INFLUENCE OF EPHEMERIS UNCERTAINTIES AND RELATIVITY MODELING ON LUNAR LASER RANGING DATA ANALYSIS

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ABSTRACT. In this paper authors pointed out that the root mean square of post-residual is two or three times larger than that of normal points. It is mainly due to the ephemeris uncertainties, i.e. the uncertainties of obliquity of the ecliptic and equinox and the inaccurary of lunar love numbers adopted. At present, with exception of the corrections of ocean tide and pole tide the theoretical modeling such as earth solid tide, lunar tide, plate motion and the relativity modeling, especially the space curvature caused by the earth, have been considered to match the present observing precision in the magnitude of centimeter. The equivalent of relativity modeling used in geocentric or barycentric reference frame is further described.

1. INTRODUCTION

Since the emplacement of retroreflectors on the lunar surface by Apollo astronauts in August 1969, laser ranging to the moon has been carried out for more than 20 years. Now the project of Lunar Laser Ranging(LLR) is being carried out at McDonald, USA(MCD) and CERGA, France (CER). Haleakala station, Hawaii, USA(HAL) stopped this project in 1990 due to a lack of funds from NASA. Satellites such as Lageos, Etalon, Starlette etc. are observed routinely and the attempts of LLR are made at Wettzell, Germany and Orroral station,Australia. One meter telescope was constructed by Communication Research Laboratory(CRL) in Japan where to range the reflectors on the moon can be done frequently. The pointing precision of 1.2 meter telescope attained to $2 - 3''$ and part of hardware for LLR was available at Yunnun Observatory, China. The present precisions of normal points (Jin et al. 1992) attain to 5 cm due to the technical improvements such as reduction of pulse width from 2.5 ns to 100 ps,use of micro channel plate etc.. There are five LLR data analysis centers in the International Earth Rotation Service(IERS), i.e. JPL, USA, CERGA, France, SHA, China, UTXMO, USA and Munich University, Germany.

The theoretical modeling of LLR data reduction is gradually perfected at Shanghai Observatory. Two lunar and planetary ephemerides were used in our work.

(A) before 1989	DE200/LE200
(B) in 1989	DE200 and lunar physical libration of Eckhardt 500
(C) after 1989	DE303/LE303

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The reason why physical libration of Eckhardt 500 was used is that in March 1989 the libration tape LE200 was ended to use . The pre-residual and post-residual of using different lunar and planetary ephemerides during the period of January 1987 to March 1989 are shown in Fig.1. There is large systematic difference by use of DE200 and the lunar physical libration of Eckhardt 500 in the pre-residuals because these are not produced by numerical integration of the equation of motions simultaneously. After adjustment of fourty parameters such as the coordinates of stations and reflectors, orbital elements of the moon and the earth, lunar moment of inertia ratios β and γ, coefficients of the third-degree harmonics in the lunar gravitational potential etc., the root mean square(rms) of post-residual should equal to that of normal point. As shown in Fig.1(b) the rms of post-residual is three or four times larger than the observing precision because of the imperfect theoretical model, uncertainties of ephemeris, the correlation between 40 elements of matrix and so on. It will be discussed in detail in the following section.

2. EPHMERIS UNCERTAINTIES

The model errors of propagation, refraction, solid earth tide, lunar tide and plate motion are almost less than 1 cm, but the lunar love numbers vary with various lunar models to a greater extent and it equals to the value itself as shown in Table 1.

Table 1. Different Lunar Love Numbers of Various Lunar Models

Author	Cheng and Toksez (1987)		Dazhang Han (1984)	Bodri (1986)	Chengzhi Zhang (1990)	
Model	homogeneous and two layers model		Bills and Ferrari	unhomogeneous density and compressible		
	A	B			luna 89-01	luna 89-02
h_2	0.0501	0.0627	0.0929	0.0503	0.04225	0.04676
k_2	0.0293	0.0335	0.0535	0.0281	0.02425	0.02680
l_2	0.0198	0.0240	0.0205	0.0133	0.01214	0.01236

The influence of lunar love numbers upon the distance from observing station to reflector was discussed in the poster paper of this meeting. When the moon was near perigee on September $6^d.23526$ UT, 1987 $\Delta\rho_2$ is 16.5 cm and 15.8 cm for reflector A14 and A15, while near the apogee of the moon on September $15^d.55072$ UT, 1987 it is 11.3 cm and 7.5 cm respectively. The average value is 12.8 cm.

Because the lunar laser ranging is sensitive to the direction of pole of the earth's rotation, i.e. sensitive to the of-date celestial equator and also sensitive to the of-date ecliptic from the solar perturbation upon the lunar orbit, the variation of theoretical distance from the station to the retroreflector on the moon is caused by the uncertainties of obliquity of the ecliptic $\Delta\varepsilon$ and that of equinox ΔE and is expressed as the following formula.

$$\begin{cases} \Delta\rho_1 = \overline{a}[(\cos\overline{\phi}\cos\overline{t}\sin\overline{\delta} - \sin\overline{\phi}\cos\overline{\delta})^2(\Delta\delta)^2 + (\cos\overline{\phi}\cos\overline{\delta}\sin\overline{t})^2]^{1/2} \\ (\Delta\overline{t})^2 = (\Delta\overline{\alpha})^2 = (\Delta E)^2 \\ \Delta\overline{\delta}\cos\overline{\delta} = (\cos\overline{\beta}\sin\overline{\lambda}\cos\epsilon - \sin\overline{\beta}\sin\epsilon)\Delta\epsilon \end{cases} \quad (1)$$

where $\overline{a}$ and $\overline{\phi}$ denote the geocentric distance and latitude of observing station; $\overline{t}$, $\overline{\alpha}$ and $\overline{\delta}$ indicate hour angle, right ascension and declination of retroreflector; $\overline{\lambda}$ and $\overline{\beta}$ indicate the longitude and latitude of reflector; ϵ denote the obliquity of the ecliptic and $\Delta\overline{t}$ and $\Delta\overline{\alpha}$ denote the uncertainty of hour angle and right ascension of reflector. For example,

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the variation of distance was calculated at 12^h UT every day of March 1987 and is shown in Fig.2 , by using formula (1) in which $\overline{a}$ is adopted as the average value of equatorial radius of the earth. the coordinates of lunar center is used instead of the coordinates of reflectors for calculation .hour angles are adopted as $\pm 0^h$ and $\pm 3^h$ respectively and $\Delta\varepsilon$ and ΔE are $\pm 0.''01$ at latitude of 20^o, 30^o and 40^o. In Fig.2 it is shown that $\Delta\rho$ induced by the uncertainties of ephemeris does not change too much with latitude while it varies with hour angle to a great extent.

In practical calculation $\Delta\varepsilon$ and ΔE are $\pm 0.''01$ and $\pm 0.''02$ which include the error of the precession constant about 0."1/cty(Standish 1987). For McDonald Observatory the maximum value of $(\Delta\rho_1)_{max}$ is 41.4cm induced by $\Delta\varepsilon$ and ΔE if t=45^o, $\lambda = -90^o$ and $\delta = \epsilon - i$ where i is the inclination of lunar orbit to the ecliptic while $(\Delta\rho_1)_a$ =33.6 cm at $5^h.3844$ UT on March 9, 1987. It is clear that the influence of ephemeris uncertainty on the distance from the observing station to reflector is 2.5 times larger than that of different lunar love numbers adopted.

Status of the precision of LLR data obtained from 1970 to 1987 is shown in Table 2(Nie et al. 1991). In this period DE200/LE200 was used for LLR data analysis at Shanghai Observatory.

Table 2. Precision of LLR unit: cm

Station	MCD	CER	MLRS	HAL	Average
N	3338	1548	620	480	
$\Delta\rho_{ob}$	13.77	19.41	7.06	4.33	13.78
$\Delta\rho_{pre}$	51.41	58.41	55.57	49.53	53.63
$\Delta\rho_{post}$	46.07	44.69	33.72	23.48	42.62

where N, $\Delta\rho_{ob}$, $\Delta\rho_{pre}$ and $\Delta\rho_{post}$ denote the number of normal point, root mean square (rms) of normal point, of pre-residual and of post-residual.

$$\begin{aligned} \overline{m} &= (\Delta\rho^2_{pre} - \Delta\rho^2_{ob})^{1/2} \\ \overline{n} &= (\Delta\rho^2_{post} - \Delta\rho^2_{ob})^{1/2} \end{aligned} \tag{2}$$

From Table 2 $\overline{m}$ and $\overline{n}$ can be calculated with formula (2) for all stations and the results are 51.83cm and 40.33 cm respectively. For MCD station the corresponding values of m and n are 49.53cm and 43.96 cm.

If the main source of error is caused by the orientation error of ephemeris reference frame and the inaccurary of lunar love numbers, the sum of the maximum error is expressed as follows

$$\Delta\overline{\rho}_{max} = [(\Delta\overline{\rho}^2_1)_{max} + (\Delta\overline{\rho}^2_2)_{max}]^{1/2}$$

The value of $\Delta\overline{\rho}_{max}$ should be consistent with the following condition:

$$\overline{n} \leq \Delta\overline{\rho}_{max} < \overline{m}$$

For general situation, $(\Delta\overline{\rho})_a = [(\Delta\overline{\rho}_1)^2_a + (\Delta\overline{\rho}_2)^2_a]^{1/2} < \overline{n}$. The data from Mcdanald observatory is 56% of all LLR data, so

$$\begin{aligned} & n \leq \Delta\rho_{max} \leq m,\ \Delta\rho_a < n \\ \text{and}\quad & \overline{n} \leq \Delta\rho_{max} \leq \overline{m},\ \Delta\rho_a < \overline{n} \end{aligned}$$

According to the previous analysis $(\Delta\rho_1)_{max} = 41.4$ cm , $(\Delta\rho_1)_a = 33.6$ cm , $(\Delta\rho_2)_{max} =$ 16.5 cm and $(\Delta\rho_2)_a = 12.8$ cm, so $(\Delta\rho)_{max} = 44.6$ cm and $\Delta\rho_a = 36.0$ cm should be obtained. These results are consistent with analysis mentioned above so the theoretical analysis concerned the error of the post-residual is correct. Although IERS recommended

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that DE200/LE200 should be used in analysis of SLR and VLBI, in LLR the parameters of earth-moon system should be fitted or a more recent lunar ephemeris should be used. Because of the uncertainties of ephemeris and the correlation between elements of matrix we have our preference to adopt later suggestion of IERS.

3. RELATIVITY MODELING IN LLR DATA ANALYSIS

Our software of reducing the LLR data has gradually perfected for the relativity model. Recently the space curvature of earth term and Lorentz contraction (Xu et al. 1993, in press) was added. In the reduction of LLR data three kinds of relativety corrections should be considered.

3.1. Transformation of Epoch and Time Interval

The epoch and time interval of a range to the moon are recorded at the station in UTC. Since the reference frame workshop of IAU was established, the concept of time has been discussed. For the barycentric and geocentric reference frame now the TCB and TCG are suggested to be used. Because the time argument of the planetary ephemeris of DE200 or DE303 is TDB, observing epoch UTC will be transformed into TDB for reduction of LLR data. There are two formulae to transform TDT to TDB, i.e. Moyer's and Fairhead's. The precision of using two formula are 1 μs and 100 ns respectively. Dr. Fairhead (Fairhead 1987) gave the more precise formula than Moyer did, but Moyer's formula has matched the present observing precision of LLR and is still used with the overlap method to obtain the range epoch in solar barycentric system.

3.2. Spatial Coordinates Transformation

Because of the computation of LLR data in solar barycentric system the positional vector $\vec{r}$ of station should be transformed from geocentric system into barycentric system. The tranformation factor of scale is about 15 cm in the height of tracking station and the Lorentz contraction is about 3 cm for station coordinates. The selenocentric coordinate of reflector should be transformed first into geocentric frame. This corrected value is less than 3×10^{-5} m, which can completly be negeleted. Then, the scale effect is 4 cm and the maximum value of Lorentz contraction effect (from special relativity) is about 1 cm for reflector coordinates to be transformed from geocentric system to barycentric system. Since the maximum influence of the moon and the major planet for Ψ,the geocentric gravitational potential, is about 2×10^{-5} m , only the solar term should be considered.

3.3. Space Curvature

Due to the gravitational influence of the sun, the earth and the moon the space curvature in different level of neighbouring space exist. As for this reason the time interval of pulse between the station and the retroreflector is increased. The gravitational time delay induced by the Sun, Earth, Moon and Jupiter are listed in Table 2.

Table 2. Estimation of Gravitational Time Delay for LLR in Two-way

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gravitational body	Sun	Earth	Moon	Jupiter
Δt_{grav}	5×10^{-6} s	2×10^{-10} s	3×10^{-12} s	1×10^{-11} s
c Δt_{grav}	16 m	7 cm	1 mm	4 mm

To match the present observing precision in the magnitude of centimeter the space curvature caused by the earth should be considered. After improving the precision of LLR to millimeter the other terms such as the gravitational time delay induced by the Moon and the Jupiter should be considered too.

Dr. Han theoretically proved the equivalent of SLR or LLR reduction in solar byrycenter and geocentric reference system with general relativistic theory (Han 1990). This point was confirmed in processing SLR observational data(Hunag 1990). Obviously the observation of LLR is in geocentric reference. If the lunar and planetary epheremis in geocentric reference frame are provided, it will be convenient for processing LLR data in geocentric frame.

4. CONCLUSION

4.1. The uncertainty of epheremis is main error source of present reduction of LLR. To use the more recent lunar and planetary ephemeris for LLR data processing is suggested.

4.2. To match the present observing precision (in magnitude of centimeter) the effect of space curvature from the earth should be considered.

4.3. Reduction of LLR data in the solar system is equivalent to that in geocentric system. According to the planteary ephermeris in solar barycentric or geocentric system the reference frame of LLR data reduction will be chosen.

5. ACKNOWLEDGEMENTS

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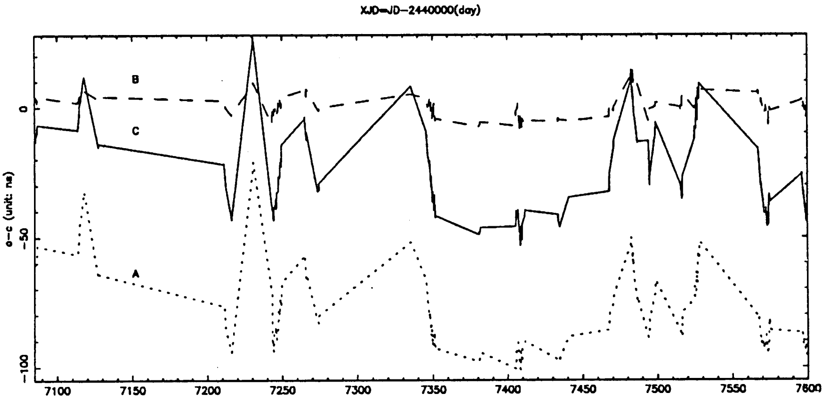

Fig.1(a) Pre-residual of reflector No.3 observed at CER station

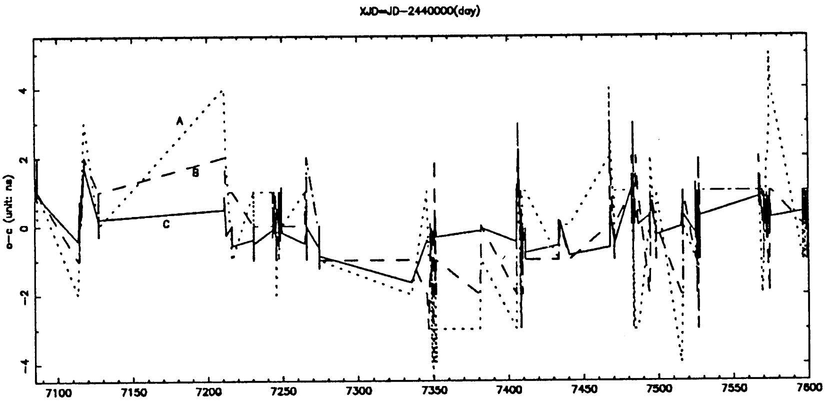

Fig.1(b) Post-residual of reflector No.3 observed at CER station

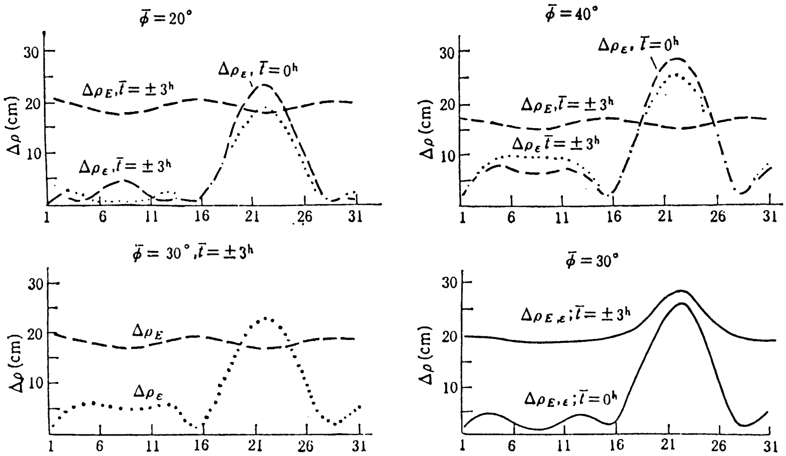

Fig.2 History of the distance errors resulting from the uncertainties of the ephemeris reference frame (The uncertainties of both the obliquity of the ecliptic and the equinox are $\pm0.''01$ and the curves of the distance errors dependent on every day of March, 1987 are also given)

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