\quad \left. - \frac{m_{\tilde{b}}^2 - m_{\tilde{g}}^2}{m_{\tilde{g}}} \left(2E_t m_{\tilde{g}} - m_t^2 - m_{\tilde{g}}^2\right) \right] \log X \\ &\quad \left. + 2\left(2E_t m_{\tilde{g}} - m_t^2 - m_{\tilde{g}}^2\right) \left[E_{\bar{b}}(\max) - E_{\bar{b}}(\min)\right] \right\}, \end{aligned} \tag{B.37f}$$

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$$G_7(m_{\tilde{g}}, m_{\tilde{t}}, m_{\tilde{b}}, m_{\widetilde{W}}) = (-1)^{\theta_{\widetilde{W}}} \frac{1}{2} m_{\widetilde{W}} m_t \int \frac{\mathrm{d}E_t}{m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\tilde{t}}^2}$$
$$\times \left\{ 2\left[E_{\bar{b}}(\max) - E_{\bar{b}}(\min)\right] - \frac{m_{\tilde{b}}^2 - m_{\tilde{g}}^2}{m_{\tilde{g}}} \log X \right\}, \tag{B.37g}$$

$$G_8(m_{\tilde{g}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\widetilde{W}})$$
$$= (-1)^{\theta_{\tilde{g}}} m_t m_{\tilde{g}} \int \mathrm{d}E_t \frac{\left(m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\widetilde{W}}^2\right)\left[E_{\bar{b}}(\max) - E_{\bar{b}}(\min)\right]}{\left(m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\tilde{t}_1}^2\right)\left(m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}} - m_{\tilde{t}_2}^2\right)}. \tag{B.37h}$$

The quantities $E_{\bar{b}}(\min,\ \max)$, p_t and X in the functions for G_i are given by,

$$\frac{(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t + m_b^2 - m_{\widetilde{W}}^2)(m_{\tilde{g}} - E_t) \mp p_t \lambda^{1/2}(m_{\tilde{g}}^2 + m_t^2 - 2m_{\tilde{g}}E_t, m_b^2, m_{\widetilde{W}}^2)}{2\left(m_{\tilde{g}}^2 + m_t^2 - 2E_t m_{\tilde{g}}\right)},$$

$$p_t = \sqrt{E_t^2 - m_t^2}, \quad \text{and}$$

$$X = \frac{m_{\tilde{b}}^2 + 2E_{\bar{b}}(\max) m_{\tilde{g}} - m_{\tilde{g}}^2}{m_{\tilde{b}}^2 + 2E_{\bar{b}}(\min) m_{\tilde{g}} - m_{\tilde{g}}^2}.$$

Finally, the limits of integration over E_t in Eq. (B.37a)–(B.37h) range from m_t to $\left(m_{\tilde{g}}^2 + m_t^2 - (m_{\widetilde{W}} + m_b)^2\right)/2m_{\tilde{g}}$, while the integration over $E_{\bar{b}}$ ranges from m_b to $[m_{\tilde{g}}^2 - (m_t + m_{\widetilde{W}})^2]/2m_{\tilde{g}}$.

B.2 Squark decay widths

The general expression for the rate for squarks to decay to gluinos, including quark masses and intra-generation mixing is given by,

$$\Gamma(\tilde{q}_{1,2} \to q\tilde{g}) = \frac{2\alpha_s}{3} m_{\tilde{q}_{1,2}} \lambda^{1/2}(1, \frac{m_{\tilde{g}}^2}{m_{\tilde{q}_{1,2}}^2}, \frac{m_q^2}{m_{\tilde{q}_{1,2}}^2})$$
$$\times \left(1 - \frac{m_{\tilde{g}}^2}{m_{\tilde{q}_{1,2}}^2} - \frac{m_q^2}{m_{\tilde{q}_{1,2}}^2} \mp 2(-1)^{\theta_{\tilde{g}}} \sin(2\theta_q) \frac{m_q m_{\tilde{g}}}{m_{\tilde{q}_{1,2}}^2}\right). \tag{B.38a}$$

For the first two generations, the quark Yukawa coupling and the concomitant intra-generation mixing can be neglected, and this reduces to

$$\Gamma(\tilde{q}_i \to q\tilde{g}) = \frac{2\alpha_s}{3} m_{\tilde{q}_i} \left(1 - \frac{m_{\tilde{g}}^2}{m_{\tilde{q}_i}^2} - \frac{m_q^2}{m_{\tilde{q}_i}^2}\right) \lambda^{1/2}(1, \frac{m_{\tilde{g}}^2}{m_{\tilde{q}_i}^2}, \frac{m_q^2}{m_{\tilde{q}_i}^2}), \tag{B.38b}$$

where $i = \mathrm{L, R}$.

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The partial width for up-type squarks to decay to neutralinos including effects of Yukawa couplings and intra-generational mixing is given by,

$$\Gamma(\tilde{t}_1 \to t\widetilde{Z}_i) = \frac{m_{\tilde{t}_1}}{8\pi}\lambda^{1/2}\left(1, \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{t}_1}}, \frac{m_t^2}{m^2_{\tilde{t}_1}}\right) \times \left\{|a|^2\left[1 - (\frac{m_t}{m_{\tilde{t}_1}} + \frac{m_{\widetilde{Z}_i}}{m_{\tilde{t}_1}})^2\right] + |b|^2\left[1 - \left(\frac{m_t}{m_{\tilde{t}_1}} - \frac{m_{\widetilde{Z}_i}}{m_{\tilde{t}_1}}\right)^2\right]\right\}, \tag{B.39}$$

where

$$a = \frac{1}{2}\left\{[\mathrm{i}A^t_{\widetilde{Z}_i} - (\mathrm{i})^{\theta_i} f_t v_1^{(i)}]\cos\theta_t - [\mathrm{i}B^t_{\widetilde{Z}_i} - (-\mathrm{i})^{\theta_i} f_t v_1^{(i)}]\sin\theta_t\right\}, \tag{B.40a}$$

$$b = \frac{1}{2}\left\{[-\mathrm{i}A^t_{\widetilde{Z}_i} - (\mathrm{i})^{\theta_i} f_t v_1^{(i)}]\cos\theta_t - [\mathrm{i}B^t_{\widetilde{Z}_i} + (-\mathrm{i})^{\theta_i} f_t v_1^{(i)}]\sin\theta_t\right\}. \tag{B.40b}$$

The formula for $\Gamma(\tilde{t}_2 \to t\widetilde{Z}_i)$ is the same, except that we must replace $m_{\tilde{t}_1} \to m_{\tilde{t}_2}$, and $\cos\theta_t \to \sin\theta_t$ and $\sin\theta_t \to -\cos\theta_t$ in the corresponding expressions for a and b.

The widths for the decays $\tilde{b}_i \to b\widetilde{Z}_i$ can be obtained from these by the substitutions,

$$m_{\tilde{t}_i} \to m_{\tilde{b}_i}, \tag{B.41a}$$
$$\tilde{A}^t_{\widetilde{Z}_i} \to \tilde{A}^b_{\widetilde{Z}_i}, \tag{B.41b}$$
$$\tilde{B}^t_{\widetilde{Z}_i} \to \tilde{B}^b_{\widetilde{Z}_i}, \tag{B.41c}$$
$$f_t \to f_b, \tag{B.41d}$$
$$v_1^{(i)} \to v_2^{(i)}, \tag{B.41e}$$
$$\theta_t \to \theta_b, \tag{B.41f}$$
$$m_t \to m_b. \tag{B.41g}$$

If quark Yukawa coupling effects are neglected (but quark masses retained), the partial widths for squark decays to neutralinos simplify to,

$$\Gamma(\tilde{u}_\mathrm{L} \to u\widetilde{Z}_i) = \frac{|A^u_{\widetilde{Z}_i}|^2}{16\pi} m_{\tilde{u}_\mathrm{L}}\left(1 - \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{u}_\mathrm{L}}} - \frac{m_u^2}{m^2_{\tilde{u}_\mathrm{L}}}\right)\lambda^{1/2}(1, \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{u}_\mathrm{L}}}, \frac{m_u^2}{m^2_{\tilde{u}_\mathrm{L}}}), \tag{B.42a}$$

$$\Gamma(\tilde{u}_\mathrm{R} \to u\widetilde{Z}_i) = \frac{|B^u_{\widetilde{Z}_i}|^2}{16\pi} m_{\tilde{u}_\mathrm{R}}\left(1 - \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{u}_\mathrm{R}}} - \frac{m_u^2}{m^2_{\tilde{u}_\mathrm{R}}}\right)\lambda^{1/2}(1, \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{u}_\mathrm{R}}}, \frac{m_u^2}{m^2_{\tilde{u}_\mathrm{R}}}), \tag{B.42b}$$

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$$\Gamma(\tilde{d}_{\mathrm{L}} \to d\widetilde{Z}_i) = \frac{|A^d_{\widetilde{Z}_i}|^2}{16\pi} m_{\tilde{d}_{\mathrm{L}}} \left(1 - \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{d}_{\mathrm{L}}}} - \frac{m^2_d}{m^2_{\tilde{d}_{\mathrm{L}}}}\right) \lambda^{1/2}(1, \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{d}_{\mathrm{L}}}}, \frac{m^2_d}{m^2_{\tilde{d}_{\mathrm{L}}}}), \tag{B.42c}$$

$$\Gamma(\tilde{d}_{\mathrm{R}} \to d\widetilde{Z}_i) = \frac{|B^d_{\widetilde{Z}_i}|^2}{16\pi} m_{\tilde{d}_{\mathrm{R}}} \left(1 - \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{d}_{\mathrm{R}}}} - \frac{m^2_d}{m^2_{\tilde{d}_{\mathrm{R}}}}\right) \lambda^{1/2}(1, \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{d}_{\mathrm{R}}}}, \frac{m^2_d}{m^2_{\tilde{d}_{\mathrm{R}}}}). \tag{B.42d}$$

The rate for third generation squarks to decay to charginos, including Yukawa coupling effects is given by,

$$\begin{aligned}\Gamma(\tilde{t}_1 \to b\widetilde{W}_i^+) &= \frac{m_{\tilde{t}_1}}{16\pi} \lambda^{1/2}(1, \frac{m^2_{\widetilde{W}_i}}{m^2_{\tilde{t}_1}}, \frac{m^2_b}{m^2_{\tilde{t}_1}}) \\ &\times \left[[(\mathrm{i}A^d_{\widetilde{W}_i} \cos\theta_t - B_{\widetilde{W}_i} \sin\theta_t)^2 + B'^2_{\widetilde{W}_i} \cos^2\theta_t] \left(1 - \frac{m^2_{\widetilde{W}_i}}{m^2_{\tilde{t}_1}} - \frac{m^2_b}{m^2_{\tilde{t}_1}}\right) \right. \\ &\left. - 4\frac{m_{\widetilde{W}_i} m_b}{m^2_{\tilde{t}_1}} (\mathrm{i}A^d_{\widetilde{W}_i} \cos\theta_t - B_{\widetilde{W}_i} \sin\theta_t) B'_{\widetilde{W}_i} \cos\theta_t \right]\end{aligned} \tag{B.43a}$$

for the lighter top squark, and

$$\begin{aligned}\Gamma(\tilde{b}_1 \to t\widetilde{W}_i^-) &= \frac{m_{\tilde{b}_1}}{16\pi} \lambda^{1/2}(1, \frac{m^2_{\widetilde{W}_i}}{m^2_{\tilde{b}_1}}, \frac{m^2_t}{m^2_{\tilde{b}_1}}) \\ &\times \left[[(\mathrm{i}A^u_{\widetilde{W}_1} \cos\theta_b - B'_{\widetilde{W}_i} \sin\theta_b)^2 + B^2_{\widetilde{W}_i} \cos^2\theta_b](1 - \frac{m^2_{\widetilde{W}_i}}{m^2_{\tilde{b}_1}} - \frac{m^2_t}{m^2_{\tilde{b}_1}}) \right. \\ &\left. - 4\frac{m_{\widetilde{W}_i} m_t}{m^2_{\tilde{b}_1}} (\mathrm{i}A^u_{\widetilde{W}_i} \cos\theta_b - B'_{\widetilde{W}_i} \sin\theta_b) B_{\widetilde{W}_i} \cos\theta_b \right],\end{aligned} \tag{B.43b}$$

for bottom type squarks. The widths for the corresponding decays of $\tilde{t}_2$ and $\tilde{b}_2$ can be obtained from those for the lighter squarks by simply replacing,

$$m_{\tilde{q}_1} \to m_{\tilde{q}_2}, \ \cos\theta_q \to \sin\theta_q, \ \text{and} \ \ \sin\theta_q \to -\cos\theta_q. \tag{B.43c}$$

The various couplings $A^f_{\widetilde{W}_i}$, $B^f_{\widetilde{W}_i}$, $B_{\widetilde{W}_i}$, and $B'_{\widetilde{W}_i}$ as well as those involving neutralino decays are as defined in Section 8.4.2.

Neglecting the couplings of the higgsino components as well as intra-generational mixing, we obtain simplified formulae for the widths:

$$\Gamma(\tilde{u}_L \to d\widetilde{W}_1^+) = \frac{g^2 \sin^2 \gamma_R}{16\pi} m_{\tilde{u}_L} \left(1 - \frac{m_{\widetilde{W}_1}^2}{m_{\tilde{u}_L}^2} - \frac{m_d^2}{m_{\tilde{u}_L}^2}\right) \lambda^{\frac{1}{2}}(1, \frac{m_{\widetilde{W}_1}^2}{m_{\tilde{u}_L}^2}, \frac{m_d^2}{m_{\tilde{u}_L}^2}), \tag{B.44a}$$

and

$$\Gamma(\tilde{d}_L \to u\widetilde{W}_1^-) = \frac{g^2 \sin^2 \gamma_L}{16\pi} m_{\tilde{d}_L} \left(1 - \frac{m_{\widetilde{W}_1}^2}{m_{\tilde{d}_L}^2} - \frac{m_d^2}{m_{\tilde{d}_L}^2}\right) \lambda^{\frac{1}{2}}(1, \frac{m_{\widetilde{W}_1}^2}{m_{\tilde{d}_L}^2}, \frac{m_u^2}{m_{\tilde{d}_L}^2}). \tag{B.44b}$$

The rates for $\tilde{u}_L \to d\widetilde{W}_2^+$ and $\tilde{d}_L \to u\widetilde{W}_2^-$ decays are the same, except that we must replace $m_{\widetilde{W}_1} \to m_{\widetilde{W}_2}$, $\sin^2 \gamma_R \to \cos^2 \gamma_R$, and $\sin^2 \gamma_L \to \cos^2 \gamma_L$.

For third generation squarks (for which we have included intragenerational mixing effects) additional two-body decays to $W^\pm$, Z or the various charged and neutral Higgs bosons may be possible. We write the partial widths for decays $\tilde{t}_i \to \tilde{b}_j W^+$ or $\tilde{b}_i \to \tilde{t}_j W^-$ as,

$$\Gamma(\tilde{q}_i \to \tilde{q}_f W) = \frac{g^2}{32\pi} \frac{1}{m_{\tilde{q}_i}^3} \frac{1}{M_W^2} \lambda^{\frac{3}{2}}(m_{\tilde{q}_i}^2, m_{\tilde{q}_f}^2, M_W^2)\Theta_i\Theta_f, \tag{B.45}$$

where Θ_i and Θ_f take into account intra-generational mixing for the initial and final squarks, $\tilde{q}_i$ and $\tilde{q}_f$, respectively. These factors are given by $\Theta_i = \cos^2 \theta_{t/b}$ ($\Theta_i = \sin^2 \theta_{t/b}$) if the parent squark is a lighter (heavier) stop/sbottom, and likewise for Θ_f. For instance, for the decay $\tilde{t}_2 \to \tilde{b}_1 W^+$, $\Theta_i = \sin^2 \theta_t$ and $\Theta_f = \cos^2 \theta_b$, etc.

The partial width for the decays $\tilde{q}_2 \to \tilde{q}_1 Z$ can be written as,

$$\Gamma(\tilde{q}_2 \to \tilde{q}_1 Z) = \frac{g^2}{64\pi} \frac{1}{\cos^2 \theta_W} \frac{1}{m_{\tilde{q}_2}^3 M_Z^2} \lambda^{\frac{3}{2}}(m_{\tilde{q}_2}^2, m_{\tilde{q}_1}^2, M_Z^2) \cos^2 \theta_q \sin^2 \theta_q. \tag{B.46}$$

Turning to the rates for squarks to decay to charged Higgs bosons we find,

$$\Gamma(\tilde{t}_i \to \tilde{b}_j H^+) = \frac{1}{16\pi} \frac{|A_{ij}|^2}{m_{\tilde{t}_i}^3} \lambda^{\frac{1}{2}}(m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2, m_{H^\pm}^2), \tag{B.47a}$$

with

$$\begin{aligned} A_{11} = \frac{g}{\sqrt{2}M_W} \big\{ & m_t m_b(\cot\beta + \tan\beta) \sin\theta_t \sin\theta_b \\ & + m_t(\mu + A_t \cot\beta) \sin\theta_t \cos\theta_b + m_b(\mu + A_b \tan\beta) \sin\theta_b \cos\theta_t \\ & + \left[(m_b^2 \tan\beta + m_t^2 \cot\beta) - M_W^2 \sin 2\beta\right] \cos\theta_t \cos\theta_b \big\}, \end{aligned} \tag{B.47b}$$

and

$$A_{12} = A_{11}(\cos\theta_b \to \sin\theta_b, \sin\theta_b \to -\cos\theta_b). \tag{B.47c}$$

The couplings A_{2j} that enter the partial widths for $\tilde{t}_2 \to H^+\tilde{b}_j$ decays can be obtained by replacing $\cos\theta_t \to \sin\theta_t$ and $\sin\theta_t \to -\cos\theta_t$ in the coefficients A_{1j} listed above. Also,

$$\Gamma(\tilde{b}_i \to H^-\tilde{t}_j) = \frac{1}{16\pi}\frac{|A_{ji}|^2}{m^3_{\tilde{b}_i}}\lambda^{\frac{1}{2}}(m^2_{\tilde{b}_i}, m^2_{\tilde{t}_j}, m^2_{H^\pm}). \tag{B.48}$$

The partial widths for the decays $\tilde{q}_2 \to \tilde{q}_1\phi$ ($\phi = h, H$, or A) are given by,[1]

$$\Gamma(\tilde{t}_2 \to \tilde{t}_1\phi) = \frac{1}{16\pi}\frac{|A_\phi|^2}{m^3_{\tilde{t}_2}}\lambda^{\frac{1}{2}}(m^2_{\tilde{t}_2}, m^2_{\tilde{t}_1}, m^2_\phi), \tag{B.49a}$$

$$\Gamma(\tilde{b}_2 \to \tilde{b}_1\phi) = \frac{1}{16\pi}\frac{|B_\phi|^2}{m^3_{\tilde{b}_2}}\lambda^{\frac{1}{2}}(m^2_{\tilde{b}_2}, m^2_{\tilde{b}_1}, m^2_\phi), \tag{B.49b}$$

where

$$A_h = \frac{gM_W}{4}\sin(\beta-\alpha)\left(1 - \frac{5}{3}\tan^2\theta_W\right)\sin 2\theta_t + \frac{gm_t}{2M_W\sin\beta}\cos 2\theta_t\,(A_t\cos\alpha - \mu\sin\alpha)\,, \tag{B.50a}$$

$$A_H = -\frac{gM_W}{4}\cos(\beta-\alpha)\left(1 - \frac{5}{3}\tan^2\theta_W\right)\sin 2\theta_t + \frac{gm_t}{2M_W\sin\beta}\cos 2\theta_t\,(A_t\sin\alpha + \mu\cos\alpha)\,, \tag{B.50b}$$

$$A_A = -\mathrm{i}\frac{gm_t}{2M_W}(A_t\cot\beta + \mu)\,, \tag{B.50c}$$

and

$$B_h = \frac{gM_W}{4}\sin(\beta-\alpha)\left(-1 + \frac{1}{3}\tan^2\theta_W\right)\sin 2\theta_b + \frac{gm_b}{2M_W\cos\beta}\cos 2\theta_b\,(A_b\sin\alpha - \mu\cos\alpha)\,, \tag{B.51a}$$

$$B_H = -\frac{gM_W}{4}\cos(\beta-\alpha)\left(-1 + \frac{1}{3}\tan^2\theta_W\right)\sin 2\theta_b + \frac{gm_b}{2M_W\cos\beta}\cos 2\theta_b\,(A_b\cos\alpha + \mu\sin\alpha)\,, \tag{B.51b}$$

[1] Although we write these for the third generation, it should be clear that these formulae also apply (with obvious changes) to the first two generations. Notice that in the absence of intra-generation mixing, these decays occur only via superpotential Yukawa interactions as pointed out in the exercise at the end of Section 13.2.

and

$$B_A = -\mathrm{i}\frac{g m_b}{2M_W}\left(A_b \tan\beta + \mu\right). \tag{B.51c}$$

Finally, if the decay to a gravitino is allowed, we find that (assuming that the gravitino is much lighter than the squark so that the decay rate can be well approximated by that to goldstinos, as discussed in the last section of Chapter 13)

$$\Gamma(\tilde{q}_i \to q\tilde{G}) = \frac{(m_{\tilde{q}}^2 - m_q^2)^4}{48\pi m_{\tilde{q}_i}^3 (M_{\mathrm{P}} m_{3/2})^2}, \tag{B.52}$$

where M_{P} is the reduced Planck mass. Notice that there is no dependence on the squark mixing angle.

B.3 Slepton decay widths

We begin by listing the partial widths for various two-body decays of the first two generations of sleptons and sneutrinos for which intrageneration mixing is negligible. For left-slepton decay to neutralinos, we have ($\ell = e$ or μ),

$$\Gamma(\tilde{\ell}_{\mathrm{L}} \to \ell\widetilde{Z}_i) = \frac{|A_{\widetilde{Z}_i}^{\ell}|^2}{16\pi} m_{\tilde{\ell}_{\mathrm{L}}} \left(1 - \frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{\ell}_{\mathrm{L}}}^2} - \frac{m_\ell^2}{m_{\tilde{\ell}_{\mathrm{L}}}^2}\right) \lambda^{1/2}\left(1, \frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{\ell}_{\mathrm{L}}}^2}, \frac{m_\ell^2}{m_{\tilde{\ell}_{\mathrm{L}}}^2}\right), \tag{B.53a}$$

while for right-slepton decay to neutralinos, we have

$$\Gamma(\tilde{\ell}_{\mathrm{R}} \to \ell\widetilde{Z}_i) = \frac{|B_{\widetilde{Z}_i}^{\ell}|^2}{16\pi} m_{\tilde{\ell}_{\mathrm{R}}} \left(1 - \frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{\ell}_{\mathrm{R}}}^2} - \frac{m_\ell^2}{m_{\tilde{\ell}_{\mathrm{R}}}^2}\right) \lambda^{1/2}\left(1, \frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{\ell}_{\mathrm{R}}}^2}, \frac{m_\ell^2}{m_{\tilde{\ell}_{\mathrm{R}}}^2}\right). \tag{B.53b}$$

The partial width for sneutrino decay to a neutralino is,

$$\Gamma(\tilde{\nu}_\ell \to \nu_\ell\widetilde{Z}_i) = \frac{|A_{\widetilde{Z}_i}^{\nu}|^2}{16\pi} m_{\tilde{\nu}_\ell} \left(1 - \frac{m_{\widetilde{Z}_i}^2}{m_{\tilde{\nu}_\ell}^2}\right)^2. \tag{B.53c}$$

For slepton decay to charginos, we have

$$\Gamma(\tilde{\ell}_{\mathrm{L}} \to \nu_\ell\widetilde{W}_i^-) = \frac{g^2 \sin^2\gamma_{\mathrm{L}}}{16\pi} m_{\tilde{\ell}_{\mathrm{L}}} \left(1 - \frac{m_{\widetilde{W}_i}^2}{m_{\tilde{\ell}_{\mathrm{L}}}^2}\right)^2, \tag{B.54a}$$

while for sneutrino decay, we have

$$\Gamma(\tilde{\nu}_\ell \to \ell\widetilde{W}_i^+) = \frac{g^2 \sin^2\gamma_{\mathrm{R}}}{16\pi} m_{\tilde{\nu}_\ell} \left(1 - \frac{m_{\widetilde{W}_i}^2}{m_{\tilde{\nu}_\ell}^2} - \frac{m_\ell^2}{m_{\tilde{\nu}_\ell}^2}\right) \lambda^{1/2}\left(1, \frac{m_{\widetilde{W}_1}^2}{m_{\tilde{\nu}_\ell^2}}, \frac{m_\ell^2}{m_{\tilde{\nu}_\ell}^2}\right). \tag{B.54b}$$

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Yukawa coupling effects can be important for decays of third generation sleptons and sneutrinos. For $\tilde{\tau}_1 \to \tau \widetilde{Z}_i$, we have

$$\Gamma(\tilde{\tau}_1 \to \tau \widetilde{Z}_i) = \frac{m_{\tilde{\tau}_1}}{8\pi}\lambda^{1/2}(1, \frac{m^2_{\widetilde{Z}_i}}{m^2_{\tilde{\tau}_1}}, \frac{m^2_\tau}{m^2_{\tilde{\tau}_1}}) \times \left\{|a|^2\left[1-(\frac{m_\tau}{m_{\tilde{\tau}_1}}+\frac{m_{\widetilde{Z}_i}}{m_{\tilde{\tau}_1}})^2\right]+|b|^2\left[1-(\frac{m_\tau}{m_{\tilde{\tau}_1}}-\frac{m_{\widetilde{Z}_i}}{m_{\tilde{\tau}_1}})^2\right]\right\}, \tag{B.55a}$$

with

$$a = \frac{1}{2}\left\{[\mathrm{i}A^\tau_{\widetilde{Z}_i} - (\mathrm{i})^{\theta_i} f_\tau v_2^{(i)}]\cos\theta_\tau - [\mathrm{i}B^\tau_{\widetilde{Z}_i} - (-\mathrm{i})^{\theta_i} f_\tau v_2^{(i)}]\sin\theta_\tau\right\},$$

$$b = \frac{1}{2}\left\{[-\mathrm{i}A^\tau_{\widetilde{Z}_i} - (\mathrm{i})^{\theta_i} f_\tau v_2^{(i)}]\cos\theta_\tau - [\mathrm{i}B^\tau_{\widetilde{Z}_i} + (-\mathrm{i})^{\theta_i} f_\tau v_2^{(i)}]\sin\theta_\tau\right\}.$$

The formula for $\tilde{\tau}_2 \to \tau \widetilde{Z}_i$ is the same, except that we must replace $m_{\tilde{\tau}_1} \to m_{\tilde{\tau}_2}$, $\cos\theta_\tau \to \sin\theta_\tau$ and $\sin\theta_\tau \to -\cos\theta_\tau$.

For stau decays to charginos, we have

$$\Gamma(\tilde{\tau}_1 \to \nu_\tau \widetilde{W}_i^-) = \frac{|\mathrm{i}A^\nu_{\widetilde{W}_i}\cos\theta_\tau - B''_{\widetilde{W}_i}\sin\theta_\tau|^2}{16\pi} m_{\tilde{\tau}_1}\left(1-\frac{m^2_{\widetilde{W}_i}}{m^2_{\tilde{\tau}_1}}\right)^2, \tag{B.55b}$$

where $B''_{\widetilde{W}_1} = -f_\tau \cos\gamma_{\mathrm{L}}$ and $B''_{\widetilde{W}_2} = f_\tau \theta_x \sin\gamma_{\mathrm{L}}$. Also,

$$\Gamma(\tilde{\tau}_2 \to \nu_\tau \widetilde{W}_i^-) = \frac{|\mathrm{i}A^\nu_{\widetilde{W}_i}\sin\theta_\tau + B''_{\widetilde{W}_i}\cos\theta_\tau|^2}{16\pi} m_{\tilde{\tau}_2}\left(1-\frac{m^2_{\widetilde{W}_i}}{m^2_{\tilde{\tau}_2}}\right)^2. \tag{B.55c}$$

Finally,

$$\Gamma(\tilde{\nu}_\tau \to \tau \widetilde{W}_i^+) = \frac{m_{\tilde{\nu}_\tau}}{16\pi}\lambda^{1/2}(1, \frac{m^2_{\widetilde{W}_i}}{m^2_{\tilde{\nu}_\tau}}, \frac{m^2_\tau}{m^2_{\tilde{\nu}_\tau}})\left\{\left[|A^\tau_{\widetilde{W}_i}|^2 + B''^2_{\widetilde{W}_i}\right] \times (1-\frac{m^2_{\widetilde{W}_i}}{m^2_{\tilde{\nu}_\tau}}-\frac{m^2_\tau}{m^2_{\tilde{\nu}_\tau}}) - 4\frac{m_{\widetilde{W}_i}m_\tau}{m^2_{\tilde{\nu}_\tau}}B''_{\widetilde{W}_i}(\mathrm{i}A^\tau_{\widetilde{W}_i})\right\}. \tag{B.55d}$$

Turning to the decays to gauge bosons, we have

$$\Gamma(\tilde{\tau}_2 \to \tilde{\nu}_\tau W) = \frac{g^2\sin^2\theta_\tau}{32\pi m^3_{\tilde{\tau}_2} M^2_W}\lambda^{3/2}(m^2_{\tilde{\tau}_2}, m^2_{\tilde{\nu}_\tau}, M^2_W), \tag{B.56a}$$

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$$\Gamma(\tilde{\nu}_\tau \to \tilde{\tau}_1 W) = \frac{g^2 \cos^2\theta_\tau}{32\pi m^3_{\tilde{\nu}_\tau} M_W^2}\lambda^{3/2}(m^2_{\tilde{\nu}_\tau}, m^2_{\tilde{\tau}_1}, M_W^2), \tag{B.56b}$$

$$\Gamma(\tilde{\nu}_\tau \to \tilde{\tau}_2 W) = \frac{g^2 \sin^2\theta_\tau}{32\pi m^3_{\tilde{\nu}_\tau} M_W^2}\lambda^{3/2}(m^2_{\tilde{\nu}_\tau}, m^2_{\tilde{\tau}_2}, M_W^2), \tag{B.56c}$$

and

$$\Gamma(\tilde{\tau}_2 \to \tilde{\tau}_1 Z) = \frac{g^2 \cos^2\theta_\tau \sin^2\theta_\tau}{64\pi \cos^2\theta_{\mathrm{W}} m^3_{\tilde{\tau}_2} M_Z^2}\lambda^{3/2}(m^2_{\tilde{\tau}_2}, m^2_{\tilde{\tau}_1}, M_Z^2). \tag{B.56d}$$

Third generation sleptons may also decay with significant rates to Higgs bosons. The partial widths for decays to charged Higgs bosons are given by,

$$\Gamma(\tilde{\nu}_\tau \to \tilde{\tau}_i H^+) = \frac{|A|^2}{16\pi m^3_{\tilde{\nu}_\tau}}\lambda^{1/2}(m^2_{\tilde{\nu}_\tau}, m^2_{\tilde{\tau}_i}, m^2_{H^+}), \tag{B.57a}$$

with

$$A(\tilde{\nu}_\tau \to \tilde{\tau}_1 H^+) = \frac{g}{\sqrt{2}M_W}\left\{\left[m_\tau^2 \tan\beta - M_W^2 \sin 2\beta\right]\cos\theta_\tau \right.$$
$$\left. + m_\tau\left[\mu + A_\tau \tan\beta\right]\sin\theta_\tau\right\} \tag{B.57b}$$

and

$$A(\tilde{\nu}_\tau \to \tilde{\tau}_2 H^+) = \frac{g}{\sqrt{2}M_W}\left\{\left[m_\tau^2 \tan\beta - M_W^2 \sin 2\beta\right]\sin\theta_\tau \right.$$
$$\left. - m_\tau\left[\mu + A_\tau \tan\beta\right]\cos\theta_\tau\right\}. \tag{B.57c}$$

Finally,

$$\Gamma(\tilde{\tau}_i \to \tilde{\nu}_\tau H^-) = \frac{|A|^2}{16\pi m^3_{\tilde{\tau}_i}}\lambda^{1/2}(m^2_{\tilde{\tau}_i}, m^2_{\tilde{\nu}_\tau}, m^2_{H^-}), \tag{B.58a}$$

with

$$A(\tilde{\tau}_i \to \tilde{\nu}_\tau H^-) = A(\tilde{\nu}_\tau \to \tilde{\tau}_i H^+). \tag{B.58b}$$

For stau decays to neutral Higgs bosons $\phi = h, H, A$, we find

$$\Gamma(\tilde{\tau}_2 \to \tilde{\tau}_1 \phi) = \frac{|A_\phi|^2}{16\pi m^3_{\tilde{\tau}_2}}\lambda^{1/2}(m^2_{\tilde{\tau}_2}, m^2_{\tilde{\tau}_1}, m^2_\phi), \tag{B.59a}$$

with

$$A_h = \frac{gM_W}{4}\sin(\beta-\alpha)\sin 2\theta_\tau[-1+3\tan^2\theta_{\mathrm{W}}]$$
$$+\frac{gm_\tau}{2M_W\cos\beta}\cos 2\theta_\tau[-\mu\cos\alpha + A_\tau\sin\alpha], \tag{B.59b}$$

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$$A_H = \frac{-gM_W}{4}\cos(\beta-\alpha)\sin 2\theta_\tau[-1+3\tan^2\theta_\mathrm{W}]$$
$$+\frac{gm_\tau}{2M_W\cos\beta}\cos 2\theta_\tau[\mu\sin\alpha + A_\tau\cos\alpha], \qquad \text{(B.59c)}$$

and

$$A_A = \frac{-\mathrm{i}gm_\tau}{2M_W}(\mu + A_\tau\tan\beta). \qquad \text{(B.59d)}$$

Finally, if the decay to a gravitino is allowed, we find that

$$\Gamma(\tilde{\ell}\to\ell\tilde{G}) = \frac{(m_{\tilde{\ell}}^2 - m_\ell^2)^4}{48\pi m_{\tilde{\ell}}^3(M_\mathrm{P}m_{3/2})^2}, \qquad \text{(B.60)}$$

again assuming that the goldstino approximation used for $\tilde{q}_i \to q\tilde{G}$ decays is valid. Here, $\tilde{\ell}$ denotes any of the sleptons or sneutrinos. Notice that there is no dependence on the slepton mixing angle.

B.4 Neutralino decay widths

B.4.1 Two-body decays

We list the partial widths for two-body decays of the neutralino, beginning with their decays to gauge bosons.

$$\Gamma(\widetilde{Z}_i \to \widetilde{W}_j^- W^+) = \frac{g^2}{16\pi m_{\widetilde{Z}_i}^3}\lambda^{1/2}(m_{\widetilde{Z}_i}^2, m_{\widetilde{W}_j}^2, M_W^2)$$
$$\times\left[(X_j^{i2}+Y_j^{i2})\left(m_{\widetilde{Z}_i}^2 + m_{\widetilde{W}_j}^2 - M_W^2 + \frac{(m_{\widetilde{Z}_i}^2 - m_{\widetilde{W}_j}^2)^2 - M_W^4}{M_W^2}\right)\right.$$
$$\left. - 6m_{\widetilde{Z}_i}m_{\widetilde{W}_j}(X_j^{i2} - Y_j^{i2})\right], \qquad \text{(B.61a)}$$

where the couplings X_j^i and Y_j^i are given in Eq. (8.103a) and (8.103b). Also,

$$\Gamma(\widetilde{Z}_i \to \widetilde{Z}_j Z) = \frac{|W_{ij}|^2}{4\pi m_{\widetilde{Z}_i}^3}\lambda^{\frac{1}{2}}(m_{\widetilde{Z}_i}^2, m_{\widetilde{Z}_j}^2, M_Z^2)$$
$$\times\left[(m_{\widetilde{Z}_i}^2 + m_{\widetilde{Z}_j}^2 - M_Z^2) + \frac{(m_{\widetilde{Z}_i}^2 - m_{\widetilde{Z}_j}^2)^2 - M_Z^4}{M_Z^2} + 6(-1)^{\theta_i}(-1)^{\theta_j}m_{\widetilde{Z}_i}m_{\widetilde{Z}_j}\right],$$
$$\text{(B.61b)}$$

with W_{ij} as defined in (8.101).

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Turning to neutralino decays to Higgs bosons, we have

$$\Gamma(\widetilde{Z}_i \to \widetilde{W}_j^- H^+) = \Gamma(\widetilde{Z}_i \to \widetilde{W}_j^+ H^-) = \frac{\lambda^{1/2}(m^2_{\widetilde{Z}_i}, m^2_{\widetilde{W}_j}, m^2_{H^+})}{16\pi m^3_{\widetilde{Z}_i}} \times \left[(a_j^2 + b_j^2)(m^2_{\widetilde{Z}_i} + m^2_{\widetilde{W}_j} - m^2_{H^+}) + 2(a_j^2 - b_j^2) m_{\widetilde{Z}_i} m_{\widetilde{W}_j}\right], \quad \text{(B.62)}$$

where

$$a_1 = \frac{1}{2}\left((-1)^{\theta_{\widetilde{W}_1}} \cos\beta A_2^{(i)} - (-1)^{\theta_i} \sin\beta A_4^{(i)}\right), \quad \text{(B.63a)}$$

and

$$b_1 = \frac{1}{2}\left((-1)^{\theta_{\widetilde{W}_1}} \cos\beta A_2^{(i)} + (-1)^{\theta_i} \sin\beta A_4^{(i)}\right). \quad \text{(B.63b)}$$

To obtain a_2 and b_2, replace $A_2^{(i)} \to \theta_y A_1^{(i)}$ and $A_4^{(i)} \to \theta_x A_3^{(i)}$ in the expressions for a_1 and b_1. The coefficients $A_j^{(i)}$ are given in Eq. (8.122a)–(8.122d).

For the partial width of the decay $\widetilde{Z}_i \to \widetilde{Z}_j h$, we have

$$\Gamma(\widetilde{Z}_i \to \widetilde{Z}_j h) = \frac{(X^h_{ij} + X^h_{ji})^2}{16\pi m^3_{\widetilde{Z}_i}} \lambda^{\frac{1}{2}}(m^2_{\widetilde{Z}_i}, m^2_{\widetilde{Z}_j}, m^2_h) \times \left[(m^2_{\widetilde{Z}_i} + m^2_{\widetilde{Z}_j} - m^2_h) + 2(-1)^{\theta_i+\theta_j} m_{\widetilde{Z}_i} m_{\widetilde{Z}_j}\right], \quad \text{(B.64)}$$

with the couplings X^h_{ij} as given in Eq. (8.117). The same formula with the replacements $m_h \to m_H$ and $X^h_{ij} \to X^H_{ij}$ yields $\Gamma(\widetilde{Z}_i \to \widetilde{Z}_j H)$. This formula also applies to $\widetilde{Z}_i \to \widetilde{Z}_j A$ if $m_h \to m_A$, $X^h_{ij} \to X^A_{ij}$ and the sign of the second term (proportional to the product of the neutralino masses) in the square brackets is flipped.

For $\widetilde{Z}_i$ decays to fermion–sfermion pairs, we have, including effects of Yukawa couplings and intra-generational mixing,

$$\Gamma(\widetilde{Z}_i \to f\bar{\tilde{f}}_k) = N_c \frac{\lambda^{1/2}(m^2_{\widetilde{Z}_i}, m^2_{\tilde{f}_k}, m^2_f)}{16\pi m^3_{\widetilde{Z}_i}} \left(|a|^2\left[(m_{\widetilde{Z}_i} + m_f)^2 - m^2_{\tilde{f}_k}\right] + |b|^2\left[(m_{\widetilde{Z}_i} - m_f)^2 - m^2_{\tilde{f}_k}\right]\right), \quad \text{(B.65)}$$

where $k = 1, 2$ and the color factor $N_c = 3$ if $f = q$, and $N_c = 1$ if $f = \ell$ or ν. The coefficients a and b are exactly the same as those that enter the decays $\tilde{f}_k \to f\widetilde{Z}_i$, and may be found in Eq. (B.40a)–(B.41g) for squarks, or directly below (B.55a)

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for sleptons. If intra-generational mixing can be neglected, these reduce to,

$$\Gamma(\widetilde{Z}_i \to f\bar{\tilde{f}}_{\rm L}) = \Gamma(\widetilde{Z}_i \to \bar{f}\tilde{f}_{\rm L})$$
$$= \frac{N_c|A^f_{\widetilde{Z}_i}|^2}{32\pi m^3_{\widetilde{Z}_i}}\lambda^{\frac{1}{2}}(m^2_{\widetilde{Z}_i}, m^2_{\tilde{f}_{\rm L}}, m^2_f)\left(m^2_{\widetilde{Z}_i} + m^2_f - m^2_{\tilde{f}_{\rm L}}\right). \quad \text{(B.66)}$$

The corresponding widths for decays to $\tilde{f}_{\rm R}$ may be obtained from this by replacing $A^f_{\widetilde{Z}_i} \to B^f_{\widetilde{Z}_i}$, and $m_{\tilde{f}_{\rm L}} \to m_{\tilde{f}_{\rm R}}$.

The widths for two-body decay to longitudinal gravitinos, which are essentially goldstinos in the limit that $m_{3/2}$ is much smaller than the mass of the decaying sparticle, are given by

$$\Gamma(\widetilde{Z}_i \to \tilde{G}\gamma) = \frac{(v_4^{(i)}\cos\theta_{\rm W} + v_3^{(i)}\sin\theta_{\rm W})^2}{48\pi m^2_{3/2}M^2_{\rm P}} m^5_{\widetilde{Z}_i}, \quad \text{(B.67)}$$

$$\Gamma(\widetilde{Z}_i \to \tilde{G}Z)$$
$$= \frac{2(v_4^{(i)}\sin\theta_{\rm W} - v_3^{(i)}\cos\theta_{\rm W})^2 + (v_1^{(i)}\sin\beta - v_2^{(i)}\cos\beta)^2}{96\pi m^2_{3/2}M^2_{\rm P}m^3_{\widetilde{Z}_i}}(m^2_{\widetilde{Z}_i} - M^2_Z)^4, \quad \text{(B.68)}$$

and

$$\Gamma(\widetilde{Z}_i \to \tilde{G}\phi) = \frac{|\kappa_\phi|^2}{16\pi m^3_{\widetilde{Z}_i}}\left(m^2_{\widetilde{Z}_i} - m^2_\phi\right)^4, \quad \text{(B.69a)}$$

where $\phi = h$, H or A, and

$$\kappa_h = -\frac{(\mathrm{i})^{\theta_i+1}}{\sqrt{6}M_{\rm P}m_{3/2}}[v_1^{(i)}\cos\alpha + v_2^{(i)}\sin\alpha], \quad \text{(B.69b)}$$

$$\kappa_H = -\frac{(\mathrm{i})^{\theta_i+1}}{\sqrt{6}M_{\rm P}m_{3/2}}[-v_1^{(i)}\sin\alpha + v_2^{(i)}\cos\alpha], \quad \text{and} \quad \text{(B.69c)}$$

$$\kappa_A = -\frac{(\mathrm{i})^{\theta_i+2}}{\sqrt{6}M_{\rm P}m_{3/2}}[v_1^{(i)}\cos\beta + v_2^{(i)}\sin\beta], \quad \text{(B.69d)}$$

as in Chapter 13 of the text. In deriving the decay rates to gravitinos, we have neglected the gravitino mass, except of course in the coupling of the goldstino.

B.4.2 $\widetilde{Z}_i \to \widetilde{Z}_j f\bar{f}$ decays

Here we present the partial width for neutralino three-body decays $\widetilde{Z}_i \to \widetilde{Z}_j f\bar{f}$, where f is a SM fermion. We neglect SM fermion masses in the evaluation of final

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state spin sums that have to be performed after squaring the matrix element, but retain these in the kinematics. This would be a poor approximation for $\widetilde{Z}_i \to t\bar{t}\widetilde{Z}_j$. However, when this decay is kinematically accessible, so are the two-body decay modes $\widetilde{Z}_i \to \widetilde{Z}_j Z$ and $\widetilde{Z}_i \to \widetilde{Z}_j h$: these two-body decays dominate the branching fraction of the neutralino, and the inapplicability of our approximation becomes essentially irrelevant.

This decay proceeds via the exchange of the two sfermion mass eigenstates $\tilde{f}_{1,2}$ or their antiparticles, via the exchange of a Z boson, or via the exchange of one of the three neutral Higgs bosons of the MSSM. The partial width can therefore be written as

$$\Gamma(\widetilde{Z}_i \to \widetilde{Z}_j f\bar{f}) = \frac{1}{2}N_c(f)\frac{1}{(2\pi)^5}\frac{1}{2m_{\widetilde{Z}_j}} \times \left(\Gamma_{\tilde{f}} + \Gamma_Z + \Gamma_{h,H} + \Gamma_A + \Gamma_{Z\tilde{f}} + \Gamma_{\phi\tilde{f}}\right), \quad \text{(B.70)}$$

where the color factor $N_c(f) = 3$ (1) for $f = b$ (τ). The Higgs and Z exchange diagrams do not interfere with each other in the approximation that the spin sums are evaluated with $m_f = 0$. For decays to the first two generations of fermions, the Higgs exchange contributions are also negligible.

The pure sfermion exchange contribution is given by

$$\Gamma_{\tilde{f}} = \Gamma_{\tilde{f}_1} + \Gamma_{\tilde{f}_2} + \Gamma_{\tilde{f}_{1,2}}, \quad \text{(B.71)}$$

where

$$\Gamma_{\tilde{f}_k} = \Gamma_{\rm LL}^{\tilde{f}_k} + \Gamma_{\rm RR}^{\tilde{f}_k} + \Gamma_{\rm LR}^{\tilde{f}_k} \quad (k = 1, 2), \quad \text{(B.72a)}$$

$$\Gamma_{\tilde{f}_{1,2}} = \Gamma_{\rm L}^{\tilde{f}_1}\Gamma_{\rm L}^{\tilde{f}_2} + \Gamma_{\rm L}^{\tilde{f}_1}\Gamma_{\rm R}^{\tilde{f}_2} + \Gamma_{\rm R}^{\tilde{f}_1}\Gamma_{\rm L}^{\tilde{f}_2} + \Gamma_{\rm R}^{\tilde{f}_1}\Gamma_{\rm R}^{\tilde{f}_2}. \quad \text{(B.72b)}$$

Here, the subscripts L and R refer to the chirality of the SM fermion coupling to the heavier neutralino $\widetilde{Z}_j$. The quantities appearing in Eq. (B.72a) and (B.72b) are:

$$\Gamma_{\rm LL}^{\tilde{f}_k} = 4\left(\alpha_{\widetilde{Z}_i}^{\tilde{f}_k}\right)^2\left\{\left[\left(\alpha_{\widetilde{Z}_j}^{\tilde{f}_k}\right)^2 + \left(\beta_{\widetilde{Z}_j}^{\tilde{f}_k}\right)^2\right]\psi(m_{\widetilde{Z}_i}, m_{\tilde{f}_k}, m_{\widetilde{Z}_j}) + (-1)^{\theta_i+\theta_j}\left(\alpha_{\widetilde{Z}_j}^{\tilde{f}_k}\right)^2\phi(m_{\widetilde{Z}_i}, m_{\tilde{f}_k}, m_{\widetilde{Z}_j})\right\}; \quad \text{(B.73a)}$$

$$\Gamma_{\rm RR}^{\tilde{f}_k} = 4\left(\beta_{\widetilde{Z}_i}^{\tilde{f}_k}\right)^2\left\{\left[\left(\alpha_{\widetilde{Z}_j}^{\tilde{f}_k}\right)^2 + \left(\beta_{\widetilde{Z}_j}^{\tilde{f}_k}\right)^2\right]\psi(m_{\widetilde{Z}_i}, m_{\tilde{f}_k}, m_{\widetilde{Z}_j}) + (-1)^{\theta_i+\theta_j}\left(\beta_{\widetilde{Z}_j}^{\tilde{f}_k}\right)^2\phi(m_{\widetilde{Z}_i}, m_{\tilde{f}_k}, m_{\widetilde{Z}_j})\right\}; \quad \text{(B.73b)}$$

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$$\Gamma^{\tilde{f}_k}_{\rm LR} = -8\alpha^{\tilde{f}_k}_{\widetilde{Z}_j}\beta^{\tilde{f}_k}_{\widetilde{Z}_j}\alpha^{\tilde{f}_k}_{\widetilde{Z}_i}\beta^{\tilde{f}_k}_{\widetilde{Z}_i} Y(m_{\widetilde{Z}_i}, m_{\tilde{f}_k}, m_{\tilde{f}_k}, m_{\widetilde{Z}_j}); \tag{B.73c}$$

$$\begin{aligned}\Gamma^{\tilde{f}_1}_{\rm L}\Gamma^{\tilde{f}_2}_{\rm L} = 8\alpha^{\tilde{f}_1}_{\widetilde{Z}_i}\alpha^{\tilde{f}_2}_{\widetilde{Z}_i}\Big\{&\Big[\alpha^{\tilde{f}_1}_{\widetilde{Z}_j}\alpha^{\tilde{f}_2}_{\widetilde{Z}_j} + \beta^{\tilde{f}_1}_{\widetilde{Z}_j}\beta^{\tilde{f}_2}_{\widetilde{Z}_j}\Big]\widetilde{\psi}(m_{\widetilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\widetilde{Z}_j})\\ &+ (-1)^{\theta_i+\theta_j}\alpha^{\tilde{f}_1}_{\widetilde{Z}_j}\alpha^{\tilde{f}_2}_{\widetilde{Z}_j}\tilde{\phi}(m_{\widetilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\widetilde{Z}_j})\Big\};\end{aligned} \tag{B.73d}$$

$$\begin{aligned}\Gamma^{\tilde{f}_1}_{\rm R}\Gamma^{\tilde{f}_2}_{\rm R} = 8\beta^{\tilde{f}_1}_{\widetilde{Z}_i}\beta^{\tilde{f}_2}_{\widetilde{Z}_i}\Big\{&\Big[\alpha^{\tilde{f}_1}_{\widetilde{Z}_j}\alpha^{\tilde{f}_2}_{\widetilde{Z}_j} + \beta^{\tilde{f}_1}_{\widetilde{Z}_j}\beta^{\tilde{f}_2}_{\widetilde{Z}_j}\Big]\widetilde{\psi}(m_{\widetilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\widetilde{Z}_j})\\ &+ (-1)^{\theta_i+\theta_j}\beta^{\tilde{f}_1}_{\widetilde{Z}_j}\beta^{\tilde{f}_2}_{\widetilde{Z}_j}\tilde{\phi}(m_{\widetilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\widetilde{Z}_j})\Big\};\end{aligned} \tag{B.73e}$$

$$\Gamma^{\tilde{f}_1}_{\rm L}\Gamma^{\tilde{f}_2}_{\rm R} = -8\alpha^{\tilde{f}_1}_{\widetilde{Z}_i}\beta^{\tilde{f}_2}_{\widetilde{Z}_i}\alpha^{\tilde{f}_2}_{\widetilde{Z}_j}\beta^{\tilde{f}_1}_{\widetilde{Z}_j} Y(m_{\widetilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\widetilde{Z}_j}); \tag{B.73f}$$

$$\Gamma^{\tilde{f}_2}_{\rm L}\Gamma^{\tilde{f}_1}_{\rm R} = -8\alpha^{\tilde{f}_2}_{\widetilde{Z}_i}\beta^{\tilde{f}_1}_{\widetilde{Z}_i}\alpha^{\tilde{f}_1}_{\widetilde{Z}_j}\beta^{\tilde{f}_2}_{\widetilde{Z}_j} Y(m_{\widetilde{Z}_i}, m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\widetilde{Z}_j}). \tag{B.73g}$$

We have already encountered the kinematic functions $\tilde{\psi}$, $\tilde{\phi}$, Y, ψ, and ϕ, that appear in the expressions above, in our discussion of the decay $\tilde{g} \to f\bar{f}\widetilde{Z}_i$; see Eq. (B.29a)–(B.29i), and the discussion immediately following these. The *real* couplings $\alpha^{\tilde{f}_k}_{\widetilde{Z}_i}$ and $\beta^{\tilde{f}_k}_{\widetilde{Z}_i}$ that enter (B.73a)–(B.73g) are given by,[2]

$$\alpha^{\tilde{f}_1}_{\widetilde{Z}_i} = \tilde{A}^{f}_{\widetilde{Z}_i}\cos\theta_f - f_f v_a^{(i)}\sin\theta_f, \tag{B.74a}$$

$$\beta^{\tilde{f}_1}_{\widetilde{Z}_i} = \tilde{B}^{f}_{\widetilde{Z}_i}\sin\theta_f + f_f v_a^{(i)}\cos\theta_f, \tag{B.74b}$$

where $a = 1$ if $T_{3f} = 1/2$ and $a = 2$ if $T_{3f} = -1/2$. The corresponding couplings for the heavy sfermions ($k = 2$) are obtained via the replacements,

$$\cos\theta_f \to \sin\theta_f, \quad \sin\theta_f \to -\cos\theta_f.$$

The couplings $\tilde{A}^{f}_{\widetilde{Z}_i}$ and $\tilde{B}^{f}_{\widetilde{Z}_i}$ are listed in (B.9a)–(B.9d) for $f = q$. For leptons, these are given by,

$$\tilde{A}^{\ell}_{\widetilde{Z}_i} = -\frac{g v_3^{(i)}}{\sqrt{2}} - \frac{g' v_4^{(i)}}{\sqrt{2}}, \tag{B.75a}$$

$$\tilde{B}^{\ell}_{\widetilde{Z}_i} = -\sqrt{2}g' v_4^{(i)}, \tag{B.75b}$$

$$\tilde{A}^{\nu}_{\widetilde{Z}_i} = \frac{g v_3^{(i)}}{\sqrt{2}} - \frac{g' v_4^{(i)}}{\sqrt{2}}, \tag{B.75c}$$

$$\tilde{B}^{\nu}_{\widetilde{Z}_i} = 0. \tag{B.75d}$$

[2] We caution the reader that these differ from these same couplings defined in Eq. (8.91a)–(8.91d) of Chapter 8 by phases that we have removed, purely for convenience. We trust that our abuse of notation in using the same symbol to denote different, though closely related, quantities will not cause a problem. These real couplings are only used in the formulae in this Appendix.

The squared Z exchange contribution, which is not affected by sfermion mixing, is given by

$$\begin{aligned}\Gamma_Z = {} & 64e^2|W_{ij}|^2\left(\alpha_f^2+\beta_f^2\right)m_{\widetilde{Z}_i}\pi^2 \\ & \times\int_{m_{\widetilde{Z}_j}}^{E_{\max}} \mathrm{d}E\,\frac{B_f\sqrt{E^2-m_{\widetilde{Z}_j}^2}}{\left(m_{\widetilde{Z}_i}^2+m_{\widetilde{Z}_j}^2-M_Z^2-2Em_{\widetilde{Z}_i}\right)^2} \\ & \times\Big\{E\left[m_{\widetilde{Z}_i}^2+m_{\widetilde{Z}_j}^2-(-1)^{\theta_i+\theta_j}2m_{\widetilde{Z}_i}m_{\widetilde{Z}_j}\right] \\ & -m_{\widetilde{Z}_i}\left(E^2+m_{\widetilde{Z}_j}^2+\frac{B_f}{3}(E^2-m_{\widetilde{Z}_j}^2)\right) \\ & +(-1)^{\theta_i+\theta_j}m_{\widetilde{Z}_j}\left(m_{\widetilde{Z}_i}^2+m_{\widetilde{Z}_j}^2-2m_f^2\right)\Big\}. \qquad \text{(B.76)}\end{aligned}$$

Here, α_f and β_f are the vector and axial vector couplings of Z to SM fermions, W_{ij} is the $Z\widetilde{Z}_i\widetilde{Z}_j$ coupling given by (8.101), with

$$B_f = \sqrt{1-\frac{4m_f^2}{m_{\widetilde{Z}_i}^2+m_{\widetilde{Z}_j}^2-2Em_{\widetilde{Z}_j}}}, \qquad \text{(B.77a)}$$

and the upper integration limit

$$E_{\max} = \frac{m_{\widetilde{Z}_i}^2+m_{\widetilde{Z}_j}^2-4m_f^2}{2m_{\widetilde{Z}_i}^2}. \qquad \text{(B.77b)}$$

The squared scalar Higgs exchange contributions can also be written as a single integral:

$$\begin{aligned}\Gamma_{h,H} = {} & 2\pi^2\left(\frac{gm_f}{M_W\cos\beta}\right)^2 m_{\widetilde{Z}_i}\int_{m_{\widetilde{Z}_j}}^{E_{\max}}\mathrm{d}E\,B_f\sqrt{E^2-m_{\widetilde{Z}_j}^2} \\ & \times\left(m_{\widetilde{Z}_i}^2+m_{\widetilde{Z}_j}^2-2m_{\widetilde{Z}_i}E-2m_f^2\right)\left[E+(-1)^{\theta_i+\theta_j}m_{\widetilde{Z}_j}\right] \\ & \times\left[\frac{\sin\alpha\left(X_{ij}^h+X_{ji}^h\right)}{m_{\widetilde{Z}_i}^2+m_{\widetilde{Z}_j}^2-2m_{\widetilde{Z}_i}E-m_h^2}+\frac{\cos\alpha\left(X_{ij}^H+X_{ji}^H\right)}{m_{\widetilde{Z}_i}^2+m_{\widetilde{Z}_j}^2-2m_{\widetilde{Z}_i}E-m_H^2}\right]^2. \qquad \text{(B.78a)}\end{aligned}$$

The couplings $X_{ij}^{h,H}$ are given by (8.117), and the upper limit of integration by (B.77b).

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The squared pseudoscalar Higgs exchange contribution can be cast in a similar form:

$$\Gamma_A = 2\pi^2 \left[\frac{g m_f \tan\beta}{M_W} \left(X_{ij}^A + X_{ji}^A \right) \right]^2 m_{\widetilde{Z}_i} \int_{m_{\widetilde{Z}_j}}^{E_{\max}} \mathrm{d}E\, B_f \sqrt{E^2 - m_{\widetilde{Z}_j}^2}$$
$$\times \frac{\left(m_{\widetilde{Z}_i}^2 + m_{\widetilde{Z}_j}^2 - 2 m_{\widetilde{Z}_i} E - 2m_f^2 \right) \left[E - (-1)^{\theta_i + \theta_j} m_{\widetilde{Z}_j} \right]}{\left(m_{\widetilde{Z}_i}^2 + m_{\widetilde{Z}_j}^2 - 2 m_{\widetilde{Z}_i} E - m_A^2 \right)^2}, \tag{B.78b}$$

where the coupling X_{ij}^A is given in (8.120).

We now turn to the various interference terms. The Z–sfermion interference contributions can be written as

$$\Gamma_{Z\tilde{f}} = \Gamma_{Z\tilde{f}_1} + \Gamma_{Z\tilde{f}_2}, \tag{B.79a}$$

with

$$\Gamma_{Z\tilde{f}_k} = 32 e \widetilde{W}_{ij} \left[\alpha_{\widetilde{Z}_i}^{\tilde{f}_k} \alpha_{\widetilde{Z}_j}^{\tilde{f}_k} \left(\alpha_f - \beta_f \right) - \beta_{\widetilde{Z}_i}^{\tilde{f}_k} \beta_{\widetilde{Z}_j}^{\tilde{f}_k} \left(\alpha_f + \beta_f \right) \right] \frac{\pi^2}{2 m_{\widetilde{Z}_i}}$$
$$\times \int_{4m_f^2}^{(m_{\widetilde{Z}_i} - m_{\widetilde{Z}_j})^2} \frac{\mathrm{d}s}{s - M_Z^2} \left\{ -\frac{1}{2} Q' \left(m_{\widetilde{Z}_i} E_Q + m_{\tilde{f}_k}^2 - m_{\widetilde{Z}_i}^2 - s - m_f^2 \right) \right.$$
$$- \frac{1}{4 m_{\widetilde{Z}_i}} \left[\left(m_{\tilde{f}_k}^2 - m_{\widetilde{Z}_j}^2 - m_f^2 \right) \left(m_{\tilde{f}_k}^2 - m_{\widetilde{Z}_i}^2 - m_f^2 \right) \right.$$
$$\left. \left. + (-1)^{\theta_i + \theta_j} m_{\widetilde{Z}_i} m_{\widetilde{Z}_j} (s - 2m_f^2) \right] \log \frac{m_{\widetilde{Z}_i} \left(E_Q + Q' \right) - \mu^2}{m_{\widetilde{Z}_i} \left(E_Q - Q' \right) - \mu^2} \right\}. \tag{B.79b}$$

Here we have introduced the quantities

$$\mu^2 = s + m_{\tilde{f}_k}^2 - m_{\widetilde{Z}_j}^2 - m_f^2, \quad E_Q = \frac{s + m_{\widetilde{Z}_i}^2 - m_{\widetilde{Z}_j}^2}{2 m_{\widetilde{Z}_i}}, \tag{B.80a}$$

and

$$Q = \sqrt{E_Q^2 - s}, \quad Q' = Q \sqrt{1 - \frac{4m_f^2}{s}}. \tag{B.80b}$$

The real coupling $\widetilde{W}_{ij}$ is defined to be,

$$\widetilde{W}_{ij} = (-\mathrm{i})^{\theta_i + \theta_j} (-1)^{\theta_j} W_{ij}. \tag{B.81}$$

Finally, the Higgs boson–sfermion interference contributions can be written as,

$$\Gamma_{\phi\tilde{f}} = \Gamma_{h\tilde{f}_1} + \Gamma_{h\tilde{f}_2} + \Gamma_{H\tilde{f}_1} + \Gamma_{H\tilde{f}_2} + \Gamma_{A\tilde{f}_1} + \Gamma_{A\tilde{f}_2}, \tag{B.82a}$$

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with

$$\Gamma_{h\tilde{f}_k} = \frac{2\pi^2}{m_{\widetilde{Z}_i}} \frac{g m_f \sin\alpha}{M_W \cos\beta} \left(X^h_{ji} + X^h_{ij}\right) \left[\alpha^{\tilde{f}_k}_{\widetilde{Z}_i} \beta^{\tilde{f}_k}_{\widetilde{Z}_j} + \alpha^{\tilde{f}_k}_{\widetilde{Z}_j} \beta^{\tilde{f}_k}_{\widetilde{Z}_i}\right] \times (-1)^{\theta_i+\theta_j} J(m_{\widetilde{Z}_i}, m_{\tilde{f}_k}, m_h, m_{\widetilde{Z}_j}, \theta_i + \theta_j), \tag{B.82b}$$

$$\Gamma_{H\tilde{f}_k} = \frac{2\pi^2}{m_{\widetilde{Z}_i}} \frac{g m_f \cos\alpha}{M_W \cos\beta} \left(X^H_{ji} + X^H_{ij}\right) \left[\alpha^{\tilde{f}_k}_{\widetilde{Z}_i} \beta^{\tilde{f}_k}_{\widetilde{Z}_j} + \alpha^{\tilde{f}_k}_{\widetilde{Z}_j} \beta^{\tilde{f}_k}_{\widetilde{Z}_i}\right] \times (-1)^{\theta_i+\theta_j} J(m_{\widetilde{Z}_i}, m_{\tilde{f}_k}, m_H, m_{\widetilde{Z}_j}, \theta_i + \theta_j), \tag{B.82c}$$

$$\Gamma_{A\tilde{f}_k} = \frac{2\pi^2}{m_{\widetilde{Z}_i}} \frac{g m_f \tan\beta}{M_W} \left(X^A_{ji} + X^A_{ij}\right) \left[\alpha^{\tilde{f}_k}_{\widetilde{Z}_i} \beta^{\tilde{f}_k}_{\widetilde{Z}_j} + \alpha^{\tilde{f}_k}_{\widetilde{Z}_j} \beta^{\tilde{f}_k}_{\widetilde{Z}_i}\right] \times (-1)^{1+\theta_i+\theta_j} J(m_{\widetilde{Z}_i}, m_{\tilde{f}_k}, m_A, m_{\widetilde{Z}_j}, 1 + \theta_i + \theta_j). \tag{B.82d}$$

The function J is defined as

$$J(m_{\widetilde{Z}_i}, m_{\tilde{f}}, m_H, m_{\widetilde{Z}_j}, \theta) = \int_{4m_f^2}^{(m_{\widetilde{Z}_i} - m_{\widetilde{Z}_j})^2} \frac{\mathrm{d}s}{s - m_H^2} \times \left[\frac{1}{2} s Q' + \frac{s m_{\tilde{f}}^2 - m_f^2(m^2_{\widetilde{Z}_i} + m^2_{\widetilde{Z}_j}) + (-1)^\theta m_{\widetilde{Z}_i} m_{\widetilde{Z}_j}(s - 2m_f^2)}{4 m_{\widetilde{Z}_j}} \times \log \frac{m_{\widetilde{Z}_i}\left(E_Q + Q'\right) - \mu^2}{m_{\widetilde{Z}_i}\left(E_Q - Q'\right) - \mu^2}\right], \tag{B.83}$$

where μ^2, E_Q, Q, and Q' have been defined previously.

B.4.3 $\widetilde{Z}_j \to \widetilde{W}_i^+ \tau^- \nu_\tau$ *decays*

The partial width for the decay $\widetilde{Z}_j \to \widetilde{W}_i^+ \tau^- \nu_\tau$ is related to that for the decay $\widetilde{W}_i^- \to \widetilde{Z}_j \tau^- \nu_\tau$, as in (B.106) of the next section. These neutralino decays are usually not very important because they are either phase space suppressed, or are dwarfed by other two-body decays of the parent neutralino.

B.5 Chargino decay widths

B.5.1 Two-body decays

We list the tree-level partial widths for all two-body decays of the charginos. The partial width for a mode and its charge conjugate are the same. Also, in the following, whether $\widetilde{W}_1$ refers to positive or negative chargino should be clear from the context.

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Starting with decays to gauge bosons, we find that

$$\Gamma(\widetilde{W}_i \to \widetilde{Z}_j W) = \frac{g^2}{16\pi m^3_{\widetilde{W}_i}} \lambda^{\frac{1}{2}}(m^2_{\widetilde{W}_i}, m^2_{\widetilde{Z}_j}, M^2_W)$$
$$\times \left[(X_i^{j2} + Y_i^{j2}) \left(m^2_{\widetilde{W}_i} + m^2_{\widetilde{Z}_j} - M^2_W + \frac{(m^2_{\widetilde{W}_i} - m^2_{\widetilde{Z}_j})^2 - M^4_W}{M^2_W} \right) \right.$$
$$\left. - 6(X_i^{j2} - Y_i^{j2}) m_{\widetilde{W}_i} m_{\widetilde{Z}_j} \right], \tag{B.84}$$

where X_i^j and Y_i^j are given in Eq. (8.103a) and (8.103b), and

$$\Gamma(\widetilde{W}_2 \to \widetilde{W}_1 Z) = \frac{e^2}{64\pi m^3_{\widetilde{W}_2}} (\cot\theta_W + \tan\theta_W)^2 \lambda^{\frac{1}{2}}(m^2_{\widetilde{W}_2}, m^2_{\widetilde{W}_1}, M^2_Z)$$
$$\times \left[(x^2 + y^2) \left(m^2_{\widetilde{W}_2} + m^2_{\widetilde{W}_1} - M^2_Z + \frac{(m_{\widetilde{W}_2} - m_{\widetilde{W}_1})^2 - M^4_Z}{M^2_Z} \right) \right.$$
$$\left. + 6(x^2 - y^2)(-1)^{\theta_{\widetilde{W}_1}}(-1)^{\theta_{\widetilde{W}_2}} m_{\widetilde{W}_1} m_{\widetilde{W}_2} \right], \tag{B.85}$$

where x and y are given in Eq. (8.100e) and (8.100f).

Turning to decays to various Higgs bosons, we find,

$$\Gamma(\widetilde{W}_i \to \widetilde{Z}_j H^-) = \frac{1}{16\pi m^3_{\widetilde{W}_i}} \lambda^{\frac{1}{2}}(m^2_{\widetilde{W}_i}, m^2_{\widetilde{Z}_j}, m^2_{H^-})$$
$$\times \left[(a^2 + b^2)(m^2_{\widetilde{W}_i} + m^2_{\widetilde{Z}_j} - m^2_{H^-}) + 2(a^2 - b^2) m_{\widetilde{W}_i} m_{\widetilde{Z}_j} \right], \tag{B.86}$$

where the coefficients a and b are exactly the same as those that enter the decay $\widetilde{Z}_j \to \widetilde{W}_i^- H^+$; these are given in (B.63a) and (B.63b), and in the discussion following for the decay $\widetilde{Z}_i \to \widetilde{W}_j^- H^+$ (so that the reader must remember to interchange i and j). Charginos may also decay to neutral Higgs bosons $\phi = h$, H or A with partial widths given by,

$$\Gamma(\widetilde{W}_2 \to \widetilde{W}_1 \phi) = \frac{g^2}{32\pi m^3_{\widetilde{W}_2}} \lambda^{\frac{1}{2}}(m^2_{\widetilde{W}_2}, m^2_{\widetilde{W}_1}, m^2_\phi)$$
$$\times \left[(S^{\phi 2} + P^{\phi 2})(m^2_{\widetilde{W}_2} + m^2_{\widetilde{W}_1} - m^2_\phi) + 2(S^{\phi 2} - P^{\phi 2}) m_{\widetilde{W}_1} m_{\widetilde{W}_2} \right], \tag{B.87}$$

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where $S^{h(H)}$ and $P^{h(H)}$ are given in (8.116c), and the corresponding couplings to A given in (8.119c).

Charginos may also decay via $\widetilde{W}_i \to \tilde{f}_j f'$ if these decays are kinematically accessible. We write the partial widths for decays to the third generation, but these can be used with obvious modifications for the first two generations. For $\widetilde{W}_i$ decay to squark plus quark, we find

$$\Gamma(\widetilde{W}_i^+ \to \tilde{t}_1 \bar{b}) = \frac{3m_{\widetilde{W}_i}}{32\pi}\lambda^{1/2}(1, \frac{m_b^2}{m_{\widetilde{W}_i}^2}, \frac{m_{\tilde{t}_1}^2}{m_{\widetilde{W}_i}^2}) \left\{ [|\mathcal{A}|^2 + B'^2_{\widetilde{W}_i} \cos^2\theta_t] \times (1 + \frac{m_b^2}{m_{\widetilde{W}_i}^2} - \frac{m_{\tilde{t}_1}^2}{m_{\widetilde{W}_i}^2}) + 4\mathcal{A}B'_{\widetilde{W}_i} \cos\theta_t \frac{m_b}{m_{\widetilde{W}_i}} \right\}, \quad \text{(B.88a)}$$

where $\mathcal{A} \equiv \mathrm{i}A^d_{\widetilde{W}_i} \cos\theta_t - B_{\widetilde{W}_i} \sin\theta_t$ is real. The width for $\widetilde{W}_i \to \tilde{t}_2 \bar{b}$ can be obtained from this by replacing $m_{\tilde{t}_1} \to m_{\tilde{t}_2}$, $\cos\theta_t \to \sin\theta_t$, and $\sin\theta_t \to -\cos\theta_t$. For $\widetilde{W}_i \to \bar{\tilde{b}}_1 t$ decay, we find

$$\Gamma(\widetilde{W}_i \to \tilde{b}_1 \bar{t}) = \frac{3m_{\widetilde{W}_i}}{32\pi}\lambda^{1/2}(1, \frac{m_t^2}{m_{\widetilde{W}_i}^2}, \frac{m_{\tilde{b}_1}^2}{m_{\widetilde{W}_i}^2}) \left\{ [|\mathcal{A}|^2 + B^2_{\widetilde{W}_i} \cos^2\theta_b] \times (1 + \frac{m_t^2}{m_{\widetilde{W}_i}^2} - \frac{m_{\tilde{b}_1}^2}{m_{\widetilde{W}_i}^2}) + 4\mathcal{A}B_{\widetilde{W}_i} \cos\theta_b \frac{m_t}{m_{\widetilde{W}_i}} \right\}, \quad \text{(B.88b)}$$

where this time $\mathcal{A} = \mathrm{i}A^u_{\widetilde{W}_i} \cos\theta_b - B'_{\widetilde{W}_i} \sin\theta_b$. Again, the replacements $m_{\tilde{b}_1} \to m_{\tilde{b}_2}$, $\cos\theta_b \to \sin\theta_b$, and $\sin\theta_b \to -\cos\theta_b$, yield the width for the decay $\widetilde{W}_i \to \tilde{b}_2 \bar{t}$.

These formulae simplify considerably if we ignore couplings to the higgsino components and intra-generation mixing. The decay rate for $\widetilde{W}_i^+ \to \tilde{u}_L \bar{d}$ can then be obtained from $\Gamma(\widetilde{W}_i^+ \to \tilde{t}_1 \bar{b})$ by replacing $m_b \to m_d$, $m_{\tilde{t}_1} \to m_{\tilde{u}_L}$ and setting $\cos\theta_t \to 1$, $\sin\theta_t \to 0$ and setting the Yukawa couplings in $B_{\widetilde{W}_i}$ and $B'_{\widetilde{W}_i}$ to zero. Similarly, the decay $\widetilde{W}_i \to \tilde{d}_L \bar{u}$ can be obtained from the formula for $\widetilde{W}_i \to \tilde{b}_1 \bar{t}$ by replacing $m_t \to m_u$, $m_{\tilde{b}_1} \to m_{\tilde{d}_L}$ and setting $\cos\theta_b \to 1$, $\sin\theta_b \to 0$ and again setting the Yukawa couplings in $B_{\widetilde{W}_i}$ and $B'_{\widetilde{W}_i}$ to zero. Of course, charginos do not decay into $\tilde{q}_R$ in this limit.

Finally, the partial widths to leptons and sleptons are given by

$$\Gamma(\widetilde{W}_i \to \tilde{\nu}_\tau \bar{\tau}) = \frac{m_{\widetilde{W}_i}}{32\pi}\lambda^{1/2}(1, \frac{m_\tau^2}{m_{\widetilde{W}_i}^2}, \frac{m_{\tilde{\nu}_\tau}^2}{m_{\widetilde{W}_i}^2}) \times \left\{ \left[|A^\tau_{\widetilde{W}_i}|^2 + B''^2_{\widetilde{W}_i} \right] \left(1 + \frac{m_\tau^2}{m_{\widetilde{W}_i}^2} - \frac{m_{\tilde{\nu}_\tau}^2}{m_{\widetilde{W}_i}^2} \right) + 4(\mathrm{i}A^\tau_{\widetilde{W}_i})B''_{\widetilde{W}_i} \frac{m_\tau}{m_{\widetilde{W}_i}} \right\}, \quad \text{(B.89a)}$$

(notice that $\mathrm{i}A_{\widetilde{W}_i}^{\tau}$ is real) and

$$\Gamma(\widetilde{W}_i \to \tilde{\tau}_1 \bar{\nu}_\tau) = \frac{|\mathcal{A}|^2}{32\pi} m_{\widetilde{W}_i} \left(1 - \frac{m_{\tilde{\tau}_1}^2}{m_{\widetilde{W}_i}^2}\right)^2, \tag{B.89b}$$

where the real coefficient $\mathcal{A} = \mathrm{i}A_{\widetilde{W}_i}^{\nu} \cos\theta_\tau - B_{\widetilde{W}_i}^{\prime\prime} \sin\theta_\tau$. The couplings $A_{\widetilde{W}_i}^{\tau}$, $A_{\widetilde{W}_i}^{\nu}$, and $B_{\widetilde{W}_i}^{\prime\prime}$ are given in (8.98a)–(8.98d). The decay width to $\tilde{\tau}_2$ is obtained via the replacements, $\cos\theta_\tau \to \sin\theta_\tau$, $\sin\theta_\tau \to -\cos\theta_\tau$, and $m_{\tilde{\tau}_1} \to m_{\tilde{\tau}_2}$ in the formula for $\Gamma(\widetilde{W}_i \to \tilde{\tau}_1 \nu_\tau)$ and the corresponding coefficient $\mathcal{A}$. How these formulae simplify if coupling via higgsino components and $\tilde{\ell}_L$–$\tilde{\ell}_R$ mixing are neglected should be evident.

B.5.2 Three-body decay: $\widetilde{W}_i \to \widetilde{Z}_j \tau \bar{\nu}_\tau$

We present a formula for the partial width for the decay $\widetilde{W}_i \to \widetilde{Z}_j \tau \nu_\tau$. We will see later that partial widths for other relevant three-body decays of the chargino can be readily obtained from this. This decay proceeds via the exchange of a W boson, a charged or neutral third generation slepton, and a charged Higgs boson. The partial width can be written as

$$\Gamma(\widetilde{W}_i \to \widetilde{Z}_j \tau^- \bar{\nu}_\tau) = \frac{1}{2} \frac{1}{(2\pi)^5} \frac{1}{2m_{\widetilde{W}_i}} \left(\Gamma_W + \Gamma_{\tilde{\nu}} + \Gamma_{\tilde{\tau}} + \Gamma_H + \Gamma_{W\tilde{\nu}} + \Gamma_{W\tilde{\tau}} + \Gamma_{\tilde{\nu}\tilde{\tau}} + \Gamma_{H\tilde{\nu}} + \Gamma_{H\tilde{\tau}}\right). \tag{B.90}$$

The Higgs and W exchange contributions do not interfere, since we neglect terms $\propto m_\tau$ when doing the Dirac algebra.

The squared W exchange contribution is given by

$$\begin{aligned}\Gamma_W = 4g^4 \frac{\pi^2}{3} \int_{m_{\widetilde{Z}_j}}^{E_{\max}} \mathrm{d}E \frac{\sqrt{E^2 - m_{\widetilde{Z}_j}^2}}{\left(m_{\widetilde{W}_i}^2 + m_{\widetilde{Z}_j}^2 - 2m_{\widetilde{W}_i} E - M_W^2\right)^2} \\ \times \left\{ \left(\left|X_i^j\right|^2 + \left|Y_i^j\right|^2\right) \left[3\left(m_{\widetilde{W}_i}^2 + m_{\widetilde{Z}_j}^2\right) m_{\widetilde{W}_i} E - 2m_{\widetilde{W}_i}^2 \left(2E^2 + m_{\widetilde{Z}_j}^2\right)\right] \right. \\ \left. - 3\left(\left|X_i^j\right|^2 - \left|Y_i^j\right|^2\right) m_{\widetilde{W}_i} m_{\widetilde{Z}_j} \left(m_{\widetilde{W}_i}^2 + m_{\widetilde{Z}_j}^2 - 2E m_{\widetilde{W}_i}\right) \right\}.\end{aligned} \tag{B.91}$$

Here X_i^j and Y_i^j are the $W\widetilde{W}_i\widetilde{Z}_j$ couplings given in (8.103a) and (8.103b), and the upper integration limit $E_{\max} = (m_{\widetilde{W}_i}^2 + m_{\widetilde{Z}_j}^2)/2m_{\widetilde{W}_i}$.

The squared sneutrino exchange contribution is given by

$$\Gamma_{\tilde{\nu}} = 2\left(\widetilde{A}^{\nu}_{\widetilde{Z}_j}\right)^2 \left[\left(\widetilde{A}^{\tau}_{\widetilde{W}_i}\right)^2 + \left(B''_{\widetilde{W}_i}\right)^2\right]^2 \psi(m_{\widetilde{W}_i}, m_{\tilde{\nu}_\tau}, m_{\widetilde{Z}_j}), \tag{B.92}$$

where $\widetilde{A}^{\nu}_{\widetilde{Z}_j}$ has been defined in (B.75d) and

$$\widetilde{A}^{\tau}_{\widetilde{W}_1} = -g\sin\gamma_{\rm R}. \tag{B.93a}$$

The pure scalar tau exchange terms can be written as

$$\Gamma_{\tilde{\tau}} = \Gamma_{\tilde{\tau}_1} + \Gamma_{\tilde{\tau}_2} + \Gamma_{\tilde{\tau}_1\tilde{\tau}_2}, \tag{B.94}$$

where

$$\Gamma_{\tilde{\tau}_k} = 2\left(\alpha^{\tilde{\tau}_k}_{\widetilde{W}_i}\right)^2 \left[\left(\alpha^{\tilde{\tau}_k}_{\widetilde{Z}_j}\right)^2 + \left(\beta^{\tilde{\tau}_k}_{\widetilde{Z}_j}\right)^2\right] \psi(m_{\widetilde{W}_i}, m_{\tilde{\tau}_k}, m_{\widetilde{Z}_j}), \tag{B.95a}$$

$$\Gamma_{\tilde{\tau}_1\tilde{\tau}_2} = 4\alpha^{\tilde{\tau}_1}_{\widetilde{W}_i}\alpha^{\tilde{\tau}_2}_{\widetilde{W}_i}\left[\alpha^{\tilde{\tau}_1}_{\widetilde{Z}_j}\alpha^{\tilde{\tau}_2}_{\widetilde{Z}_j} + \beta^{\tilde{\tau}_1}_{\widetilde{Z}_j}\beta^{\tilde{\tau}_2}_{\widetilde{Z}_j}\right] \tilde{\psi}(m_{\widetilde{W}_i}, m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}, m_{\widetilde{Z}_j}), \tag{B.95b}$$

and where

$$\alpha^{\tilde{\tau}_1}_{\widetilde{W}_1} = -g\sin\gamma_{\rm L}\cos\theta_\tau + f_\tau\cos\gamma_{\rm L}\sin\theta_\tau, \tag{B.96a}$$

$$\alpha^{\tilde{\tau}_2}_{\widetilde{W}_1} = -g\sin\gamma_{\rm L}\sin\theta_\tau - f_\tau\cos\gamma_{\rm L}\cos\theta_\tau, \tag{B.96b}$$

$$\alpha^{\tilde{\tau}_1}_{\widetilde{W}_2} = (-g\cos\gamma_{\rm L}\cos\theta_\tau - f_\tau\sin\gamma_{\rm L}\sin\theta_\tau)\theta_x, \tag{B.96c}$$

$$\alpha^{\tilde{\tau}_2}_{\widetilde{W}_2} = (-g\cos\gamma_{\rm L}\sin\theta_\tau + f_\tau\sin\gamma_{\rm L}\cos\theta_\tau)\theta_x. \tag{B.96d}$$

The squared charged Higgs boson exchange contribution is

$$\Gamma_H = \pi^2 m_{\widetilde{W}_i}\left(\frac{g m_\tau\tan\beta}{M_W}\right)^2 \int_{m_{\widetilde{Z}_j}}^{E_{\max}} {\rm d}E\sqrt{E^2 - m^2_{\widetilde{Z}_j}}\left(m^2_{\widetilde{W}_i} + m^2_{\widetilde{Z}_j} - 2Em_{\widetilde{W}_i}\right)$$
$$\times \frac{\left\{E\left[\left(\alpha^{(j)}_{\widetilde{W}_i}\right)^2 + \left(\beta^{(j)}_{\widetilde{W}_i}\right)^2\right] + 2(-1)^{\theta_{\widetilde{W}_i}+\theta_j} m_{\widetilde{Z}_j}\alpha^{(j)}_{\widetilde{W}_i}\beta^{(j)}_{\widetilde{W}_i}\right\}}{\left(m^2_{\widetilde{W}_i} + m^2_{\widetilde{Z}_j} - 2Em_{\widetilde{W}_i} - m^2_{H^+}\right)^2}. \tag{B.97}$$

Here, $E_{\max}$ is as defined just after (B.91), and

$$\alpha^{(j)}_{\widetilde{W}_1} = \cos\beta A^{(j)}_2, \tag{B.98a}$$

$$\beta^{(j)}_{\widetilde{W}_1} = -\sin\beta A^{(j)}_4, \tag{B.98b}$$

$$\alpha^{(j)}_{\widetilde{W}_2} = \cos\beta A^{(j)}_1\theta_y, \tag{B.98c}$$

$$\beta^{(j)}_{\widetilde{W}_2} = -\sin\beta A^{(j)}_3\theta_x, \tag{B.98d}$$

with the coefficients $A^{(j)}_i$ as defined in (8.122a)–(8.122d).

The W–sneutrino interference contribution is not affected by $\tilde{\tau}_L$–$\tilde{\tau}_R$ mixing and contributions $\propto f_\tau$; it can be written as

$$\Gamma_{W\tilde{\nu}} = -4\sqrt{2}g^2(-1)^{\theta_{\widetilde{W}_i}+\theta_j}\widetilde{A}^{\tau}_{\widetilde{W}_i}\widetilde{A}^{\nu}_{\widetilde{Z}_j} \times \left[\left(X_i^j - Y_i^j\right) I_1(m_{\widetilde{W}_i}, m_{\tilde{\nu}_\tau}, m_{\widetilde{Z}_j}) - \left(X_i^j + Y_i^j\right) I_2(m_{\widetilde{W}_i}, m_{\tilde{\nu}_\tau}, m_{\widetilde{Z}_j})\right], \tag{B.99}$$

where we have introduced the functions

$$I_1(m_{\widetilde{W}}, m_{\tilde{f}}, m_{\widetilde{Z}}) = \frac{\pi^2}{2m_{\widetilde{W}}}\int \frac{ds}{s - M_W^2}\left[-\frac{1}{2}Q\left(m_{\widetilde{W}}E_Q + m_{\tilde{f}}^2 - m_{\widetilde{W}}^2 - s\right) - \frac{\left(m_{\tilde{f}}^2 - m_{\widetilde{Z}}^2\right)\left(m_{\tilde{f}}^2 - m_{\widetilde{W}}^2\right)}{4m_{\widetilde{W}}}\log\frac{m_{\widetilde{W}}\left(E_Q + Q\right) - \mu^2}{m_{\widetilde{W}}\left(E_Q - Q\right) - \mu^2}\right], \tag{B.100a}$$

$$I_2(m_{\widetilde{W}}, m_{\tilde{f}}, m_{\widetilde{Z}}) = \frac{\pi^2}{8m_{\widetilde{W}}}\int \frac{ds}{s - M_W^2} m_{\widetilde{Z}} s \log\frac{m_{\widetilde{W}}\left(E_Q + Q\right) - \mu^2}{m_{\widetilde{W}}\left(E_Q - Q\right) - \mu^2}, \tag{B.100b}$$

and the limits of integration on I_1 and I_2 run from zero to $(m_{\widetilde{W}} - m_{\widetilde{Z}})^2$. The quantities μ^2, E_Q, and Q are defined in (B.80a) and (B.80b) but with $m_{\widetilde{Z}_i} \to m_{\widetilde{W}}$, $m_{\widetilde{Z}_j} \to m_{\widetilde{Z}}$, and $m_{\tilde{f}_k} \to m_{\tilde{f}}$.

The same functions also appear in the W–scalar tau interference contributions:

$$\Gamma_{W\tilde{\tau}} = \Gamma_{W\tilde{\tau}_1} + \Gamma_{W\tilde{\tau}_2}, \tag{B.101a}$$

where

$$\Gamma_{W\tilde{\tau}_k} = 4\sqrt{2}g^2\alpha^{\tilde{\tau}_k}_{\widetilde{W}_i}\alpha^{\tilde{\tau}_k}_{\widetilde{Z}_j}\left[\left(X_i^j + Y_i^j\right) I_1(m_{\widetilde{W}_i}, m_{\tilde{\tau}_k}, m_{\widetilde{Z}_j}) - \left(X_i^j - Y_i^j\right) I_2(m_{\widetilde{W}_i}, m_{\tilde{\tau}_k}, m_{\widetilde{Z}_j})\right]. \tag{B.101b}$$

The sneutrino–scalar tau interference terms can be written as

$$\Gamma_{\tilde{\nu}\tilde{\tau}} = \Gamma_{\tilde{\nu}\tilde{\tau}_1} + \Gamma_{\tilde{\nu}\tilde{\tau}_2}, \tag{B.102a}$$

where

$$\Gamma_{\tilde{\nu}\tilde{\tau}_k} = -4\widetilde{A}^{\nu}_{\widetilde{Z}_j}\alpha^{\tilde{\tau}_k}_{\widetilde{W}_i}\left[B''_{\widetilde{W}_i}\beta^{\tilde{\tau}_k}_{\widetilde{Z}_j}Y(m_{\widetilde{W}_i}, m_{\tilde{\nu}_\tau}, m_{\tilde{\tau}_k}, m_{\widetilde{Z}_j}) -(-1)^{\theta_i+\theta_j}\widetilde{A}^{\tau}_{\widetilde{W}_i}\alpha^{\tilde{\tau}_k}_{\widetilde{Z}_j}\tilde{\phi}(m_{\widetilde{W}_i}, m_{\tilde{\nu}_\tau}, m_{\tilde{\tau}_k}, m_{\widetilde{Z}_j})\right]. \tag{B.102b}$$

The functions Y and $\tilde{\phi}$ have already been defined in (B.29a) and (B.29i), respectively.

The charged Higgs–sneutrino interference term is given by

$$\Gamma_{H\tilde{\nu}} = 2\sqrt{2}\widetilde{A}^{\nu}_{\widetilde{Z}_j} B''_{\widetilde{W}_i} \frac{g m_\tau \tan\beta}{m_W} I_H(m_{\widetilde{W}_i}, m_{H^+}, m_{\tilde{\nu}_\tau}, m_{\widetilde{Z}_j}), \tag{B.103}$$

where we have introduced the function

$$I_H(m_{\widetilde{W}_i}, m_H, m_{\tilde{f}}, m_{\widetilde{Z}_j}) = \frac{\pi^2}{2m_{\widetilde{W}_i}} \int_0^{(m_{\widetilde{W}_i}-m_{\widetilde{Z}_j})^2} \frac{\mathrm{d}s}{s - m_H^2} \tag{B.104}$$

$$\times \left\{ \frac{1}{2} s Q \beta^{(j)}_{\widetilde{W}_i} + \frac{1}{4m_{\widetilde{W}_i}} \left[\beta^{(j)}_{\widetilde{W}_i} s m_{\tilde{f}}^2 + (-1)^{\theta_{\widetilde{W}_i}+\theta_j} \alpha^{(j)}_{\widetilde{W}_i} m_{\widetilde{W}_i} m_{\widetilde{Z}_j} s \right] \right.$$

$$\left. \times \log \frac{m_{\widetilde{W}_i}(E_Q + Q) - \mu^2}{m_{\widetilde{W}_i}(E_Q - Q) - \mu^2} \right\}. \tag{B.104}$$

The coupling $\widetilde{A}^{\nu}_{\widetilde{Z}_j}$ is as defined in (B.75d), and the quantities μ^2, E_Q, and Q are as defined below (B.100b).

The same function also appears in the charged Higgs–scalar tau interference contributions:

$$\Gamma_{H\tilde{\tau}} = \Gamma_{H\tilde{\tau}_1} + \Gamma_{H\tilde{\tau}_2}, \tag{B.105a}$$

where

$$\Gamma_{H\tilde{\tau}_k} = 2\sqrt{2}\alpha^{\tilde{\tau}_k}_{\widetilde{W}_i} \beta^{\tilde{\tau}_k}_{\widetilde{Z}_j} \frac{g m_\tau \tan\beta}{M_W} I_H(m_{\widetilde{W}_i}, m_{H^+}, m_{\tilde{\tau}_k}, m_{\widetilde{Z}_j}). \tag{B.105b}$$

The partial widths for the analogous neutralino to chargino decays are given by crossing. The partial width for the neutralino to chargino decay can be obtained from the formula for the corresponding width for the chargino decay by simply interchanging the masses. In other words,

$$\Gamma(\widetilde{Z}_j \to \widetilde{W}_i^+ \tau^- \bar{\nu}_\tau) = \Gamma(\widetilde{W}_i^- \to \widetilde{Z}_j \tau^- \bar{\nu}_\tau)(m_{\widetilde{W}_i} \leftrightarrow m_{\widetilde{Z}_j}). \tag{B.106}$$

Note that $\widetilde{Z}_j$ can also decay into $\widetilde{W}_i^- \tau^+ \nu_\tau$ final states, with equal probability. However, these neutralino decays are usually not very important, since they are either phase space suppressed, or have to compete with two-body decays of the heavy neutralinos.

Our formula for $\Gamma(\widetilde{W}_i \to \widetilde{Z}_j \tau^- \bar{\nu}_\tau)$ decay can be readily adapted to three-body chargino decays into fermion–antifermion pairs of the first two generations. Ignoring Yukawa couplings and intra-generation mixing, we have just three contributions to this amplitude: W exchange, and the exchanges of the "left-handed" up and down type sfermions. We retain only the W, $\tilde{\tau}_1$, and sneutrino exchange contributions and set $\cos\theta_\tau = 1$ to obtain the partial width for the decays $\widetilde{W}_i \to \ell \bar{\nu}_\ell \widetilde{Z}_j$. The replacements $\tilde{\nu}_\ell \to \tilde{u}_{\mathrm{L}}$, $\tilde{\ell}_{\mathrm{L}} \to \tilde{d}_{\mathrm{L}}$ in the formula for $\Gamma(\widetilde{W}_i \to \widetilde{Z}_j \ell \bar{\nu}_\ell)$ will

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yield $\Gamma(\widetilde{W}_i \to \widetilde{Z}_j d\bar{u})$ if we remember to include the color factor of 3. The decay $\widetilde{W}_i \to b\bar{t}\widetilde{Z}_j$ always has a small branching fraction, since two-body decays $\widetilde{W}_i \to W\widetilde{Z}_j$ are also accessible whenever the three-body decay to top is.

B.6 Top quark decay to SUSY particles

If charged Higgs bosons or sparticles are light enough, new two-body decays of the top quark may be allowed. These include, $t \to bH^+$, $t \to \tilde{t}_1\widetilde{Z}_i$, and $t \to \tilde{b}_1\widetilde{W}_i$. The partial width for the decay $t \to fS$, where f is a spin $\frac{1}{2}$ fermion and S a spin zero particle, is given by,

$$\Gamma(t \to fS) = \frac{m_t}{16\pi}\lambda^{\frac{1}{2}}(1, \frac{m_f^2}{m_t^2}, \frac{m_S^2}{m_t^2}) \times \left[(|\alpha|^2 + |\beta|^2)(1 + \frac{m_f^2}{m_t^2} - \frac{m_S^2}{m_t^2}) + 2(|\alpha|^2 - |\beta|^2)\frac{m_f}{m_t}\right], \tag{B.107}$$

where α and β are the scalar and pseudoscalar couplings of S to the t–f system.

For the decay $t \to bH^+$, $f = b$ and $S = H^+$ and we have,

$$\alpha = \frac{g}{2\sqrt{2}M_W}(m_b \tan\beta + m_t \cot\beta),$$
$$\beta = \frac{g}{2\sqrt{2}M_W}(m_b \tan\beta - m_t \cot\beta). \tag{B.108a}$$

For the decay $t \to \tilde{t}_1\widetilde{Z}_i$, $f = \widetilde{Z}_i$, $S = \tilde{t}_1$, and

$$\alpha = \frac{1}{2}\left\{\left[\mathrm{i}A^t_{\widetilde{Z}_i} - (\mathrm{i})^{\theta_i} f_t v_1^{(i)}\right]\cos\theta_t - \left[\mathrm{i}B^t_{\widetilde{Z}_i} - (-\mathrm{i})^{\theta_i} f_t v_1^{(i)}\right]\sin\theta_t\right\}$$

and

$$\beta = \frac{1}{2}\left\{\left[-\mathrm{i}A^t_{\widetilde{Z}_i} - (\mathrm{i})^{\theta_i} f_t v_1^{(i)}\right]\cos\theta_t - \left[\mathrm{i}B^t_{\widetilde{Z}_i} + (-\mathrm{i})^{\theta_i} f_t v_1^{(i)}\right]\sin\theta_t\right\}. \tag{B.108b}$$

Finally, for the decay $t \to \tilde{b}_1\widetilde{W}_i$, $f = \widetilde{W}_i$, $S = \tilde{b}_1$, and

$$\alpha = \frac{1}{2}\left[\mathrm{i}A^t_{\widetilde{W}_i}\cos\theta_b - B'_{\widetilde{W}_i}\sin\theta_b + B_{\widetilde{W}_i}\cos\theta_b\right],$$
$$\beta = \frac{1}{2}\left[-\mathrm{i}A^t_{\widetilde{W}_i}\cos\theta_b + B'_{\widetilde{W}_i}\sin\theta_b + B_{\widetilde{W}_i}\cos\theta_b\right]. \tag{B.108c}$$

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