Riemann's "Partielle Differentialgleichungen;" Heine's "Kugelfunctionen," vol. ii., pp. 302-332; Todhunter's "Functions of Laplace, Lamé, and Bessel;" Jordan's "Cours d'Analyse, vol. iii., chap. iii., Part iv.

The notice now given has, of course, no pretentions to being exhaustive; but it may perhaps serve a useful purpose in helping one to follow the development of the theory whose basis Fourier so thoroughly established.

Second Meeting, December 14th, 1888.

GEORGE A. GIBSON, Esq., M.A., President, in the Chair.

On the general equation of the second degree representing a pair of straight lines.

By DAVID MUNN, M.A.

Kötters synthetic geometry of algebraic curves— Part I., imaginary curves.

By Rev. NORMAN FRASER, M.A.

[See Index.]

Third Meeting, January 11th, 1889.

GEORGE A. GIBSON, Esq., M.A., President, in the Chair.

Note on a Formula in Quaternions.

By R. E. ALLARDICE, M.A.

The formula referred is the condition for the coplanarity of the extremities of four coinitial vectors; namely, if α, β, γ, δ, are the vectors, then

$$a\alpha + b\beta + c\gamma + d\delta = 0, \text{ where } a + b + c + d = 0.$$

(See Kelland and Tait's Quaternions, p. 62.)

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In the first place, it may be pointed out that these equations do involve one condition on the four vectors α, β, γ, δ. For, since the vanishing of a vector involves three conditions, and only the three ratios of the four quantities a, b, c, d, are involved, it is always possible to determine these four quantities so that the condition $a\alpha + b\beta + c\gamma + d\delta = 0$ shall be satisfied. But, in general, the equation $a + b + c + d = 0$ will not be satisfied; and thus one condition is involved.

The object of this note is to point out the geometrical interpretation of the above condition, and to furnish an independent proof of the theorem in Solid Geometry that is involved in it.

Let the four points ABCD be coplanar; and let α, β, γ, δ, denote the vectors OA, OB, OC, OD. Take A′B′C′D′ on OA, OB, OC, OD, respectively, so that $OA' = a\alpha$, &c. Then, writing the condition for coplanarity in the form $a\alpha - b\beta + c\gamma - d\delta = 0$ where $a - b + c - d = 0$, we have from the latter equation $OA'/OA - OB'/OB + OC'/OC - OD'/OD = 0$. Also the first equation may be written $a\alpha - b\beta = d\delta - c\gamma$; and hence B′A′ is parallel and equal to C′D′, and therefore A′B′C′D′ is a parallelogram. Hence the condition for the coplanarity of A, B, C, D, is equivalent to the following proposition:—

If four straight lines OA, OB, OC, OD, are met by a plane in the points A, B, C, D, and another plane meets the straight lines in the points A′, B′, C′, D′, so that A′B′C′D′ is a parallelogram, then

$$OA'/OA - OB'/OB + OC'/OC - OD'/OD = 0.$$

Geometrical Proof.

Let the plane of the paper (fig. 1) be the plane of ABCD and let O be the point of concurrence of the four lines. A parallelogram is obtained by drawing a plane parallel to OE and OF. E′, E″, F′, F″ are the points where this plane meets the line EA, EC, FB, FD Let fig. 2 represent the plane determined by AB and O.

Then $\quad OA'/OA = EE'/EA$ and $OB'/OB = EE'/EB$;

$\therefore \quad OB'/OB - OA'/OA = EE'.AB/EA.EB.$

Similarly $\quad OC'/OC - OD'/OD = EE''.\ CD/EC.ED.$

We have now to show that $EE'.AB/EA.EB = EE''.CD/EC.ED.$

Now, $EE'/EE'' = \sin CEK/\sin BEH = \sin GED/\sin GEA =$
$(LD/ED)/(AL/AE).$

This reduces the above relation to

$$\frac{LD.AB}{ED.EA.EB} = \frac{AL.CD}{EA.EC.ED}; \text{ that is}$$

$$LD.AB.EC = AL.CD.EB;$$

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which is seen to follow from Ceva's Theorem on considering the triangle ADE and the point G.

The corresponding theorem *in plano* is :—

If a transversal ABC meets three concurrent lines OA, OB, OC, and A′, B′, C′, are points in these lines such that OA′B′C′ is a parallelogram, then

$$OA'/OA - OB'/OB + OC'/OC = 0\,;$$

a theorem which is very easily proved.

If we invert the four points A, B, C, D, of the theorem proved above into the points P, Q, R, S, taking O as centre and k as radius of inversion, we have

$$OA.OP = k^2\,;\ \therefore\ OA = k^2/OP.$$

Substituting in the relation

$$OA'/OA - OB'/OB + OC'/OC - OD'/OD = 0,$$

we get $$OA'.OP - OB'.OQ + OC'.OR - OD'.OS = 0.$$

Hence if four points P, Q, R, S, lie on the same sphere with the point O and a plane cuts OP, OQ, OR, OS, in A′, B′, C′, D′, so that A′B′C′D′ is a parallelogram, then the above relation holds.

The condition that the extremities of four vectors lie on a sphere passing through the origin, may be written

$$\frac{a}{a^2}.a + \frac{b}{\beta^2}.\beta + \frac{c}{\gamma^2}.\gamma + \frac{d}{\delta^2}.\delta = 0, \text{ where } a + b + c + d = 0\,;$$

or, $$pa + q\beta + r\gamma + s\delta = 0, \text{ where } pa^2 + q\beta^2 + c\gamma^2 + d\delta^2 = 0.$$

On the number of elements in space.

By Rev. Norman Fraser, M.A.

On the solution of the equation $x^p - 1 = 0$ (p being a prime number).

By J. Watt Butters.

[At the first meeting of this Session a paper was read on the value of cos $2\pi/17$, which evidently may be made to depend on the

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