Dynamics of Populations of Planetary Systems
Proceedings IAU Colloquium No. 197, 2005
Z. Knežević and A. Milani, eds.

DOI: 10.1017/S1743921304008841

Resonances near the orbit of 2003 VB_{12} (Sedna)

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Abstract. 2003 VB_{12} (Sedna) is as much distinguished by its considerable size as by its extremely unusual orbit, which has perihelion at about $q = 76$ AU with semi-major axis $a = 533$ AU (Brown *et al.* 2004, JPL Horizons†). Thus it is effectively decoupled from both Neptune and the Galactic tide (Fernandez 1997). Brown *et al.* (2004) and Morbidelli & Levison (2004) maintain that only scattering by a so-far-unobserved "Planet X" or by an errant star could produce such a high-perihelion orbit for a scattered-disk KBO. While a close encounter is plausible, given the Sun's likely birth in an open cluster, such an interaction would profoundly disturb the Oort cloud and would require fundamental revision to the present theories of its formation.

Although the planets cannot significantly affect VB_{12}'s orbit through close approaches, resonant perturbations could conceivably produce secular effects on it. To explore this possibility, we have numerically integrated test particles with $480 < a < 580$ AU and a fixed $q = 76$ AU. Including the four giant planets, but ignoring the Kuiper Belt and the inner Oort Cloud, as well as the Galactic tide, we find multiple resonances, some of which perturb significantly the test particles' eccentricity more strongly than the leading secular terms. We identify these resonances as variants of the very high-order ($n_N > 60\ n$) mean-motion commensurabilities between Neptune and VB_{12}. Although unprecedented, these extremely high-order resonances can be significant due to VB_{12}'s very high eccentricity ($e = 0.86$). Even powers of eccentricity beyond sixty are still on the order of 10^{-4}, which is comparable to the strength of low-order resonances involving near-circular orbits. We extrapolate the possible long-term drift rate and estimate the likelihood of such resonances producing an "inner Oort cloud" population consistent with VB_{12} over the age of the Solar System. Finally we discuss how planetary migration and the Kuiper-Belt's depletion might have affected VB_{12}'s putative resonance.

Keywords. Celestial mechanics, Kuiper Belt, minor planets, asteroids, Oort Cloud, solar system: formation

1. Introduction

Minor planet 2003 VB_{12} (unofficially known as "Sedna", hereafter "VB_{12}") is a unique Solar System object, and its discovery convinced many researchers that present theories of the Solar System's formation need some rethinking (Brown *et al.* 2004). After nine major planets, VB_{12} is most likely the largest known body that orbits the Sun; its very eccentric orbit has a perihelion that is too high for direct interactions with Neptune (Gladman *et al.* 2002), and an aphelion too low to be affected by the Galactic tide (Fernandez 1997). Given that its present orbit is extremely stable, it is unclear how it could have evolved into its present state.

Morbidelli & Levison (2004, ML04) discuss four possible solutions to the mystery of VB_{12}'s orbit: scattering by a more eccentric Neptune, scattering by an unknown distant major planet, secular interactions with an extended planetesimal disc and effects of a

† Orbital elements were obtained on August 13$^{\text{th}}$ 2004, through Jet Propulsion Laboratory's Horizons on-line ephemeris service, http://ssd.jpl.nasa.gov/

close stellar encounter. The authors show that, even if Neptune at some point had eccentricity as large as 0.4, VB_{12} could not attain such a high aphelion distance. ML04 find interaction with an unknown planet to be a more promising mechanism, although one that "raises more problems than it solves". They demonstrate that an Earth-mass planet would require billions of years to increase VB_{12}'s aphelion to almost 1000 AU, making it almost certainly both decoupled from Neptune and still extant. Neither scattering by Neptune nor in-situ formation are likely to produce such a body. Interactions with an extended planetesimal disk conserve the component of angular momentum normal to the disk, $H = \sqrt{\mu a(1-e^2)}\cos i$, making it impossible for low-i objects like VB_{12} to have had a past eccentricity large enough to interact strongly with Neptune. ML04 find that a slow passage by a solar-mass star, with the perihelion distance of 800 AU, can explain the present characteristics of VB_{12}'s orbit, as well as that of another "extended scattered disk" object, 2000 CR_{105}.

Such an encounter is not unlikely given that most stars form in clusters; a similar encounter has been invoked by Ida *et al.* (2000) to explain the dynamical excitation of the "classical" Kuiper Belt. However, ML04 think that the existence of objects like 2000 CR_{105} and 2003 VB_{12} is the only compelling evidence for such a passage. A slow stellar passage would likely strip the Sun of the much of the existing Oort cloud, making its formation even more difficult (H. Levison 2004, personal communication). Therefore, before we can accept the necessity of a stellar passage early in the Solar System history, it is essential to consider all the alternatives.

In this paper, we will take a first look at the intermediate-term (3×10^7 yr) dynamics of objects with VB_{12}-like orbits, that is, with perihelia at 76 AU and semimajor axes in 480–580 AU range. In Section 2, we will present the results of a rough dynamical survey using a symplectic integrator. In Section 3, we will concern ourselves with features that are weakly dependent on mean motion, while the resonant features will be discussed in Section 4, with results summarized in Section 5.

2. Numerical Experiment

In order to study the secular dynamics of the inner Oort cloud, we have integrated orbits of 100 massless test particles for 3×10^7 yr period, starting at the epoch of midnight, September 24$^{\text{th}}$ 2003. The Sun and four giant planets were fully included into the integration, while Pluto, the Kuiper belt and any possible extended disk were ignored. The initial conditions were varied so that the range of average semimajor axes was 478–578 AU, with the step size of 1 AU. The perihelion distance was fixed at 76 AU for all particles, while the inclination and other angular variables were taken to be the same as those for VB_{12} (generated through JPL Horizons on August 13$^{\text{th}}$ 2004, for the above epoch). A home-made symplectic integrator based on the standard algorithm of Wisdom & Holman(1991) was used. The large dynamic range of mean motions made our integrations comparably inefficient: the timestep was dictated by Jupiter's orbital period, which is 1000 times shorter than those of our test particles. However, we could not avoid integrating directly all four giant planets, since we were interested in detecting any perturbations arising from frequencies associated with the planets' mean motions (some important perturbations, notably the near-resonance of Uranus and Neptune, have periods longer than 10^3 yr).

Since we are above all interested in perturbations that can change the perihelion distance of VB_{12}-like objects, in Fig. 1 we plot the change of pericenter distance in the course of the simulation, as a function of a particle's semimajor axis. Right away, despite the low resolution of our survey, we can see two major features of the plot: the

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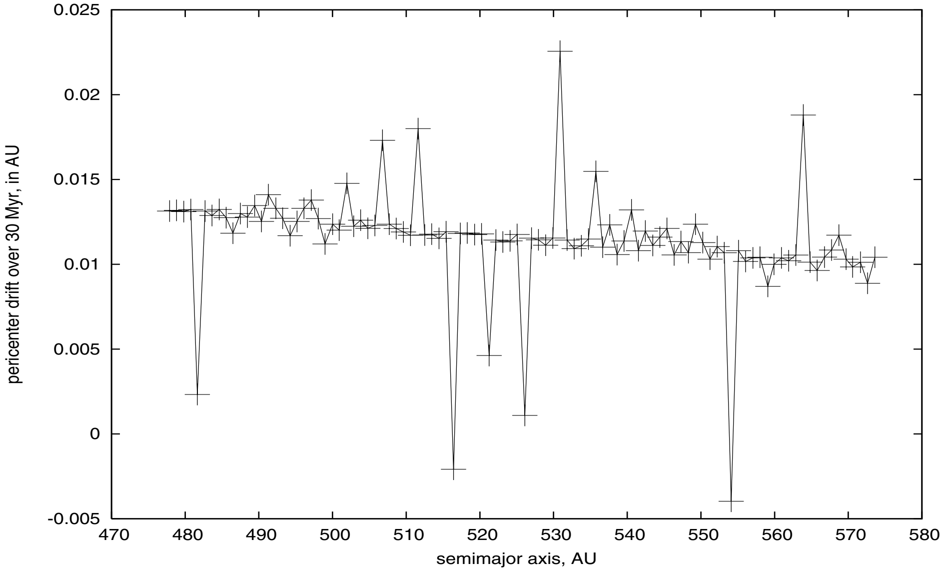

Figure 1. Change in perihelion distance (in AU) during the integration (3×10^7 yr) as the function of a test particle's semimajor axis. The change was computed as the difference between the average perihelia over the first and the last 5×10^5 yr, while the a shown is the average over the first of those intervals. The continuous background and discrete features are clearly visible. Varying intensities of the resonances are in large part due to rough sampling (1 AU).

constant background drift of the pericenter and multiple discrete features, within which the perihelion behaves significantly different from the background. The background drift is positive, amounting to about 10^{-2} AU over the whole integration, and it decreases noticeably for larger semimajor axes. Despite its secular appearance in our survey, this drift is just a consequence of a long-period oscillation in eccentricity, caused by the J_4 moment of the Solar System. Its direction is determined solely by the phase of the particle's argument of pericenter ω, and we will derive all of its important features in the next section.

Discrete features fill the whole range in a covered by the survey, with spacing of about 5 AU between neighboring ones. Their regularity and large number hint at each being a high-order mean-motion resonance with one of the giant planets, Neptune being most likely. Indeed, spacing between two neighbouring high-order resonances with Neptune of type $1 : k$ and $1 : (k+1)$ (where $k >> 1$) is expected to be

$$\frac{\Delta a}{a} = \frac{2\Delta n}{3n} \approx \frac{2n}{3n_N} = \frac{2}{3}\left(\frac{a}{a_N}\right)^{-3/2},$$

where a and n are the semimajor axis and mean motion of the particle, while a_N and n_N are those for Neptune. For $a = 530$ AU, the spacing will be $\Delta a = 4.8$ AU. Finally, we have plotted the resonant argument $\Psi = \lambda_N - k\lambda + (k-1)\varpi$ (where k is an integer) for the number of particles (an example is given in Fig. 2) and observed episodes of the libration of Ψ for all "excited" bodies, while Ψ circulated rapidly for all "non-excited" ones. In section 4 we will address the particulars of these resonances in more detail.

3. Secular Drift

Secular effects of the planets on the orbit of a "inner Oort cloud" body like VB_{12} are relatively simple. We are interested in timescales much longer than the precession periods of the planet's perihelia and nodes, so the Solar System potential can be considered azimuthally symmetric. Stated this way, the problem of the secular evolution of VB_{12}'s orbit is very similar to that of a very eccentric artificial satellite orbit evolving in the field of an oblate planet. Brouwer (1959) has analytically studied that problem in some detail, and we will apply his results to the problem of the secular drift in VB_{12}'s eccentricity. More recently, Yokoyama *et al.* (2003) have independently derived the secular disturbing function to the fourth order in a/a' for a more general case of two well-separated bodies; when the inner body's eccentricity equals zero, their result becomes identical to that of Brouwer (1959).

The J_2 moment of the Solar system, defined here as $J_2 = (1/2)\sum_{k=1}^{4}(m_k/M)(a_k/a_4)^2$, where m_k and a_k are masses and distances of planets Jupiter through Neptune, and M is the Sun's mass, will cause only simple precession of a perturbed particle's orbit, with no effect on its e or i (see, e.g., Danby 1992). Therefore, the terms in the Hamiltonian involving $J_4 = (3/8)\sum_{k=1}^{4}(m_k/M)(a_k/a_4)^4$ need to be taken into account (the moment $J_3 = 0$ as the problem exhibits north-south symmetry). According to Brouwer (1959), the secular part of the disturbing potential arising from J_4, expressed in Delaunay elements, is:

$$\begin{aligned} U_4 = \frac{\mu^6 k_4}{L^6 G^7}&\Big(\frac{3}{8} - \frac{15}{4}\frac{H^2}{G^2} + \frac{35}{8}\frac{H^4}{G^4}\Big)\Big(\frac{5}{2} - \frac{3}{2}\frac{G^2}{L^2}\Big) \\ &+\Big(-\frac{5}{6} + \frac{20}{3}\frac{H^2}{G^2} - \frac{35}{6}\frac{H^4}{G^4}\Big)\Big(\frac{3}{4} - \frac{3}{4}\frac{G^2}{L^2}\Big)\cos(2g). \end{aligned} \tag{3.1}$$

In Eq. 3.1, $\mu = GM$, $k_4 = (3/8)J_4$ while the Delaunay variables L, G, H and g have their usual meaning (Murray & Dermott 1999), except that the planetary radius (in our case replaced by Neptune's a) is used as the unit of length. The only part of (3.1) that is of interest to us is the one containing $\cos(2g)$, as the remaining terms can produce no change in e. Expressed in standard orbital elements, the term in question becomes:

$$U_4' = \frac{3}{8}\frac{\mu\ J_4\ a_N^4 e^2}{a^5(1-e^2)^{7/2}}\Big(\frac{15}{4}\sin^2 i - \frac{35}{8}\sin^4 i\Big)\cos(2\omega). \tag{3.2}$$

Through the Lagrange equations (Danby 1992), we get the secular rate of change in eccentricity:

$$\dot{e} = \frac{15}{16} n\ J_4\Big(\frac{a_N}{a}\Big)^4 \frac{e}{(1-e^2)^3}\Big(3\sin^2 i - \frac{7}{2}\sin^4 i\Big)\sin(2\omega). \tag{3.3}$$

Using $n = 5.1\times10^{-4}$rad/yr, $J_2 = 2.4\times10^{-5}$, $a = 533$AU, $e = 0.857$, $i = 12°$, $\omega = 312°$, we obtain $\dot{e} = -7.5\times10^{-13}\mathrm{yr}^{-1}$. The corresponding perihelion drift rate during our integration is $\Delta q = -a\dot{e}(3\times10^7 yr) = 1.2\times10^{-2}AU$, which agrees well with result plotted in Fig. 1. According to the above formulae, the magnitude of Δq should decrease with semimajor axis proportional to approximately $a^{-1.5}$, also in agreement with Fig. 1. This decrease is a smooth function of a and cannot be related to the discrete features in Fig. 1. Therefore those features must be caused by a different mechanism which cannot be explained by a purely secular theory.

It is interesting to note that secular evolution of a body perturbed by the system's J_4 moment is somewhat similar to that described by Kozai (1962). In the latter case, perturbee is orbiting inside the perturber's orbit, which need not be eccentric (classical application of Kozai's theory is the secular motion of a high-inclination asteroid perturbed

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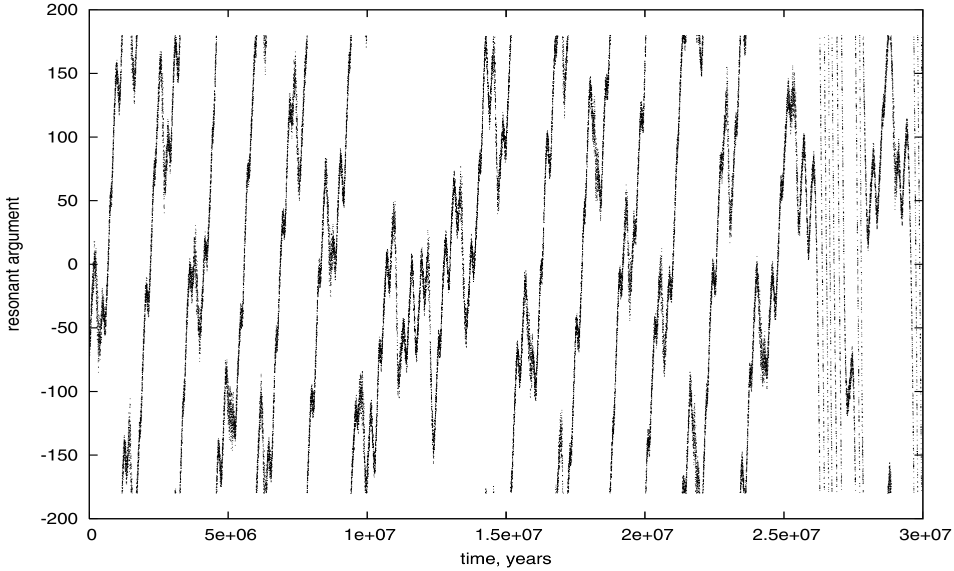

Figure 2. The resonant argument $\Psi = \lambda_N - 74\lambda + 73\varpi$ over the course of the integration for the particle associated with the resonant feature at 531 AU

by Jupiter). I both cases, e and i oscillate in counter-phase, and go through two cycles during one full precession period of ω. However, the terms in the disturbing potential which cause oscillations due to the Kozai mechanism and J_4 are not the same. Kozai mechanism relies on term $U_2 \sim a_1^2/a_2^3$ (subscripts 1 and 2 refer to inner and outer bodies), which becomes zero if $e_1 = 0$ (since ω_1 is then meaningless). Similarly, $U_3 \sim a_1^3/a_2^4$ is zero if either of the eccentricities is zero. Only $U_4 \sim a_1^4/a_2^5$ will not be zero if $e_1 = 0$ (which is an inevitable side effect of using the J_2/J_4 approximation). So it is somewhat incorrect to refer to oscillations in a particle's e and i caused by J_4 as "the Kozai behavior" in the usual sense, but the label "quasi-Kozai" would be more appropriate.

4. Resonances

To illustrate the resonant behavior, Fig. 2 plots the resonant argument $\Psi = \lambda_N - 74\lambda + 73\varpi$ for the test particle that produces the positive peak at $a = 531$ AU in Fig. 1. During most of the integration, Ψ shows intermittent episodes of slow circulation and libration. The transitions between different regimes toward the end of the integrations is correlated with abrupt (if small) changes in a and e. Based on the locations and spacing of these resonances, as well as the evidence from the resonant arguments, we have no doubt that they are very high-order mean-motion commensurabilities with Neptune.

The existence of meaningful resonances with orders as high as 73 (as that in Fig. 2) is somewhat surprising. Resonances with orders higher than a few are rare in the Solar System (Murray & Dermott 1999). This is mainly because every resonant term in the Hamiltonian containing a body's secular angles ϖ and Ω has to multiplied by a monomial in the same body's e and $\sin i$, respectively. The power of e or $\sin i$ that factors the resonant term in question is equal to the whole-number coefficient that multiplies the corresponding secular angle in the resonant argument. The total sum of all such whole-number coefficients factoring the secular angles in the resonant argument is, in turn, equal to the order of the resonance. Therefore, a term in the Hamiltonian associated with the mean-motion resonance of order n has to contain a factor $e^j \sin^k i$, where $j + k = n$. Since

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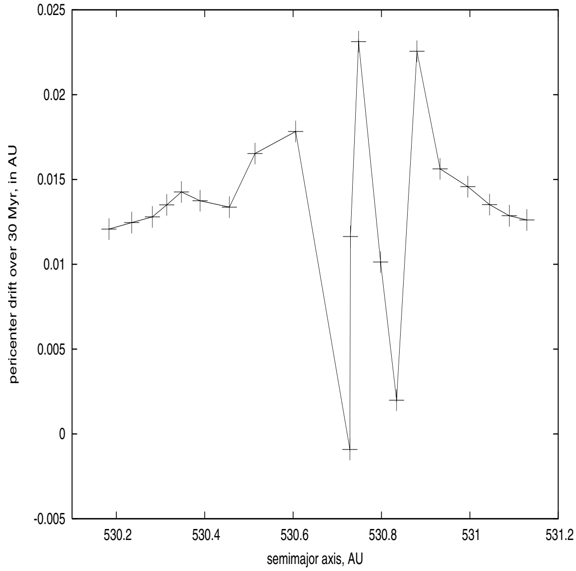

Figure 3. Same as Fig. 1, only for the region around 1:74 mean motion resonance with Neptune at 531 AU.

both e and $\sin i$ are small for most Solar System objects, the magnitudes of high-order resonant terms are usually vanishingly small. These relations describing resonant terms are commonly known as d'Alembert rules (Murray & Dermott 1999).

However, in the case of VB_{12}, eccentricity is by no means small, and even a high power of it like e^{73} is larger than 10^{-5}. This factor is quite comparable to the one a low-order resonance among bodies on low-eccentricity orbits (e.g., the major satellites of Saturn, Murray & Dermott 1999) would have. Therefore, even if a 73^{th}-order resonance intuitively appears impossible, it cannot be ruled out on analytical grounds.

To probe in more detail the structure of one of these resonances, we also ran a higher-resolution probe with twenty test particles at semimajor axes within 1 AU surrounding the body featured in Fig. 2. Figure 3 is the same as Fig. 1, just restricted to the region around 531 AU, and shows the result of the higher-resolution probe. We can now see that the "wings" of the resonance are associated with a positive change in q, the magnitude of which increases toward the center, while the core itself appears more chaotic, without any preferred sign of the perturbation in q. In terms of the resonant argument, we have noticed a prevalence of Ψ's slow circulation among the particles affected by the "wings" of the resonance (most of the behavior shown in Fig. 2 is of this type), while the particles in the "core" show a chaotic mix of episodes of fast circulation and libration, the latter resulting in abrupt changes to the particles' a and e. Therefore we can tentatively conclude that while the bodies closest to the centers of the resonances usually have chaotically changing perihelia, it is likely that a uniform increase in q can be sustained for very long periods of time for some bodies.

How significant is this drift over the age of the Solar System? We will make a naive approximation that should give us some handle on the timescales needed for a significant change in the orbital elements of VB_{12} to occur. If we designate $C_e = de/dt$ at the present epoch ($e_0 = 0.857$), and assume that $a = const$ and $de/dt = (e/e_0)^{73} C_e$, then the time

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needed to change the perihelion to 45 AU ($e_1 = 0.916$) is given by:

$$T = \frac{1}{C_e}\int_{e_1}^{e_0}\left(\frac{e_0}{e}\right)^{73} de = -\frac{e_0}{72C_e}\left[1-\left(\frac{e_0}{e_1}\right)^{72}\right]. \tag{4.1}$$

Based on 4.1, we can estimate what C_e is required to change the eccentricity from e_1 to e_0 in $T = 4.5\times 10^9$ yr:

$$C_e \simeq -\frac{e_0}{72T} = -2.64\times 10^{-12}\mathrm{yr}^{-1}, \tag{4.2}$$

which corresponds to $\Delta q = 4.2\times 10^{-2}$ AU over the course of our integration. While this drift is several times larger than the deviation of resonant points from the background in Fig. 1, it is likely that a more careful sampling could find locations within the resonances where a sustained positive drift in Δq could be produced.

Given the extremely high orders involved, it appears unlikely that any one particular resonance would be much stronger than its immediate neighbors. However, some of them might be affected by secondary resonances; in particular, frequencies associated with multiples of the 4200-yr period of the "Lesser Inequality" (LI) of Uranus and Neptune might change the nature of local mean-motion resonances. Bodies with $a \simeq 544$ AU have orbital periods close to three times the LI; while we have not seen any primary resonances associated with this or any other harmonic of the LI, we still have to explore the possible secondary resonances associated with the LI. The frequency of the LI leaves a strong signal in Neptune's a, so it is conceivable that it could affect its mean-motion resonances, too.

5. Conclusions

In previous sections we have seen that high-order mean-motion resonances with Neptune can slowly but continuously change the perihelion of bodies on orbits similar to that of 2003 VB_{12}. Although of very high order (> 70), these resonances are still significant due to considerable eccentricity of 2003 VB_{12} ($e \simeq 0.86$). Also, assuming that the strength of the resonance is a simple function of the perturbee's eccentricity, the strength of the observed resonances is of the right order of magnitude to produce the current orbit of 2003 VB_{12} over the age of the Solar System.

While our present results are intriguing, they by no means prove that 2003 VB_{12} is in a mean-motion resonance with Neptune or that such a resonance produced its current orbit. Much more work, both theoretical and observational, is needed to test this hypothesis. More detailed surveys and longer integrations are needed to better understand the dynamics of these resonances. To fully explore all the possible paths 2003 VB_{12} could have taken to its present dwelling place, simulations including migration of Neptune are also needed (especially so if the Lesser Inequality of Uranus and Neptune is also involved).

Further observations of 2003 VB_{12} would be valuable not only to determine its precise mean motion, but also to put some constraints on its physical properties. If 2003 VB_{12} is as large as currently thought, it implies a large mass for the "inner Oort Cloud", which then should be taken into account by all theories of Solar System formation. On the other hand, if the "observed" 2003 VB_{12} refers to its coma or an extended atmosphere of some kind, the implied mass and the significance of similar bodies could be much less. In order to decide among the different models for the formation and migration of 2003 VB_{12}, it will be absolutely necessary to have some estimate of the total mass of all bodies on similar orbits. In any case, it is likely that "Sedna" and her yet unknown sisters will hold more surprises for observers and theorists alike.

Acknowledgements

The author would like to thank Joseph A. Burns for help and support since the beginning of this project, as well as for his comments on an early draft of this paper.

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