On the centre of spherical curvature of a curve

By C. E. WEATHERBURN, University of Western Australia.

The position of the centre S of spherical curvature at a point P of a given curve C may be found in the following manner, regarding S as the limiting position of the centre of a sphere through four adjacent points P, P_1, P_2, P_3 on the curve, as these points tend to coincidence at P. The centre of a sphere through P and P_1 lies on the plane which is the perpendicular bisector of the chord PP_1; and so on. Thus the centre of spherical curvature is the limiting position of the intersection of three normal planes at adjacent points. Let s be the arc-length of the curve C, $\mathbf{r}$ the position vector of the point P, and $\mathbf{t}$, $\mathbf{n}$, $\mathbf{b}$ unit vectors in the directions of the tangent, principal normal and binormal at P. Then if $\mathbf{s}$ is the position vector of the current point on the normal plane at P, the equation of this plane is

$$(\mathbf{s}-\mathbf{r})\cdot\mathbf{t}=0. \tag{1}$$

Since $\mathbf{r}$ and $\mathbf{t}$ are functions of s, the limiting position of the line of intersection of the normal planes at P and an adjacent point (*i.e.* the polar line) is determined by (1) and the equation obtained by differentiating this with respect to s, viz.

$$\kappa(\mathbf{s}-\mathbf{r})\cdot\mathbf{n}-1=0$$

which is equivalent to

$$(\mathbf{s}-\mathbf{r})\cdot\mathbf{n}=\rho \tag{2}$$

where κ is the curvature at P, and ρ the radius of curvature. The ultimate intersection of three adjacent normal planes is then found from (1), (2) and the equation obtained by differentiating (2) with respect to s. This third equation is

$$(\mathbf{s}-\mathbf{r})\cdot(\tau\,\mathbf{b}-\kappa\,\mathbf{t})=\rho'$$

where τ is the torsion of the curve at P, and the prime denotes differentiation with respect to s. In virtue of (1) this relation is equivalent to

$$(\mathbf{s}-\mathbf{r})\cdot\mathbf{b}=\sigma\rho' \tag{3}$$

where σ is the reciprocal of τ. The vector $\mathbf{s}$ satisfying (1), (2) and (3) is clearly

$$\mathbf{s}=\mathbf{r}+\rho\,\mathbf{n}+\sigma\rho'\,\mathbf{b} \tag{4}$$

and this is the required position vector of the centre of spherical curvature.

https://doi.org/10.1017/S1757748900002486 Published online by Cambridge University Press

Mr A. S. Ramsey kindly drew my attention to a flaw in the proof which I gave in my *Differential Geometry*, Vol. 1, p. 22. This may be rectified by considering S as the limiting position of a point which is equidistant from the four points P, P_1, P_2, P_3 as these tend to coincidence at P. In other words S is the limiting position of the intersection of four equal spheres with centres at these four points. Let R be their common radius. Then, if $\mathbf{s}$ is the position vector of the current point on the sphere whose centre is at P, the equation of the sphere is

$$(\mathbf{s}-\mathbf{r})^2 = R^2. \qquad (5)$$

The position of the ultimate point of intersection of the spheres is then found from the three equations obtained by three differentiations of (5) with respect to s.

The advantage of differentials in the technique of differentiation

By E. G. Phillips, University College of North Wales, Bangor.

In the last few years the topic of *differentials* has occupied the attention of a good many teachers of mathematics. Since the appearance in 1931 of my article[1] advocating the teaching of the Differential Calculus from the differential standpoint right from the start, very divergent views have been expressed on this question and there was a general discussion on the topic at the annual meeting of the Mathematical Association in 1934. This article, the subsequent correspondence and the discussion have certainly brought into evidence the fact that many mathematicians still have a very obvious distrust of "differentials" and they seem unable to clear their minds of some early prejudice against the employment of differentials for any purpose.

The advantages of the differential notation are most evident in the technique of differentiation of functions of more than one variable. The superiority of the method of differentials over the

[1] *Math. Gazette* XV (1931), 401.