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The useless introduction of the new words "wobble" and "sway" has produced a flourishing rise in confusing terminology in most of the recent papers. These two words even if picturesque are fundamentally unprecise. They perhaps help to describe -- in the English language -- the nature of the movement but in no way correspond to the classical terminology of fundamental mechanics. They have apparently no convenient translation into the other usual languages.

Because of this new fashion introduced by some geophysicists, many astronomers have become really confused and, instead of keeping the very clear Poinsoot-Tisserand terminology, started to stammer into the following extensive terminologies, never clearly defined, which I found in one paper: wobble, free wobble, forced nutation, sway, nutation, nearly diurnal wobble, diurnal wobble, nearly diurnal free nutation, nutation in space, nearly diurnal free wobble. I am pleased to say that this paper is perfectly correct insofar as its theoretical contents are concerned (which proves the author's competence) but its discussion is very confusing. There are also other definitions for the same thing like "Oppolzer terms," "forced diurnal nutation" and even "dynamical variation of the latitude." I wonder if the Euler nutation is not dynamical also as it involves clear dynamical quantities like the moments of inertia. Some authors recently even called the Oppolzer terms "wobble."

Another curious example is given by an author who calls "forced nutation" the herpolhody corresponding to the free polhode as if the polhode motion forces a herpolhode motion. It is as well the herpolhode which forces the polhode! As a matter of fact both motions are concomitant and free and are not produced one by the other. The title of this item is "complementary nutation in space," a new thing. I have also found somewhere the expression "indirect effect of the nearly diurnal wobble" which should be the equivalent of the "sway." Finally some author defines the polhode as the body cone and the herpolhode as the space cone, which is clearly an abuse of language as the polhode is the path of the pole on the inertia ellipsoid and the herpolhode is the path on a fixed plane in space.

This general confusion in terminology is sufficient to explain contradictions in the interpretation of observations or in the comparisons between observations and theory. I therefore propose and strongly recommend a return to the clear and simple classical definitions as they were used by the most famous mathematicians (Poincot, Klein, Sommerfeld) and astronomers (Tisserand, Newcomb). Such a proposition would help the many authors who believe themselves obliged to put the new expressions in quotation marks. I propose to use only the word nutation and distinguish the nutation in space from the nutation inside the body with an indication of the period of the concerned phenomenon.

Definitions

Every purely harmonic free or forced oscillation of the axis of rotation is a nutation and is represented by a system of two circular cones rolling on each other without slipping (Poincot representation). One cone rotates with respect to the Earth's body. The other cone simultaneously rotates with respect to the inertial frame of reference, that is with respect to the stars. The axis of rotation is the contact line of the two cones. In the case of the free motion the intersection of the cone with the ellipsoid of inertia is the polhode. Its intersection with the plane upon which the ellipsoid of inertia rotates is the herpolhode. There are two or perhaps more free nutations and there are many forced nutations, each one being a combination of two circular cones.

In their very good paper, Rochester, Jensen and Smylie (1974, p. 350) said: "As was pointed out by Munk and MacDonald, the distinction can be made on the basis of observational consequences: wobble leads to variation of latitude, nutation to changes in the declinations of stars" (here wobble means polhode). It is a good remark, but with all respect to the beautiful book of Munk and MacDonald it must be said that this statement had been made clear at the time of Peters, Fergola and Küstner in the last century! All astronomers had known this from time immemorial, and the introduction of the Kimura term (1905) in the least-squares solution of the latitude observations took care of this basic distinction.

FREE NUTATIONS will be described here in terms of rheological models.

- a) Euclidean body (infinite rigidity) -- Euler free nutation
 In this case we should observe a polhody with a period $(2\pi/\omega)(A/C-A) \approx 305$ days revolving in the same direction as ω , that is prograde. The herpolhody should have a period $(2\pi/\omega)(A/C) \sim 1$ day and be prograde also. The radii are in the ratio $H/P = 1/305$. For the Eulerian nutation the cones are determined by the rotation of the ellipsoid of inertia on an invariable plane. The terms polhody and herpolhode strictly can only be used in that case.

b) Hooke body (perfect elasticity) -- Chandler free nutation

It has been shown by Newcomb and later on by Love that the period of the polhode is apparently lengthened to about 430 days while the herpolhode period is unchanged and the radii are in the ratio $H/P = 1/430$.

c) Poincaré body (mantle with infinite rigidity containing a perfectly liquid core)

In this case there are two free nutations:

1. The Euler prograde free nutation of the mantle

The polhody has a period $(2\pi/\omega)(A_0/C_0 - A_0) \sim (2\pi/\omega)(A_0/C - A) \sim 270$ days where (A_0, C_0) are the moments of inertia of the mantle alone. The herpolhody is diurnal $[(2\pi/\omega)(A_0/C)]$.

2. The retrograde free principal core nutation

The polhody has a nearly diurnal period $-(2\pi/\omega)[1 + (A\varepsilon_1/A_0)]^{-1}$ (retrograde) where ε_1 is the geometrical flattening of the core which is about 1/400 in the Clairaut theory and gives a period equal to $(1 - 1/400) 24$ hours. This is due to an oscillation of the angular momentum of the fluid core. Then as the total angular momentum must remain fixed in space, the mantle must oscillate with respect to it. The herpolhody is also retrograde and its diameter is $A_0/A\varepsilon_1 \sim 400$ times the polhody diameter. Its period is 400 days.

d) Jeffreys body (perfect elastic mantle with perfect fluid core).

There are again two free nutations

1. The Chandler prograde free nutation of the elastic mantle

The polhody has a period of 430 days. The herpolhody is diurnal. Their diameters are in the ratio $H/P = 1/430$.

2. The retrograde principal free core nutation

The polhody has a nearly diurnal period $-(2\pi/\omega)[1 + (1/490)]$. The herpolhody has a period of about 490 days. Their diameters are in the ratio $H/P = 490$.

FORCED NUTATIONS

We distinguish two main kinds of forced nutations:

- a) Geophysical nutations forced by the atmosphere and oceans in their circulation with respect to the Earth's body as defined in the free nutation analysis. Munk and MacDonald have perfectly described this class of forced nutations in their book. Therefore we will limit ourselves to mentioning that there is an annual forced nutation with a diurnal nutation in space and probably sub-harmonics like a semi-annual forced nutation.

b) Astronomical nutations forced by the tidal potential of the Moon and the Sun. These have been fully described in my papers (Melchior and Georis 1968, and Melchior 1971). See Figure 1.

The essential feature is that, the tidal spectrum being symmetric with respect to a central line which corresponds to the sidereal velocity of rotation of the Earth, there will always be two circular nutations of equal frequencies but opposite in sign and this is the reason why we observe that the forced astronomical nutations are elliptic. In fact it is the angular momentum H which responds to the external torques and

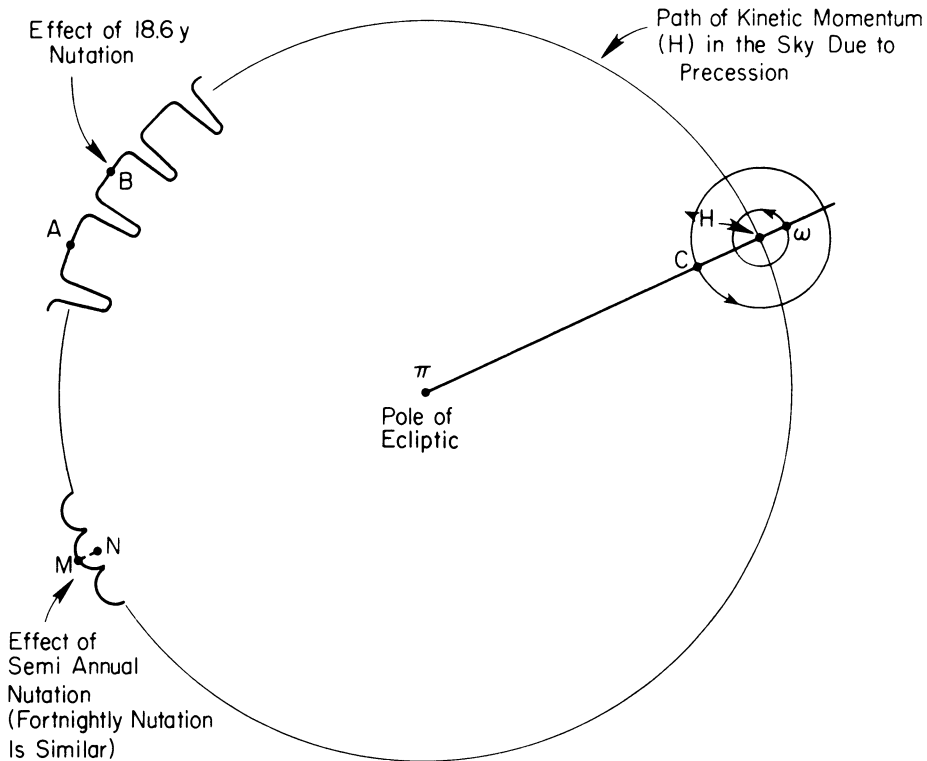


Figure 1. Forced precession and nutations. The scale of motions is such that

$$\pi H = 23^{\circ}27'$$

$$H\omega \cong 0''.001$$

$$HC \cong 0''.3$$

$$AB \sim 940'' \text{ in } 18.6 \text{ y}$$

$$MN \sim 1''.10 \text{ for semi-annual nutation}$$

$$0''.09 \text{ for fortnightly nutation}$$

describes a retrograde cone around the pole of the ecliptic π , taking with it the axis of rotation ω and the axis of inertia C which must remain very near to it except for the free nutation effect.

It is very unlikely that the mantle internal friction will be observable by a lag in the nutations as it produces such a small lag in the tidal deformation which itself is so small with respect to the Earth's ellipticity ($50 \text{ cm}/21 \text{ km} = 2 \times 10^{-5}$). On the other hand the viscosity of the core is so low according to Gans (1972) that it is unlikely that this will give an observable contribution. Earth tide results confirm that point. As concerns the free nutation, the internal friction should produce a dephasing in the position of OC outside of the plane (H, ω) and should make the Poincaré analysis no longer valid.

References

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