IV. ORIGINS OF THE MOON AND SATELLITES

(Edited by G. Contopoulos)

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S. K. Runcorn (Chairman), E. Anders, A. Dollfus, G. H. Pettengill, J. Ring, H. Wood

ON THE GROWTH OF THE EARTH-MOON SYSTEM

F. L. WHIPPLE

Smithsonian Astrophysical Observatory and
Harvard College Observatory, Cambridge, Mass., U.S.A.

This paper elaborates the postulate that the Earth and Moon became a binary system during their accretional development and that the Moon's growth was essentially completed before the assumed solar nebula dissipated. The solar nebula was still hot enough at the formation of the two bodies that both consisted largely of the refractory and relatively low-density minerals now characteristic of the Moon. During the subsequent condensation, agglomeration and accretion of siderophile and more volatile higher density minerals, the Earth grew very much faster than the Moon because of (a) its much greater gravitational capture area coupled with retention by a sizeable atmosphere and (b) the Moon's velocity, with respect to the solar nebula, which produced a wind that aerodynamically blew away volatiles and smaller debris resulting from hypervelocity impacts of larger planetesimals. This 'impact differentiation' process favored the retention of the refractory minerals on the Moon (Figure 1). The Moon's surprisingly high moment of inertia follows naturally from the basic postulate.

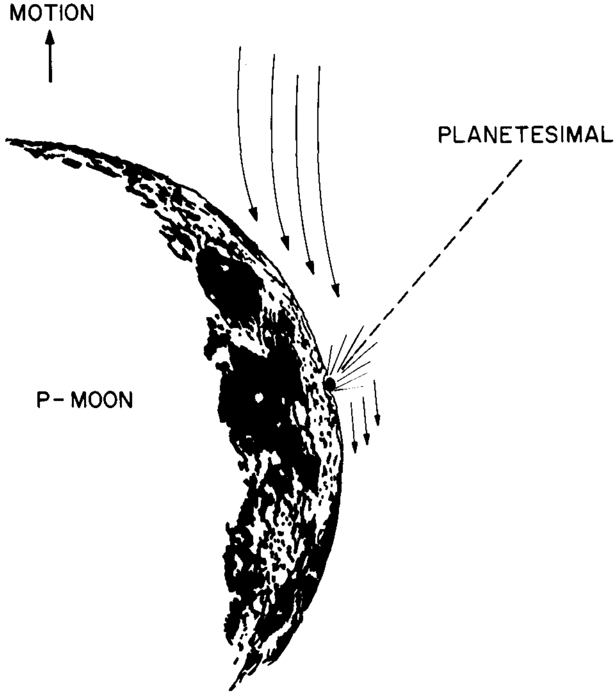

Fig. 1. Impact differentiation.

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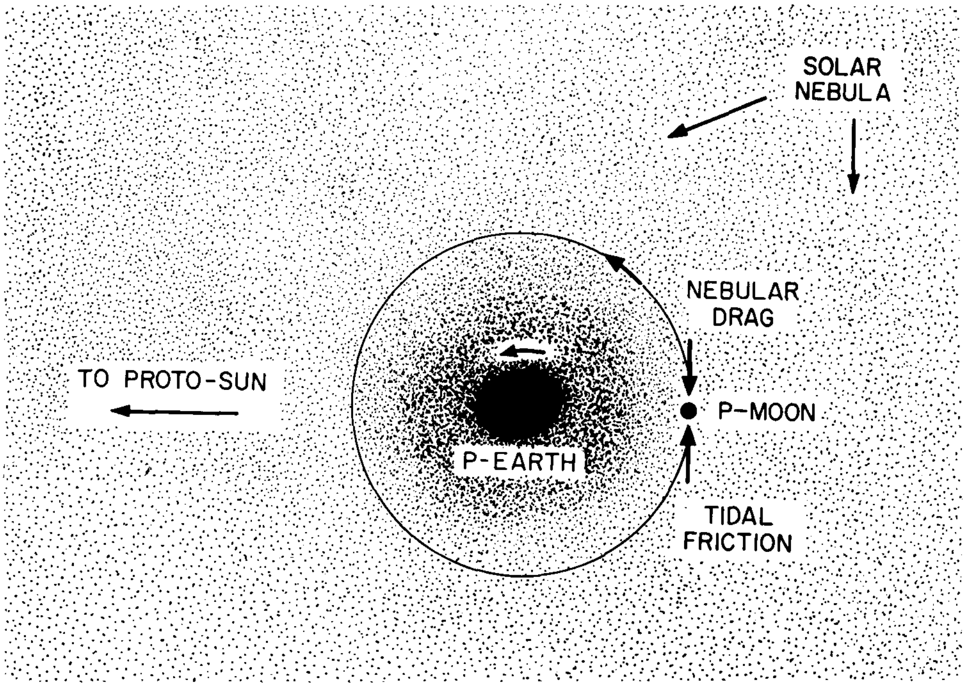

Fig. 2. Proto-Earth and Moon in Solar nebula.

A sizeable velocity of the Moon with respect to the solar nebula may have arisen in two ways, which could have been sequential. If the proto-Moon formed near the proto-Earth's orbit, the perturbations by the Earth's mass would have forced it into a rather eccentric and inclined orbit before subsequent capture. (See detailed discussion of such motions by Safronov, 1969.) After capture, possibly by collision with the Earth as discussed by Öpik, its orbital velocity was significant as will be discussed below. The proto-Moon may conceivably have been foımed in a ring-system about the proto-Earth while the solar nebula was still hot.

Regardless of the early history, the proto-Moon, once in orbit about the proto-Earth, was subject to drag by the solar nebula causing it to spiral in towards the proto-Earth. The opposed force of 'tidal friction,' however, produced a quasi stable orbit in which drag and tidal-friction forces were balanced (Figure 2).

The drag acceleration on the proto-Moon (mass, M_m; density, ϱ_m; radius, R_m) at distance, a, and velocity V_m with respect to the proto-Earth (M_e, ϱ_e, R_e) in a solar nebula of density ϱ, would be of the classical Newtonian form:

$$\text{Acceleration} = -\frac{\varrho v_m^2}{6\varrho_m R_m} = -\frac{\mu\varrho}{6a\varrho_m R_m}, \tag{1}$$

where $\mu = G(M_e + M_m)$ neglecting the effect of lunar gravity on the density and velocity of the nebular gas.

Hence the rate of change of a in a nearly circular orbit became

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{\mu^{1/2}\varrho a^{1/2}}{3\varrho_m R_m}, \tag{2}$$

neglecting terms in M_m/M_e.

The tidal acceleration by the proto-Earth with polar axis at some moderate angle to the orbital pole and rotation period much shorter than the orbital period would (e.g. MacDonald, 1964) produce an acceleration in a given by

$$\frac{da}{dt} = 0.9\,\frac{\mu^{1/2}}{a^{11/2}}\,\frac{M_m}{M_e}\,R_e^5 \sin 2\delta, \tag{3}$$

for Love Number$=0.30$, where δ is the assumed tidal lag and terms in M_m/M_e are neglected.

By equating the nebular drag to the tidal friction as given by Equations (2) and (3) respectively, we solve for the orbital parameter, a_s, in the case of quasi-stable equilibrium:

$$a_s^6 = 2.7\,\frac{\varrho_m^2}{\varrho\varrho_e}\,R_m^4 R_e^2 \sin 2\delta. \tag{4}$$

The tidal lag, $\sin 2\delta$, in Equation (4) could alternatively be expressed as an energy dissipation term, but neither expression for tidal friction can be confidently evaluated under the assumed physical circumstances. The present-day value of δ is $2\overset{\circ}{.}16$, which as MacDonald shows, leads back in time to the Roche limit in 1.7×10^9 yr, much shorter than the age of the Earth and Moon. With little uncertainty in $a_s (a_s \sim \delta^{1/6})$ we may reasonably adopt a tidal friction rate for the proto-Earth and Moon one tenth the present rate so that the dimensionless term $2.7 \sin 2\delta$ in Equation (4) becomes 0.020.

The condition that the proto-Moon cannot exist for a_s within the Roche limit of the proto-Earth provides an upper limit to R_e/R_m in Equation (4) shortly after capture or at formation of the binary system. Let us adopt $a_s > 2.455\,(\varrho_e/\varrho_m)^{1/3} R_e$ for the Roche limit and apply it to Equation (4). Then

$$\left(\frac{R_e}{R_m}\right)^4 < \frac{0.020}{(2.455)^6}\,\frac{\varrho_m^4}{\varrho\varrho_e^3}, \tag{5}$$

represents an upper limit to the ratio R_e/R_m for the quasi-equilibrium condition stated by Equation (4).

Basic to any numerical calculations is the density, ϱ, to be assumed for the solar nebula in the neighborhood of the proto-Earth. Abundances of the elements by Urey (1972) lead to the mass distribution and molecular weights given in Table I for materials divided into the classes *gaseous* (H, He, noble gases), *icy* (hydrides of C, N, O) and *earthy* (oxides of heavier elements). At the proto-Earth, following Larimer and Anders (1967), the temperature is taken as 550K. At this temperature the gas plus ice mixture would be gaseous with an adopted mean molecular weight of 2.37 (Table I).

The assumption that the Earth-Moon system was essentially complete at the time of dissipation of the solar nebula leads to a minimum density nebula, if we assume a central mass about equal to the present Sun. Thus the total surface density across the solar nebular disk at the Earth's distance is the order of 6×10^3 gm cm^{-2}. at $T = 550$K during most of the accumulation of the Earth if the mass fraction of *Earth* in the nebula is 0.004. This corresponds to an Earth mass in the zone half-way to Venus and to Mars. The space density of the *gas* and *ice* mixture at the proto-Earth's distance then becomes $\varrho = 3.7 \times 10^{-9}$ gm cm^{-3}.

TABLE I

Material	Solar abundances	
	Mass	Mol. Wt.
Gaseous	0.976	2.33
Icy	0.020	17.2
Earthy	0.004	45.0
Gaseous and icy	0.996	2.37

Application of Equation (5) with the value of ϱ leads to a minimum ratio of the radii, proto-Earth to proto-Moon, at the earliest stage of their binary formation, $R_e/R_m < 13.3$. (Assumed: $\varrho_e = 4$, $\varrho_m = 3$, gm cm^{-3}). Smaller bodies in orbit about the proto-Earth would spiral in rather rapidly because of drag by the solar nebula.

For a spherical body of radius s, density ϱ_s and in a circular orbit of radius a_0 about a proto-Earth of mass M_e the spiral time, t, to the proto Earth is given by

$$t = \frac{6\varrho_s s}{\varrho G^{1/2} M_e^{1/2}} \left[a_0^{1/2} - (R_e + s)^{1/2}\right]. \tag{6}$$

For a proto-Earth mass of one-tenth the Earth's mass, a body of $s = 10$ km, $\varrho_s = 3$ gm cm^{-3} would spiral to the proto-Earth from $a_0 = 100000$ km in 2000 yr because of the nebular drag. It is difficult to see how a ring system could develop or persist in a nebula, at least about a terrestrial planetary mass.

With the density now assumed for the solar nebula we can apply Equation (4) to determine the quasi-stable separation of the proto-Moon and proto-Earth under the opposed forces of nebular drag and tidal friction. Table II lists values of this quantity distance, a_s, the orbital period, the circular velocity and the velocity of escape from the proto-Moon, all for various values of proto-Earth and proto-Moon masses and densities.

The resultant separations of the binary in Table II are the order of 30–40000 km or 6–9 of the various proto-Eartn radii over a small range in proto-Moon masses, consistent with commonly favored early distances.

The calculations from Equations (4), (5), and (6) neglect gravitation effects of the proto-Earth and proto-Moon that will increase the density of the solar nebula near the proto-Earth. The drag force on the proto-Moon will be increased both by this increased density and by the increased drag coefficient induced by the gravitational attraction of the proto-Moon in its passage through the gas. The ability of the nebular wind about the proto-Moon to carry away impact debris from planetesimal collisions will be increased by the increased velocity and particularly by the increased density near the proto-Moon caused by these gravitational effects. Correspondingly the impact velocities will also be increased, producing more impact debris to be blown away.

TABLE II

Calculations for assumed conditions of proto-Earth and proto-Moon (Equation (4))

M_e/M_e (present)	1.0	0.5	0.25	0.1
ϱ_e gm cm^{-3}	5.5	5.0	4.5	4.0
M_m/M_m (present)	1.0	1.0	0.8	0.5
ϱ_m gm cm^{-3}	3.3	3.3	3.3	3.0
$a_s(10^3$ km)	40	38	35	29
a_s/R_e	6.3	7.3	8.1	8.8
Period (hr.)	22	29	35	42
v(circ.) km s^{-1}	3.1	2.3	1.7	1.1
v_m(escape) km s^{-1}	2.4	2.4	2.2	1.8

If we neglect these gravitational effects the present Moon moving about the present Earth in the assumed solar nebula would appear, from crude theory, to lose rather than gain mass by planetesimal accretion. About an appreciably less massive proto-Earth the effect would be reversed were it not for the above-mentioned gravitational processes. Hence it appears worthwhile to continue this research into the more difficult theoretical area of gravitating spheres moving through compressible nebular gas of significant density. Quite possibly the Moon has not gained or lost a significant fraction of mass since the binary system developed, either in the nebular stage or subsequently. The Earth may have gained enormously in mass during the same interval.

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