

recent graduate, or enterprising final-year undergraduate, in mathematics. The reviewer's only adverse criticism—and that a minor one—is that the author has perhaps been too severe in limiting the scope of the book. Some further developments and indications of how the integral can be used (for instance, in constructing the  $L_1$ -algebra of a group) would have been welcome. But what the author has given us is certainly an attractive and useful account of the basic facts of the subject. His aim of arousing the reader's interest and stimulating him to further study should undoubtedly be achieved.

J. H. WILLIAMSON

HELSON, H., *Lectures on Invariant Subspaces* (Academic Press Inc., New York), \$5.00.

The material of this book is part classical analysis and part functional analysis; it is also in close relation with other branches of mathematics (for instance, stochastic processes). The basic problem is to characterise, for the space of square-integrable functions on the unit circle, those subspaces that are invariant under multiplication either by  $\exp ix$  and  $\exp -ix$  or by  $\exp ix$  alone. Previous treatments of this and similar problems have for the most part been function-theoretic in character (involving the interior of the unit circle). The author aims to develop methods that involve as few excursions from the unit circle as possible—to replace function-theory by harmonic analysis, essentially—and then to exploit these methods as far as he can. The appropriate tools are found in Hilbert space theory; elementary general theorems about subspaces, together with a little extra information about the special case under consideration, readily yield the required results. The treatment is an elegant example of what can be done by abstract methods in a concrete situation.

Of the eleven "lectures" the first four are devoted to the classical problem described above, and some further related matters. The fifth lecture deals with the similar problem in which the unit circle is replaced by the real line; here the results are similar, but the proofs required are different (and more complicated). The remainder of the book deals again with functions on the unit circle, but now taking values in a separable Hilbert space. Appropriately formulated analogues of the scalar theorems can be proved. Special mention should perhaps be made of the topic discussed in the tenth lecture; this is the general invariant subspace problem for a bounded operator in a separable Hilbert space, a subject of considerable current interest. It is shown that the general problem is equivalent to an apparently more special problem, in spaces of the type considered.

The writing is pleasantly informal, and there is no excess of detail to obscure the main lines of the argument. The book is to be welcomed as a well presented account of some recent research in an interesting and important field.

J. H. WILLIAMSON

HERMES, H., *Enumerability, Decidability, Computability. An Introduction to the Theory of Recursive Functions* (Mathematischen Wissenschaften Band 127, Springer-Verlag, 1965), x+245 pp., DM39.

There are several textbooks on recursive functions available in English and this translation from the German covers much the same material as in *Computability and Unsolvability* by M. Davis. The book begins with some introductory reflections on algorithms, then defines computable functions, recursive functions and shows their equivalence, and ends with some undecidability results and miscellaneous topics. The treatment of recursive functions is good.

The translation is good, the few faults are of little consequence. One of the good points of the text is that illustrative examples are introduced at an early stage. The result is a very readable book.

R. M. DICKER