A REVIEW OF GEODETIC PARAMETERS

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RÉSUMÉ. — L'auteur recommande qu'on choisisse comme valeurs conventionnelles provisoires des paramètres, celles qui sont le plus utilisées dans les calculs d'orbites, plutôt que les meilleures possibles, dont elles diffèrent d'ailleurs peu. Il remarque que les erreurs les plus importantes d'origine géodésique affectant les constantes astronomiques portent sur les positions des stations d'observation. Il propose des méthodes types pour décrire les positions de ces stations.

ABSTRACT. — It is recommended that the parametric values which are currently most used in orbital computation be adopted as provisional standards rather than those which may be the best available and from which they differ only slightly. It is remarked that the most serious geodetic errors affecting astronomy are tracking station positions. Standard methods of describing positions are suggested.

ZUSAMMENFASSUNG. — Es wird empfohlen, die Werte der Parameter vorläufig anzunehmen, die in der Bahnrechnung am häufigsten verwendet werden, und nicht diejenigen Werte, die gerade als die besten bekannt sind, zumal sie von diesen nur wenig abweichen. Ferner wird erwähnt, dass die geodätischen Fehler, welche sich auf die astronomischen Ergebnisse am stärksten auswirken, diejenigen in den Örtern der Beobachtungsstationen sind. Zur Angabe dieser Örter werden verbindliche Methoden vorgeschlagen.

Резюме. — Автор рекомендует выбирать в качество предварительных условных значений параметров, не возможно наилучшие значения, а предпочтительно тѐ, которые чаще всего употребляются в вычислениях орбит. К тому-же эти значения мало отличаются друг от друга. Автор отмечает, что самые значительные ошибки геодезического происхождения, которые влияют на астрономические постоянные, являются положения станций где ведутся наблюдения. Он предлагает стандартные методы для определения этих положений.

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Introduction. — This review recommends which geodetic parameters should be adopted as standard, the manner in which the parameters should be expressed, and the values which should be adopted. In making these recommendations, current practice, available determinations, and anticipated improvements will be considered.

Gravitational parameters. — For the notation of the Earth potential, recommendations have already been made by Commission No. 7 on Celestial Mechanics, of the International Astronomical Union [1] :

$$(1) \quad U = \frac{\mu}{r}\left[1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^n P_n^m(\sin\beta)\,(C_{n,m}\cos m\lambda + S_{n,m}\sin m\lambda)\right],$$

where $\mu = GM_{\oplus}$, r is the distance from the center of the Earth, R is the mean equatorial radius of the Earth, P_n^m is the associated Legendre polynomial, β is the latitude, and λ is the longitude. Alternative notations recommended for the gravitational coefficients are

$$(2) \qquad J_n = -C_{n,0}$$

and

$$(3) \qquad (A_{n,m}, B_{n,m}) = \left[\frac{(n+m)!}{(n-m)!}\right]^{\frac{1}{2}} (C_{n,m}, S_{n,m}).$$

These two additions are suggested :

1. Define

$$(4) \qquad (\overline{C}_{n,m}, \overline{S}_{n,m}) = \left[\frac{(n+m)!}{(n-m)!\,(2n+1)\,(2-\delta_m^0)}\right]^{\frac{1}{2}} (C_{n,m}, S_{n,m}),$$

where the Kronecker delta δ_m^0 is 1 for $m = 0$ and 0 for $m \neq 0$. The $\overline{C}_{n,m}$, $\overline{S}_{n,m}$ are coefficients of harmonics which have a mean square amplitude of 1 for all values of n and m.

2. Define the mean equatorial radius more precisely as the equatorial radius of the mean Earth ellipsoid, i. e., the ellipsoid of revolution which best fits the geoid. This definition is consistent with geodetic practice and involves the equatorial radius with only two of the set of orthogonal parameters defining the radius vector of the geoid — the zeroth and second degree zonal harmonics. (The more literal definition of the mean equatorial radius as the radius of the circle which best fits an equatorial section through the geoid would connect the radius to the infinite set of even degree zonal harmonics.) An alternative possibility for the equatorial radius in equation (1) is the mean radius of the entire Earth which, since it differs by a factor of 10^{-3}, would affect the value of J_2. The mean

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radius seems slightly preferable aesthetically, but current practice overwhelmingly favors the equatorial radius; a perusal of some papers on close satellite dynamics and orbit analysis found ten workers using the equatorial radius but none using the mean radius (in addition, five theoreticians did not define their radius).

To be consistent with the connection of equatorial radius to the mean Earth ellipsoid, it is recommended that the following be the relationships between the astronomical parameters $\mu = GM_{\oplus}$ and $J_2 = -C_{2,0}$ and the geodetic parameters $R = a_e$, the equatorial radius; γ_e, the equatorial gravity; f, the flattening; and ω, the rate of the Earth's rotation with respect to inertial space ([2], [3], [4]) :

$$(5) \qquad GM_{\oplus} = a_e^2 \gamma_e \left[1 + \frac{3}{2} m - f - \frac{15}{14} mf - \frac{1}{294} mf^2 - O(f^4) \right],$$

$$(6) \qquad J_2 = \frac{2}{3} f \left(1 - \frac{1}{2} f \right) - \frac{1}{3} m \left[1 - \frac{3}{2} m - \frac{2}{7} f + \frac{9}{4} m^2 + \frac{11}{49} f^2 + O(f^3) \right].$$

where

$$(7) \qquad m = \frac{\omega^2 a_e}{\gamma_e}.$$

The values of $GM_{\oplus}$ and J_2 which are probably the most extensively used at orbit computation centers in the United States are ([5], [6], [7]) :

$$(8) \qquad \begin{cases} GM_{\oplus} = 3.986032 \pm 0.000030 \times 10^{20} \text{ cm}^3.\text{s}^{-2}, \\ J_2 = 1082.30 \times 10^{-6}. \end{cases}$$

In the alternative notation of Herrick, Baker, and Hilton [8] and Makemson, Baker, and Westrom [9] :

$$(9) \qquad k_e = (GM_{\oplus})^{\frac{1}{2}} = 0.01996 5049 \text{ Mm}^{\frac{3}{2}}.\text{s}^{-1}.$$

The values of $GM_{\oplus}$ and J_2 in equation (8) are consistent with these values for the geodetic parameters

$$(10) \qquad \begin{cases} a_e = 6\,378\,165 \pm 25.0 \text{ m}, \\ \gamma_e = 978.0300 \pm 0.0012 \text{ cm}.\text{s}^{-2}, \\ f = \dfrac{1}{298.30}, \\ \omega = 0.72921 15085 \times 10^{-4} \text{ s}^{-1}. \end{cases}$$

The value for a_e is a compromise between the solutions of Fischer [10], and Kaula [11], and other values which are unpublished. The γ_e value differs from that of the International Formula and the Potsdam System ($978.0490\ \mathrm{cm.s^{-2}}$) in three ways :

1. Correction to Potsdam System absolute g [12] : -0.0128 ± 0.0003;

2. Change of flattening from $\frac{1}{297}$ to $\frac{1}{298.3}$: -0.0051;

3. Change of mean gravity over the Earth's surface [12] : -0.0005 ± 0.0012.

The correction to absolute g is a provisional value and has not been adopted by the International Union of Geodesy and Geophysics; an improved value should be forthcoming within the next few years from several determinations in progress [13]. The correction to mean gravity is negative, mainly because correlation between gravity and topography was used to estimate anomalies for the areas without observations, which are predominantly oceans. Solutions by Uotila which fit observed gravimetry and do not use correlation with topography give positive corrections ranging from $+0.0004$ to $+0.0019\ \mathrm{cm.s^{-2}}$ [14]. Rather slow improvement is expected; problems in observing gravity at sea are not entirely solved ([15], [16]). Some improvement may also come from using the better statistical techniques which larger capacity computers permit.

The value of $GM_\oplus$ may also be obtained through the modified Kepler equation by using the radar mean distance of the Moon A and the Moon's mean motion n :

$$GM_\oplus = \frac{n^2(1+\beta)^3}{1+\frac{\mu_M}{\mu_E}} A^3, \tag{11}$$

where β is the solar perturbation of the mean semi-major axis and $\frac{\mu_M}{\mu_E}$ is the ratio of the Moon's mass to the Earth's mass, equal to the lunar inequality [17]. The most recently published value for A is $384\,400.2 \pm 1.1$ km [18]. This value is dependent on an assumed lunar radius of 1738.0 km, which is approximately equal to the mean radius of the lunar limb. Geometrical determinations of the radius toward the Earth vary considerably; Baldwin's conclusion [19] leads to 1740.2 km, whereas Schrutka-Rechtenstamm [20] concludes that the bulge is too small to be determined. However, we are not interested in just the long axis of a best-fitting triaxial ellipsoid, but rather in the mean radius of the area contributing to the leading edge of the radar return pulse, which would fall within the $\pm 7°$ area of libration. Contour maps of the Moon ([21], for example) indicate that the average radius of this $\pm 7°$ area could differ by as much as 2 km from the best-fitting ellipsoid. If the lunar surface is assumed to be an equipotential surface, then using the moments of inertia obtained from the physical libration yields 1738.7 km as the radius toward the Earth. Letting

$$A = 384\,400.9 \pm 1.1 \text{ km}, \qquad \beta = 0.00090768, \qquad n = 2.6616997 \times 10^{-6}\ \text{s}^{-1} \quad [22],$$

and

$$\frac{\mu_M}{\mu_E} = \frac{1}{81.3015 \pm 0.0033} \quad [23]$$

gives

$$(12) \qquad GM_{\oplus} = 3.986057 \pm 0.00004 \times 10^{20}\ \text{cm}^3.\text{s}^{-2}.$$

The value $\frac{1}{81.3015}$ obtained from analysis of radio tracking of *Mariner II* (1962 αρ 1) falls near the center of the range of values obtained by combining various determinations of the lunar inequality from Eros with the astronomical unit derived from radar measurements : $\frac{1}{81.36}$ [24] to $\frac{1}{81.26}$ [25]. Hence the lunar radius is probably the major source of uncertainty in $GM_{\oplus}$.

The recent rediscussion of the triangulated distance of the Moon by Fischer [26] obtains 384 413.2 km from Crommelin's data and 384 400.9 km from O'Keefe's data. Although there are difficulties of interpretation of Crommelin's paper, no convincing explanation was found for the discrepancy.

A new source of information about $GM_{\oplus}$ is the mean motion of artificial satellites and probes. For vehicles tracked by radio, as used by Hamilton and others [23] and by Anderle and Oesterwinter [27], the scale factor comes from the velocity of light, which induces an uncertainty of about $\pm 5 . 10^{-6}$ in the derived $GM_{\oplus}$; observational errors, including ionospheric refraction, are probably a larger source of error, however. For vehicles tracked by camera, as used by Kaula [28], the scale factor comes from the geodetic triangulation connecting the tracking stations, so the determination is not independent from those based on terrestrial data. The determinations by Anderle and Oesterwinter and by Kaula are by-products of analyses for tesseral harmonics of the gravitational field and for tracking station positions, so non-uniform distribution of observations may introduce some distortion. Error in the tracking station position used by Hamilton and others may have some effect if observations taken while the probe was less than a few thousand kilometers away have appreciable weight.

Various recent determinations of $GM_{\oplus}$ are summarized in table I. It appears that the value of $3.986032 \times 10^{20}\ \text{cm}^3.\text{s}^{-2}$ is a good compromise, and that the Anderle and Oesterwinter and the Fischer-Crommelin determinations are the first places to look for systematic error.

TABLE I.

Determinations of $GM_\oplus$.

Method.	Reference.	Sources of Error.	Result (10^{20} cm^3.s^{-2}).
Terrestrial Geodesy	Kaula [11] Kaula and Uotila [14]	Triangulation Gravimetry	3.986020 ± 0.000028 3.986043
Lunar motion and radar distance...	Yaplee and others [18]	Lunar radius	3.986057
Lunar motion and triangulated distance	Fischer-Crommelin [26] Fischer-O'Keefe	Lunar radius Triangulation	3.986457 3.986078
Lunar probe and Doppler....	Hamilton and others [23]	Observational Station position	3.986016 ± 0.000025
Close satellite and Doppler....	Anderle and Oesterwinter [27]	Observational Orbit perturbations	3.985889
Close satellite and Camera....	Kaula [5] : 1960 ι 2 Kaula [28] : 1961 αδ	Triangulation Orbit perturbations	3.986037 ± 0.000012 3.985993 ± 0.000011

The uncertainties quoted in table I are those given by the author as based upon internal consistency. For the solutions utilizing the lunar motion and distance, $\frac{\mu_M}{\mu_E} = \frac{1}{81.3015}$ and a lunar radius of 1738.7 km were used; if it is assumed that the uncertainty resulting in the lunar distance is ± 1.2 km, then the uncertainty in the derived $GM_\oplus$ is $\pm 4.10^{15}$ cm^3.s^{-2}.

In addition to $GM_\oplus$ and J_2, standard orbit computation programs usually incorporate J_3 and J_4. The values which are probably most commonly used at United States computation centers are [6] :

$$(13) \qquad \begin{cases} J_3 = -2.3 \times 10^{-6}, \\ J_4 = -1.8 \times 10^{-6}. \end{cases}$$

At present, the best values of the odd zonal harmonics are undoubtedly those of Kozai [7] :

$$(14) \qquad \begin{cases} J_3 = -2.562 \pm 0.012 \times 10^{-6}, & J_5 = -0.064 \pm 0.019 \times 10^{-6}, \\ J_7 = -0.470 \pm 0.021 \times 10^{-6}, & J_9 = 0.117 \pm 0.025 \times 10^{-6}. \end{cases}$$

There are two major recent determinations of the even zonal harmonics by Kozai [7] and by King-Hele, Cook and Rees [29] :

Kozai.	King-Hele, Cook and Rees.
$J_2 = 1082.48 \pm 0.06 \times 10^{-6}$,	$J_2 = 1082.86 \pm 0.1 \times 10^{-6}$,
$J_4 = -1.84 \pm 0.08 \times 10^{-6}$,	$J_4 = -1.03 \pm 0.2 \times 10^{-6}$,
$J_6 = 0.39 \pm 0.12 \times 10^{-6}$,	$J_6 = 0.72 \pm 0.2 \times 10^{-6}$,
$J_8 = -0.02 \pm 0.02 \times 10^{-6}$,	$J_8 = 0.34 \pm 0.2 \times 10^{-6}$.

King-Hele, Cook and Rees also estimated a J_{10} and a J_{12}. It is disappointing at this late date that the two determinations do not agree better. The principal difference in method is that Kozai uses secular motions of both perigees and nodes, while King-Hele, Cook and Rees use only nodal motion. Since the difference is small for practical application, N. A. S. A. computing centers are continuing to use the values specified by equations (8) and (13) until these differences are resolved.

Most of the current close satellite orbit analyses for geodetic purposes seek tesseral harmonic perturbations. In view of the smallness of these perturbations, it does not seem appropriate to adopt standardized values for the tesseral harmonics $C_{n,m}$, $S_{n,m}$. The one exception might be $C_{2,2}$, $S_{2,2}$, for which an upper limit would be useful because of its effect on supplemental energy requirements for 24 h orbits. The most recent determinations of tesseral harmonics obtain for these two coefficients :

Kaula [28] :

$$C_{2,2} = 1.05 \times 10^{-6}, \qquad S_{2,2} = -0.98 \times 10^{-6};$$

Izsak [30] :

$$C_{2,2} = 0.97 \times 10^{-6}. \qquad S_{2,2} = -0.40 \times 10^{-6};$$

Anderle and Oesterwinter [27] :

$$C_{2,2} = 1.84 \times 10^{-6}, \qquad S_{2,2} = -0.98 \times 10^{-6}.$$

Geometrical parameters. — As shown by analyses involving large systems of observations (10], [11], [26]), the equatorial radius is a derived, rather than a fundamental, quantity : accurate knowledge of the radius is not necessary to obtain other parameters, such as the lunar distance, geoid undulations, or datum positions by fitting of the astro-geodetic to the gravimetric geoid. However, for astronomical purposes, it is desirable to have a reference ellipsoid correct within $\pm$ 50 m in order to obtain reasonably correct positions of isolated tracking stations from astronomic latitude and longitude. Also it is convenient to have a unit of length approximating the Earth's radius for use in the potential formula [equation (1)] and for use as a base line to compare or combine parallax observations. For these astronomical purposes, the value of 6 378 165.0 m given in equation (10) should be entirely adequate. Marked improvement is not expected for about 5 years, by which time satellite observations should contribute significantly to the strengthening of triangulation systems and to the interconnection of geodetic datums.

By far the most annoying problems in the astronomical application of geodetic data pertain to tracking station positions. Errors in the adopted values of station positions, in conjunction with drag and non-uniform distribution of observations, prevent accurate determination of tesseral harmonics and are even believed to be a major cause of discrepancies in space probe trajectories [31]. These station position errors are due to both inadequate data and mistaken treatment of data; in descending order of reprehensibility they include :

1. Weak, erroneous, or nonexistent connection of tracking stations to local geodetic control (this includes the moving of antennas by stations without informing the computing center);

2. Failure to state the datum or ellipsoid to which tracking station positions refer;

3. Use of obsolete or erroneous standard datum and ellipsoid;

4. An incomplete or ambiguous statement about how datum or ellipsoid transformations were made;

5. Failure to provide for geoid-ellipsoid difference in calculating heights;

6. Neglecting systematic error due to incorrect observation (for example, no Laplace stations) or incorrect adjustment (for example, arbitrary scale changes or rotations) of geodetic control connecting tracking stations more than, say, 1000 km apart;

7. Actual observational error of position.

In view of the number of geodetic datums and corrections thereto, they do not seem to be appropriate parameters to be adopted as standard by an international organization, except possibly for the large continental triangulation systems. The corrections to coordinates u, v, w with positive axes directed respectively toward latitude and longitude (0°, 0°), (0°, 90°E), (90° N) obtained in the world geodetic system solution of Kaula [11] are listed in table II, where NAD, ED, and TD refer to the North American, European, and Tokyo datums, respectively. The uncertainties in this table are based on estimates of the errors due to interpolation and representation in the astro-geodetic and gravimetric geoids, and are probably a fair measure of item 7 on the above list, but may neglect significant effects falling under item 6. The relationships of the rectangular co-ordinates u, v, w to the geodetic latitude φ, longitude λ, and elevation h, referred to an ellipsoid of parameters a_e and f, are

$$(15) \qquad \left\{ \begin{aligned} u &= (\nu + h)\cos\varphi\cos\lambda, \\ v &= (\nu + h)\cos\varphi\sin\lambda, \\ w &= [(1 - e^2)\,\nu + h]\sin\varphi, \end{aligned} \right.$$

where

$$\nu = \frac{a_e}{(1 - e^2 \sin^2 \varphi)^{\frac{1}{2}}} \quad \text{and} \quad e^2 = 2f - f^2.$$

TABLE II.

Corrections to u, v, w from [11] (meters).

Datum shift.	Δu.	Δv.	Δw.
WGS-NAD..........	-23 ± 26	$+142 \pm 22$	$+196 \pm 22$
WGS-ED...........	-57 ± 23	-37 ± 29	-96 ± 23
WGS-TD............	-89 ± 40	$+551 \pm 53$	$+710 \pm 40$

To help minimize the number of unnecessary errors in categories 1 through 5 on the above list, it is suggested that organizations be urged to publish the following information pertaining to each tracking station for which they publish any precise observations of artificial satellites or probes, or orbital data based thereon :

1. The names and co-ordinates of local geodetic control points, both horizontal and vertical, to which the tracking station is connected;

2. The geodetic datum and ellipsoid to which the horizontal co-ordinates refer;

3. The organization which established the local geodetic control points;

4. The manner in which the horizontal and vertical survey connections were made from the local control points to the tracking station;

5. The date of the survey connection and a description of the termination point of the survey;

6. The geodetic (φ, λ, h) and rectangular (u, v, w) co-ordinates of the station referred to the local geodetic datum;

7. A statement of the geoid height, if any, estimated for the station and the basis for the estimate;

8. If the tracking station position has been shifted for the purpose of referring observations (direction cosines or altitude and azimuth) or calculating orbits, the geodetic and rectangular co-ordinates after the shift and the ellipsoid to which the new co-ordinates refer.

Every item on this list is an action which must be accomplished for any tracking station, but thus far the Smithsonian Astrophysical Institute Baker-Nunn camera network is the only one for which even part of the list has been published [32]. It is symptomatic of the difficulties which occur that, since this publication, the co-ordinates for at least four of the twelve Baker-Nunn cameras have been found to be in error by 20 m or more. These geometrical details of tracking station position are rather uninteresting, but they must be examined carefully and determined correctly if the full potentialities of modern tracking techniques are to be realized.

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