ON THE MATHIEU GROUP M_{23}

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1. Introduction

Until 1965, when Janko [7] established the existence of his finite simple group J_1, the five Mathieu groups were the only known examples of isolated finite simple groups. In 1951, R. G. Stanton [10] showed that M_{12} and M_{24} were determined uniquely by their order. Recent characterizations of M_{22} and M_{23} by Janko [8], M_{22} by D. Held [6], and M_{11} by W. J. Wong [12], have facilitated the unique determination of the three remaining Mathieu groups by their orders. D. Parrott [9] has so characterized M_{22} and M_{11}, while this paper is an outline of the characterization of M_{23} in terms of its order.

MAIN THEOREM. *Let G be a non-abelian simple group of order* 10,200,960. *Then G is isomorphic to* M_{23}.

2. Some known results

1. The results used in the proof of the main theorem were obtained by R. Brauer [1], [2], [3], H. F. Tuan [4] and applied by R. G. Stanton [10], D. Parrott [9] and S. K. Wong [11]. Some of the important theorems are given here without proof.

2. If G is a group of order $|G|$ containing k classes $K_1, \cdots, K_k$ of conjugate elements, then there exists exactly k distinct irreducible characters $\zeta_1(g), \cdots, \zeta_k(g)$ where g denotes a variable element of G. Let p be a prime which divides $|G|$, then the k characters are distributed into a certain number of p-blocks $B_1(p)$, $B_2(p), \cdots$. The *principal p-block* $B_1(p)$ is always taken as the block containing the 1-character $\zeta_1(g) = 1$ for all $g \in G$. Suppose $p^y \top |G|$; if for all characters ζ_μ of $B_\sigma(p)$ the degrees z_μ of ζ_μ is divisible by p^α while at least one of the degrees z_μ is not divisible by $p^{\alpha+1}$ then $B_\sigma(p)$ is a block of *defect* $(y-\alpha)$, or *type* α. In particular if $p \top |G|$ a p-block $B_\sigma(p)$ is of defect 0 (highest type) or of defect 1 (lowest type).

An element g is *p-regular* if its order is prime to p, otherwise g is called *p-singular*.

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3. We assume in this section that $p \top |G|$. Let G_p be a Sylow p-subgroup of G. Then $C_G(G_p) = G_p \times V_p$. If V_p has l conjugate classes in the group $N_G(G_p)$ then G has l blocks of defect 1. Let t denote the number of conjugate classes of elements of order p in G. To each of the l p-blocks $B_\sigma(p)$ of defect 1 there corresponds a certain multiple t_σ of t, where $t_\sigma | p-1$, such that $B_\sigma(p)$ has $(p-1)/t_\sigma$ characters ζ_μ which are p-conjugate only to themselves and one exceptional family of t_σ p-conjugate characters.

Theorem 2.1 ([2]. Theorem 11). *For the block $B_1(p)$, we have $t_1 = t$. The degrees z_μ of the characters ζ_μ of $B_1(p)$ satisfy:*

$$z_\mu \equiv \delta_\mu = \pm 1 (\mathrm{mod}\, p), \quad 1 \leqq \mu \leqq \omega = (p-1)/t \tag{2.1}$$

$$t z_{\omega+1} \equiv \delta_{\omega+1} = \pm 1 (\mathrm{mod}\, p), \tag{2.2}$$

where $z_{\omega+1}$ is the degree of a representative of the exceptional family.

$$\sum_{\mu=1}^{\omega+1} \delta_\mu z_\mu = 0 \qquad (\delta_1 = z_1 = 1). \tag{2.3}$$

Moreover, for p-singular elements P of G we have

$$\zeta_\mu(P) = \delta_\mu \qquad (1 \leqq \mu \leqq \omega).$$

Corollary 1. *Let G be a group of order $pq^b g^*$ where p and q are distinct primes, b and g^* positive integers and $(pq, g^*) = 1$. Suppose that G has an element of order pq, then q^b cannot divide the degree of any irreducible character ζ_μ in $B_1(p)$.*

We shall say a character ζ of $B_1(p)$ is of type 0 for the prime p if $\zeta(1) \equiv 1 (\mathrm{mod}\, p)$ or if ζ belongs to the exceptional family of $B_1(p)$ and $\zeta(1) \equiv -(p-1)/t\, (\mathrm{mod}\, p)$; ζ is of type 1 if $\zeta(1) \equiv -1 (\mathrm{mod}\, p)$ or if χ belongs to the exceptional family and $\zeta(1) \equiv +(p-1)/t (\mathrm{mod}\, p)$.

Theorem 2.2 ([10] Lemma 6). *Let G be a group of order $|G|$. Assume p and p' are distinct primes which divide $|G|$ to the first power only and that G has no elements of order pp'. Let a_{ij} be the number of characters in $B_1(p) \cap B_1(p')$ which are of type i for p and type j for p', the indices i and j being 0 or 1 as described above. Then*

$$a_{00} + a_{11} = a_{01} + a_{10}.$$

It is clear that a character ζ in $B_1(p) \cap B_1(p')$ cannot be exceptional for both primes p and p'.

Theorem 2.3 ([4], Lemma 1). *Let G be a finite group which is identical with its commutator group G', and assume that the principal p-block $B_1(p)$ contains an irreducible faithful character ζ of degree $z < 2p$. Then the order of the centralizer $C_G(G_p)$ of a Sylow p-subgroup G_p of G is a power of p.*

3. The Sylow 23-normalizer of G

We assume from now on, that G is an non-abelian finite simple group of order $10{,}200{,}960 = 2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 23$.

Let S_{23} be a Sylow 23-subgroup of G and let $n_{23} = |G : N_G(S_{23})|$. Then n_{23} has the following possibilities: (1) $2^7 \cdot 3^2 \cdot 5 \cdot 7$, (2) $2^6 \cdot 5 \cdot 11$, (3) $2^6 \cdot 3$, (4) $2^4 \cdot 3 \cdot 5 \cdot 7$, (5) $2^3 \cdot 3^2 \cdot 7 \cdot 11$, (6) $2^3 \cdot 3$, (7) $2 \cdot 3^2 \cdot 5 \cdot 11$, (8) $2 \cdot 5 \cdot 7$, (9) $3 \cdot 7 \cdot 11$.

We know that G has either 1, 2, or 11 classes of elements of order 23 according as t for prime 23 (written as $t_{(23)}$) is 1, 2, or 11. Using equations (2.1), (2.2), and (2.3), and Theorem 2.3 $t_{(23)} = 11$ is ruled out, consequently $|N_G(S_{23})/(C_G(S_{23})| = 11$ or 22. Hence cases (2), (5), (7), and (9) above, for n_{23} are not possible. The impossibility of cases (4) and (8) follows almost as quickly, because otherwise G has no elements of order $5 \cdot 23$, $7 \cdot 23$, or $11 \cdot 23$ thus facilitating the use of Stanton's block intersection theorem (Theorem 2.2). Suppose $n_{23} = 2^3 \cdot 3$, case (6). Then $|N_G(S_{23})| = 2^4 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$. G then contains elements of order $2 \cdot 23$, $3 \cdot 23$, $5 \cdot 23$, and $7 \cdot 23$. From this it follows that 528 is the only possible degree of a nonexceptional character and 264 the only possible exceptional degree. But both of these degrees are even, and for $(2 \cdot 3)$ to be satisfied $B_1(23)$ must contain a character of odd degree. Case (3) is ruled out similarly. Hence we have proved

LEMMA 3.1. *The Sylow* 23-*normalizer* $N_G(S_{23})$ *is a Frobenius group of order* $23 \cdot 11$.

COROLLARY 3.1. *The principal* 23-*block* $B_1(23)$ *is the only* 23-*block of defect* 1, *and consists of* 11 *non-exceptional characters and a family of* 2 *exceptional characters. All other characters of G have degrees divisible by* 23.

4. The Sylow 11-normalizer of G

Let S_{11} be a Sylow 11-subgroup of G and $n_{11} = |G : N_G(S_{11})|$. Lemma 3.1 reduces the possible values for n_{11} to the following: (1) $3^2 \cdot 5 \cdot 23$, (2) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 23$, (3) $2^2 \cdot 3 \cdot 23$, (4) $2^2 \cdot 7 \cdot 23$, (5) $2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 23$, (6) $2^4 \cdot 3^2 \cdot 23$, (7) $2^5 \cdot 3 \cdot 7 \cdot 23$, (8) $2^6 \cdot 5 \cdot 23$, (9) $2^7 \cdot 3^2 \cdot 7 \cdot 23$.

Using the same methods as for the prime 23, one proves quickly that $t_{(11)} \neq 5$ and so $|N_G(S_{11})/C_G(S_{11})| = 5$ or 10. This in turn eliminates cases (1), (2), (5) and (8), from the above list for n_{11}.

Suppose $|N_G(S_{11})| = 2^5 \cdot 3 \cdot 5 \cdot 7 \cdot 11$, case (3). Then $|C_G(S_{11})| = 2^5 \cdot 3 \cdot 7 \cdot 11$ or $2^4 \cdot 3 \cdot 7 \cdot 11$.

If $|C_G(S_{11})| = 2^5 \cdot 3 \cdot 7 \cdot 11$, then $t_{(11)} = 2$ and $B_1(11)$ consists of 5 non-exceptional characters 1_G, χ_2, χ_3, χ^4 and χ^5 and a family of 2 exceptional charac-

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ters with representative χ_6. Since G has elements of order $2 \cdot 11$, $3 \cdot 11$ and $7 \cdot 11$, the possible degrees for the non-exceptional characters are

TABLE 1

1,	23,	276	$\equiv +1 \pmod{11}$
230,	736,	2760	$\equiv -1 \pmod{11}$

while the possible degrees for χ_6 are

TABLE 2

115,	368,	1380	$\equiv +5 \pmod{11}$
138,	160,	1920	$\equiv -5 \pmod{11}$

Then the degrees in $B_1(23) \cap B_1(11)$ are 1 and 160, and so $\chi_6(1) = 160$. Applying theorem 2.2 to $B_1(11) \cap B_1(5)$ we see that only degrees 1 and 736 lie in this intersection. Let $\chi_2(1) = 736$. Substitute the values 1, 160 and 736 in the degree equation (2.3). Then

$$\delta_3 z_3 + \delta_4 z_4 + \delta_5 z_5 = -(1-736+160) = 575$$

and so $z_3 = 23$, $z_4 = z_5 = 276$. The characters 1_G, χ_2, χ_3 and χ_6 are real on 11-regular elements, but this implies that in the tree for $B_1(11)$, two characters having the same sign $\delta = +1$ are joined by one edge contrary to a result of Brauer ([2], Theorem 5).

Thus $|C_G(S_{11})| = 2^4 \cdot 3 \cdot 7 \cdot 11$, and so $t_{11} = 1$ and $B_1(11)$ consists of 10 non-exceptional characters whose possible degrees are given by Table I. But then the only character which could lie in the principal 23-block and the principal 11-block is the principal character which is impossible.

Using similar arguments cases (4), (6) and (8) are removed and so we have

LEMMA 4.1. *The Sylow* 11*-normalizer* $N_G(S_{11})$ *is a Frobenius group of order* $5 \cdot 11$.

COROLLARY 4.1. *The principal* 11*-block* $B_1(11)$ *is the only* 11*-block of defect* 1. *All other characters of G have degrees divisible by* 11, *and lie in* 11*-blocks of defect* 0.

5. The determination of degrees and blocks of characters of G

We know now that G has no elements of order $23 \cdot 11$, $23 \cdot 7$, $23 \cdot 5$, $23 \cdot 3$, $11 \cdot 7$, $11 \cdot 5$ or $11 \cdot 3$. Applying Theorem 2.2 to the intersection of $B_1(23)$ and $B_1(5)$ we see that both blocks contain a character of degree 896. This character is then the exceptional character for $B_1(11)$ and using the degree equation (2.3) together with Theorem 2.2, we have

LEMMA 5.1. *The principal* 11-*block* $B_1(11)$ *contains only characters with the following degrees* 1, 45, 45, 1035, 230, 896. *All other characters of G have degrees which are divisible by* 11.

Since a character of degree $896 = 2^7 \cdot 7$ lies in $B_1(5)$ then G has no elements of order $7 \cdot 5$, or $2 \cdot 5$. As shown earlier, G has no elements o orderf $23 \cdot 5$ or $11 \cdot 5$ and so a Sylow 5-subgroup S_5 of G can be centralized only by elements of order 3 or 9. Further $|N_G(S_5)/C_G(S_5)| \leqq 4$, whence $|N_G(S_5)| = 2 \cdot 5$ or $2^2 \cdot 3 \cdot 5$. But in $B_1(5)$ we have already 3 non-exceptional characters and so $|N_G(S_5)| = 2^2 \cdot 3 \cdot 5$. Hence $t_{(5)} = 1$ and $B_1(5)$ contains exactly 5 characters. These are found easily using equation (2.3).

LEMMA 5.2. $|N_G(S_5)| = 2^2 \cdot 3 \cdot 5$. *$B_1(5)$ consists of 5 characters with the following degrees*: 1, 896, 896, 231, 2024.

Using the same methods we have

LEMMA 5.3. *The principal* 23-*block* $B_1(23)$ *contains only characters with the following degrees*: 1, 22, 45, 45, 231, 231, 231, 896, 896, 990, 990 *and* 770. *All other degrees of characters of G are divisible by* 23.

LEMMA 5.4. $|N_G(S_7)/C_G(S_7)| = 3$. *The principal* 7-*block* $B_1(7)$ *contains only characters with the following degrees*: 1, 2024, 1035 *and* 990.

We have determined 16 characters of G, the sum of squares of degrees is $(10200960-64009)$. Further, the degrees of the remaining characters must be divisible by both 23 and 11. However $(11 \cdot 23)^2 = 64009$, so G has only one more character and that is of degree $253 = 11 \cdot 23$.

LEMMA 5.5. *G has* 17 *characters with the following degrees*: 1, 22, 45, 45, 230, 231, 231, 231, 253, 770, 770, 896, 896, 990, 990, 1035 *and* 2024.

It is thus clear there are two 7-blocks of defect 1, and hence two conjugate classes of 7-regular elements of $C_G(S_7)$ in $N_G(S_7)$. Further since $|N_G(S_7)/C_G(S_7)| = 3$, $|N_G(S_7)|$ has the following possible orders, $2^7 \cdot 3 \cdot 7$, $2^4 \cdot 3 \cdot 7$ and $2 \cdot 3 \cdot 7$, but only when $|N_G(S_7)| = 2 \cdot 3 \cdot 7$, are there the required two classes of 7-regular elements. Finally, there is only one 3-block of defect 2 and so a Sylow 3-subgroup is self centralizing.

6. Conclusion

The group G has 17 conjugate classes and we have so far determined 16 of them, as is shown in the table below.

Order of element	1	23	11	7	14	5	15	6	4	3	2
No. of classes	1	2	2	2	2	1	2	1	1	1	1

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There is at least one class of involutions, and at least one class of elements of order 3 with one class to be determined.

By Sylow theorems, the order of the normaliser of a Sylow 3-subgroup of G is either $2^2 3^2$ or $2^4 \cdot 3^2$, and consequently a Sylow 3-subgroup is elementary abelian. Suppose G has two classes of elements of order 3. Let R be a Sylow 3-subgroup of G. We know that R is self centralising and that $|N_G(R)| = 2^2 \cdot 3^2$, and so $N_G(R)/R$ is cyclic of order 4. Let Q be a subgroup of order 3 in R and $C_G(Q)$ the centraliser of Q in G. Then since $N_{C_{G(Q)}}(R) = R$, we have by Burnside's result ([5], p. 252) that $C_G(Q)$ has a normal 3-complement, say N. Let $\tilde{Q}$ be the subgroup of order 3 of R which is centralised by an element of order 5.

Then $C_G(\tilde{Q}) = R\tilde{N}$ where $\tilde{N}$ is the normal 3-complement in $C_G(\tilde{Q})$ and $5||\tilde{N}|$. But then by the Frattini argument ([5], p. 12), $9||N_G(G_5)|$ where G_5 is a Sylow 5-subgroup of G, which is false. Hence G has only one class of elements of order 3 and so we have proved

LEMMA 6.1. *The group G has one class of elements of order* 3. *A Sylow* 3-*subgroup is normalised by a semi-dihedral group of order* 16, *and so G has only one class of involutions and one class of elements of order* 8.

Let t be the involution in the normaliser of a Sylow 7-subgroup G_7 of G, and consider the centraliser of t in G, $C_G(t)$. It follows immediately that $N_G(G_7) \subset C_G(t)$. Since G has no elements of order $2 \cdot 23$, $2 \cdot 11$, or $2 \cdot 5$, then $C_G(t)$ has order $2^\alpha \cdot 3^\beta \cdot 7$, where $\alpha \leqq 7$ and $\beta \leqq 2$. We know that G has only one class of involutions, and because $|C_G(t) : N_G(G_7)| \equiv 1 (\text{mod } 7)$, the order of $C_G(t)$ is $2^7 \cdot 3 \cdot 7$.

Suppose the group $C_G(t)$ is soluble. Let G_2 be a Sylow 2-subgroup of G which is contained in $C = C_G(t)$. Let $O_2(C)$ be the maximal normal subgroup of 2-power order in C. Then the factor group $C/O_2(C)$ is soluble. Let $\bar{N}$ be a minimal normal subgroup of $C/O_2(C)$. Then $\bar{N}$ has order 7 and so $O_2(C) = G_2$. But then $C_G(t)$ is 2-closed and so by a result of Suzuki ([5], p. 466). G is one of known list of finite simple groups. However, none of these have the order 10, 200, 960, a contradiction.

Hence we conclude that $C_G(t) = C$ is insoluble. Write $E = O_2(C)$. Because we must have $|C/E : N_{C/E}(\bar{G}_7)| \equiv 1 (\text{mod } 7)$ where $\bar{G}_7$ is a Sylow 7-subgroup in C/E, we have $|E| = 2$ or 16.

Suppose we have $|E| = 2$. Since $2^6 \cdot 3 \cdot 7$ is not the order of any simple group, C/E contains a normal subgroup. Let $\bar{N}$ be a minimal normal subgroup of C/E, then $\bar{N}$ is either elementary abelian or a direct product of isomorphic simple groups. Clearly $\bar{N}$ cannot be an elementary abelian 2-group. Further, $\bar{N}$ cannot be of order 3 for then G would have elements of order 21, and $\bar{N}$ cannot be of order 7 for this would imply that $|N_G(G_7)| > 2 \cdot 3 \cdot 7$. So we conclude that $|\bar{N}| = 2^3 \cdot 3 \cdot 7$, and $\bar{N} \simeq PSL(2, 7)$. Write $N = O_2(C)\bar{N}$, then we have $N \lhd C = C_G(t)$. Let N_7

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be a Sylow 7-subgroup of N. By the Frattini argument $C = NN_C(N_7)$ and so $C/N \simeq N_C(N_7)/N_N(N_7)$. But then order of the normaliser of a Sylow 7-subgroup is greater than $2 \cdot 3 \cdot 7$, which is a contradiction.

Thus we conclude that $|O_2(C)| = 16$. Since $C_G(t)$ is insoluble, $C_G(t)$ is an extension of $E = O_2(C)$ of order 16 by $PSL(2, 7)$. Suppose that $E = O_2(C)$ is non-abelian. Let $Z(E)$ be the centre of E. It follows that $|Z(E)| \neq 4$ for otherwise the order of the centraliser of a Sylow 7-subgroup in C is $4 \cdot 7$. Hence $Z(E) = \langle t \rangle$. Let $\Phi(E)$ be the Frattini subgroup of E, then $\Phi(E)$ has order 4 or 2. If $|\Phi(E)| = 4$ then $\Phi(E) \lhd C_G(t)$ and again we have that a Sylow 7-subgroup of C has a normalizer of order 4.7. So $\Phi(E) = Z(E) = E' = \langle t \rangle$ and hence E is an extra special 2-group, but this is impossible as $|E| = 2^4$. So E is abelian.

By a result of Suzuki ([5], p. 177) a Sylow 7-subgroup H of C acts as an automorphism group of E, and so $E = \langle t \rangle Z$ where $\langle t \rangle \cap Z = \langle 1 \rangle$ and Z is an H-admissible subgroup of E. The group Z is then of order 8 and so is elementary abelian. Hence E is elementary abelian.

Let T be a Sylow 2-subgroup of $C_G(t)$. Clearly the centre of T, $Z(T)$, is contained in E. If $Z(T)$ is of order 8, then at least two involutions say z and z' in $Z(T) \setminus \langle t \rangle$ are conjugated in C by an element of order 7. But this contradicts the result of Burnside ([5], p. 240) since they are not conjugate in $N_C(T) = T$. Suppose $Z(T)$ is of order 4 and let z be an element in $E \setminus \langle t \rangle$. Since z has 7 conjugates in C, $C_C(z)$ has order $2^7 \cdot 3$. Let Q be a Sylow 3-subgroup of $C_C(z)$ and let $\tilde{T}$ be a Sylow 2-subgroup of $C_C(z)$. It is clear that $\tilde{T}$ is also a Sylow 2-subgroup of G. We have $E \lhd \tilde{T}$ and so $\langle t, z \rangle = Z(\tilde{T}) = C_E(Q)$. Further we have $|C_C(Q)| = 2^2 \cdot 3$ and hence $N_C(Q)$ has order $2^3 \cdot 3$.

Let F^* be a Sylow 2-subgroup of $C_G(Q)$ which contains $\langle t, z \rangle$ and suppose by way of contradiction that $\langle t, z \rangle < F^*$ has a subgroup F_1 which contains $\langle t, z \rangle$ properly and $|F_1 : \langle t, z \rangle| = 2$. Since F_1 does not lie in C, F_1 is contained in $C_G(z)$ or in $C_G(tz)$ and so $|C_{C(z)}(Q) > 2^2 \cdot 3$ or $|C_{C(tz)}(Q)| > 2^2 \cdot 3$. But G has only one class of involutions and so this is impossible. Hence $C_G(Q)$ has order $2^2 \cdot 3^2 \cdot 5$. By a result of Gaschütz ([5], p. 26) Q splits in $C_G(Q)$ and so we may write $C_G(Q) = Q \times L$ where L is a group of order 60. From the order of the normalizer of a Sylow 5-subgroup of G (lemma 5.6) it follows that L is insoluble, and so L is simple. But then $L \cong A_5$ where A_5 is the alternating group on 5 letters. By a result of Gaschütz we may write $N_G(Q) = QK$ where $|K| = 2^3 \cdot 3 \cdot 5$, and so $L \lhd K$, where $L \cong A_5$ and $L \subseteq C_G(Q)$.

Let F be the Sylow 2-subgroup of $N_G(Q)$, then F must be Abelian since a dihedral group of order 8 cannot normalize a group of order 3. Consequently $K = L \times S$ where S is a group of order 2. But then G has elements of order 10, which is impossible. Hence a Sylow 2-subgroup of G has cyclic centre of order 2. We have proved:

Lemma 6.2. *The centralizer C of an involution t in the centre of a Sylow*

2-subgroup T of G is an extension of an elementary abelian group E of order 16 by a group H, $H \cong PSL(2, 7)$. Further the centre of T is cyclic.

It now follows from a result of Janko [8] that $G \cong M_{23}$.

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