# Radiative Fluxes and Forces in Non-spherical Winds

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**Abstract.** For the modelling of the radiation fields in stellar winds and the resulting forces new efficient algorithms are presented. In the first one, the radiative transfer equation for moving 3D media is solved analytically with the assumption that the source function is known, eg. from the solution of the NLTE rate equations. For a wind with inhomogeneities an a-posteriori error controlled finite element code is described that takes scattering explicitly into account. Finally,we present possibilities for the accurate inclusion of an arbitrary number of spectral lines in a deterministic and in a stochastic way.

## 1 Introduction

It is now well known for more than twenty years that radiative transfer is of crucial importance for stellar winds from hot stars (Lucy and Solomon, 1970; Castor, Abbot and Klein, 1975) since it essentially determines the thermodynamical state of the matter involved and the force that acts on every volume element. Unfortunately, the radiative transfer equation differs from the hydrodynamic equations that describe the winds in that it is usually an integral-differential equation and therefore may involve the coupling of very distant regions. Hence special techniques for the solution have to be invoked, in particular if many lines are present and/or the geometry is not simple. Since 'classical methods' (cf. Kalkofen, 1987) either involve many severe simplifications or are by far too expensive computationally new types of solutions of the transfer equation are required.

In this contribution some analytical and numerical solutions of the transfer equation in moving media will be reviewed that are relevant to stellar winds and that have recently been obtained in Heidelberg in an interdisciplinary effort.

In the next section the transfer equation is given and interpreted as an equation in configuration⊗frequency⊗direction space. This allows a very convenient specific formulation e.g. for rotating winds not only of the transfer equation itself but also of the solutions if the source if given. In section 3

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a finite element algorithm is described that allows the accurate solution for coherently scattering media with arbitrary density and temperature distributions. Section 4 is devoted to the inclusion of many lines and the corresponding evaluation of frequency integrated quantities relevant for hydrodynamics. We close with brief discussion and outlook.

## 2 The Radiative Transfer Equation and its Analytical Solution for Rotating Winds

We base our discussion on the transfer equation for unpolarized time independent radiation (cf. Oxenius, 1986, for a derivation and its inherent limitations)

$$\frac{dI(\mathbf{s})}{d\mathbf{s}} = -\chi(\mathbf{s})(I(\mathbf{s}) - S(\mathbf{s})) \tag{1}$$

with $I$ = specific intensity, $\mathbf{s}$ = vector in ray direction $\mathbf{n}$ =, $\nu$ = frequency, $\chi$ = extinction coefficient, $S$ = source function. If the medium is static the coordinate system for $\mathbf{s}$ can be positioned in such a way that $\mathbf{s}$ reduces essentially to a scalar $s$. Frequency and angles can be considered then just as parameters.

In order to take velocities $\beta = (\beta_x(\mathbf{x}, \beta_y(\mathbf{x}\beta_z(\mathbf{x}))$ ($\mathbf{x}$ = geometrical coordinates $(x, y, z)$) we apply a Lorentz transformation to Eq. 1 (cf. Baschek, et al., 1997a). The resulting general expression is very complicated but it becomes very convenient if advection/aberration terms as well as terms involving $\beta^n$ with $n > 1$ are neglected. For the relativistically invariant intensity in terms of the logarithmic wavelength

$$\xi = \ln \lambda \tag{2}$$

it reads

$$\begin{aligned} \mathbf{n} \cdot \nabla I(\mathbf{x}, \mathbf{n}, \xi) + \mathbf{n} \cdot \nabla(\mathbf{n} \cdot \beta(\mathbf{x}))I(\mathbf{x}, \mathbf{n}, \xi) \\ = -\chi(\mathbf{x}, \xi)(I(\mathbf{x}, \mathbf{n}, \xi) - S(\mathbf{x}, \xi)) \end{aligned} \tag{3}$$

The solution of Eq. 3 can either be derived from the standard solution of Eq. 1

$$I(\mathbf{s}) = \exp(-\tau(\mathbf{0}, \mathbf{s}))I(\mathbf{0}) + \int_{\mathbf{0}}^{\mathbf{s}} \exp(-\tau(\mathbf{s}', \mathbf{s}))\chi(\mathbf{s}')S(\mathbf{s}')d\mathbf{s}' \tag{4}$$

with

$$\tau(\mathbf{s}', \mathbf{s}) = \int_{\mathbf{s}'}^{\mathbf{s}} \chi(\mathbf{s}'')d\mathbf{s}'' \tag{5}$$

and $I(\mathbf{0})$ is the incident intensity and the subsequent application of the appropriate Lorentz transformation to this solution or the solution can be obtained directly from Eq. 3 by means of characteristics. In both cases the solution is given by Eq. 4 but all integrals have to be considered as *path integrals* (cf. Wehrse and Baschek, 1998) along the curve $\mathcal{C}(s) = (\mathbf{x}_0 + \mathbf{n}s, \mathbf{n} \cdot \beta(\mathbf{x}_0 + \mathbf{n}s))$.

It is now straightforward to use the velocity field for a rotating wind using a Cartesian coordinate system

$$v(x,y,z) = v_{exp}(\sqrt{x^2+y^2+z^2})(x,y,z)^t/\sqrt{x^2+y^2+z^2} + v_{rot}(\sqrt{x^2+y^2+z^2})(x,y,0)^t/\sqrt{x^2+y^2}. \quad (6)$$

The results will be presented and discussed in detail in a separate paper (Wehrse and Müller, 1998).

## 3 Numerical Solution of the Transfer Equation for Arbitrary 3D Geometries

Whenever the wind is clumpy a **finite element** algorithm is advantageous since it takes the inhomogeneities into account in a natural way. Kanschat (1996) has developed a corresponding code for the transfer equation 3 with

$$S = \epsilon B + \frac{1-\epsilon}{4\pi} \int_{4\pi} p(\mathbf{n}, \mathbf{n}' I(\mathbf{x}, \mathbf{n}')d\omega'$$

($B$ =Planck function, $p$ =phase function, $\omega$ =solid angle). The code employs unstructured grids that are adaptively refined by means of an a-posteriori error estimate which is obtained from the solution of the dual problem. In this way it is guaranteed that the numerical solution of the transfer equation does not deviate from the exact analytical one by more than a prescribed value and simultaneously the computation time is minimized. An additional reduction of the CPU time is achieved by the use of a variant of the conjugate gradient method in place of the usual Jacobi iteration ($\Lambda$ or *approximate* $\Lambda$ iteration in the astronomical literature): since in this scheme the unknowns are eliminated according to the absolute value of corresponding eigenvalue starting with the largest one, convergence problems for optically thick media are completely avoided. The code is written in C++ in order to reduce the book-keeping concerning the interaction of the various cells and is designed for multiple instruction parallel! machines but runs also on workstations. In spite of the high numerical efficiency the machine requirements are quite high since always all radiative couplings have to evaluated. Therefore, the CPU time and memory vary strongly with required accuracy and the dimension and the complexity of the medium. Typical values are in the range of a few Gigabyte and several hours on an IBM SP2.

Fig. 1 shows, as an example, the radiative accelerations in an inhomogeneous medium that is illuminated from below. Note that the forces seem to compress the regions of higher density. Consequences for stellar winds appear to be obvious but firm conclusions cannot be drawn before this transfer code is coupled with a hydrodynamic one.

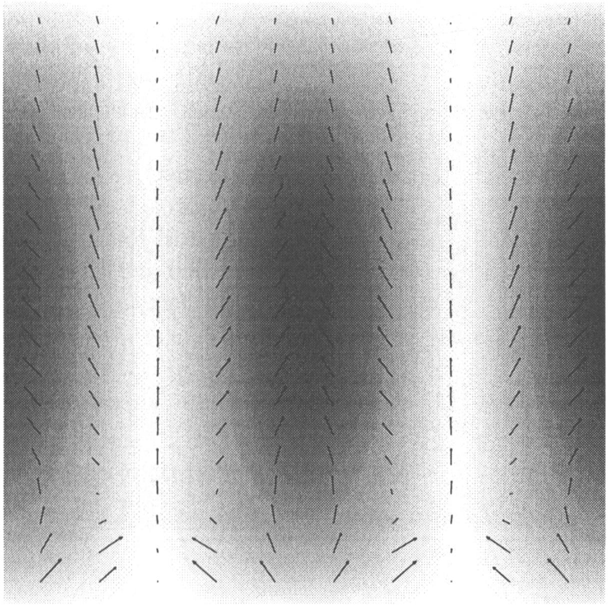

**Fig. 1.** Example for the complex behavior of radiative forces in an inhomogeneous, static scattering 2D medium with small thermal emission that is illuminated from below (Kanschat, 1996). Only one frequency is taken into account. The shading indicates the density distribution and the arrows indicate strengths and direction of the radiative acceleration. Differential motion, wavelength dependent extinction and thermal emission would introduce additional complications.

## 4 Many Lines

Since it seems that the winds from hot stars are not accelerated by just a few lines but by the combined action of all lines (cf. Kudritzki's contribution in this volume) it is important to include in simulations the line absorption as completely as possible; in particular, weak lines should be represented adequately. Unfortunately, even very fast present-day computers do not allow such a representation if the well known solutions of the transfer equation are employed.

In our search for alternative methods it turned out (Baschek et al., 1997b) that the solution of eq. 3 for a layered moving medium can easily be expressed in terms of the *spectral thickness*

$$\psi(\xi) = \int_{\xi_0}^{\xi} \chi(\xi')d\chi'. \tag{7}$$

The spectral thickness is much better suited for the evaluation of integrals over wavelength than the extinction coefficient itself since it is much smoother due to integration; in fact; in most cases it can be well approximated by a piece-wise linear function. Since, in addition, it was found that the integral over the Planck function over an arbitrary wavelength interval can conveniently be expressed in terms of poly-logarithmic functions, the computation times for total fluxes and radiative accelerations could be reduced by more than 5 dex with a loss in accuracy of only 1 percent.

As an alternative to this deterministic approach we also developed a method in which the line positions are assumed to follow a Poisson process with mean density $\rho(\xi)$(Wehrse et al., 1998). The line strengths and broadening parameters may be correlated with wavelength and may obey some suitable distribution, as e.g.

$$\rho(\xi, \vartheta) = \rho_0 f(\vartheta) \tag{8}$$

where $f$ is a suitable function to describe the line strengths and widths (combined in the parameter $\vartheta$). The expectation values of the specific intensity for a shell of geometrical thickness $z << R$ with constant acceleration $w$ is then given by

$$\langle I \rangle = \int_0^\infty I\mathbf{P}(dI) \tag{9}$$

$$= \langle I_0 \rangle - \int_{\xi - wz}^{\xi} \left\langle \exp\left(-\frac{1}{w}\int_\eta^\xi \chi(\zeta)d\zeta\right)\right\rangle \frac{d}{d\eta} S(z - \frac{\xi-\eta}{w}, \eta)d\eta$$
$$+S(z,\xi) - S(0, \xi - wz)\left\langle \exp\left(-\frac{1}{w}\int_{xi-wz}^{\xi} \chi(\zeta)d\zeta\right)\right\rangle \tag{10}$$

where the crucial term

$$\left\langle \exp\left(-\frac{1}{w}\int_\eta^\xi \chi(\zeta)d\zeta\right)\right\rangle = \exp\left(-\chi_c(\xi-\eta)/w\right)\Omega(\xi,\eta) \tag{11}$$

with

$$\Omega(\xi,\eta) = \exp\left(\int_S \rho(\xi',\vartheta)\left\{\exp\left(-\frac{1}{w}\int_\eta^\xi \chi_l(\xi',\vartheta,\zeta-\xi')d\zeta\right) - 1\right\} d\xi' d\vartheta\right). \tag{12}$$

In these expressions we have splitted the extinction coefficient in an continuum (subscript c) and a line contribution (subscript l). $S$ s the combined set of wavelengths and line parameters; the three arguments of the line absorption coefficient $\chi$ indicate the actual wavelength, the line parameters, and the center of line. The general formula for $\Omega$ given here looks rather complicated; in actual cases, however, one integration can often be performed analytically so that quite handy expressions can be obtained. Fig. 2 gives as an example the expectation value of the intensity emerging from a rotating stellar wind under the assumption that the invidual lines have a $\delta$-function profile.

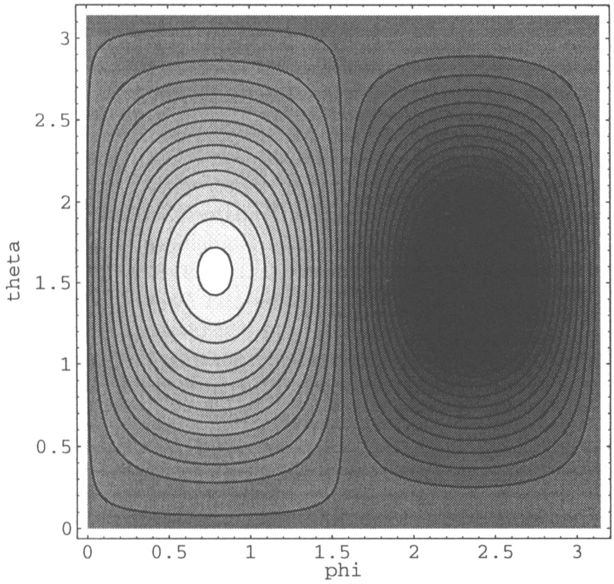


**Fig. 2.** Typical expectation value of the comoving frame specific intensity from an rotating wind as a function of latitude $\theta$ and longitude $\phi$. Spectral lines are included in a stochastic description, see text.

## 5 Discussion and Outlook

In this contribution some powerful new methods for the modelling of radiation fields in non-spherical wind have been discussed. Up to now they have been applied to rather simple situations only in order to *understand* the way radiation fields operate in such complicated systems as non-spherical and rotating winds. The next step will be the proper determination of the opacity input data, in particular the distribution functions mentioned in the previous section.

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### Discussion

**J. Bjorkman**: One of the more difficult aspects of radiation transfer is that the opacity depends on the radiation field. In the model you presented, the opacity constants are independent of the radiation field. How would one modify your model to account for these effects?

**R. Wehrse**: In the analytical solutions for moving media presented here it is indeed assumed that the source function is known in advance. However, the solutions can also be used advantageously in NLTE calculations by inserting them into the rate equations.

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