ON THE FORMATION OF CLOSE BINARY WHITE DWARFS

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I. INTRODUCTION

Single stars of initial main-sequence mass larger than about 0.8-1.0$_\odot$ and less than approximately 6-8M$_\odot$ evolve into carbon-oxygen white dwarfs of mass in the range 0.55-1.1M$_\odot$ in less than the Hubble time. Single stars do not make helium white dwarfs in a Hubble time. These statements are based on both observational and theoretical considerations. It is not yet established, but there are suspicions that single stars of initial mass in the range 8-10M$_\odot$ may evolve into oxygen-neon white dwarfs of mass in the range 1.1-1.4M$_\odot$. As a single star of the appropriate initial mass develops a hot, electron-degenerate core composed of carbon and oxygen or of oxygen and neon, it becomes large ($R > 200R_\odot$) and luminous ($L > 6000L_\odot$), with the luminosity being generated most of the time by hydrogen burning, interrupted quasiperiodically by a helium shell flash. The star is said to be an asymptotic giant branch (AGB) star. Within 10^5-10^6yr after arriving on the AGB, an envelope instability, whose nature is still being explored observationally and theoretically, leads to the ejection of most of the hydrogen-rich envelope of the star. The remnant shortly becomes the central star of a planetary nebula composed of the ejected material, and then cools into a white dwarf configuration.

In a close binary, the process of white dwarf formation can be radically different, as mass can be stripped from either star in a Roche-lobe overflow event and either transferred to the other star or lost from the system or both. Thus, the mass of the star which ultimately becomes a white dwarf can be quite different from that of the initial main-sequence progenitor, and the evolution of the white dwarf remnant of each Roche-lobe overflow event can be quite different from that of a single star of the same initial mass. Hence, the mapping between final white dwarf mass and composition and the main-sequence mass of the progenitor in a close binary is not the same as the mapping for single stars. For example, one can envision systems which form helium white dwarfs with masses in the range ~0.1-0.5M$_\odot$, something which single stars cannot do in a Hubble time. The upper limits on the masses of progenitors which can form carbon-oxygen and oxygen-neon white dwarfs is also influenced by presence in a close binary.

The evolution of white dwarfs formed in a close binary can also be different from the evolution of single white dwarfs. For example, the thicknesses of the hydrogen and helium layers in binary white dwarfs will be different from those of single white dwarfs because the mechanism of envelope ejection is different in the two cases. This can affect cooling rates (through opacity) and spectral evolution (through diffusion, radiative levitation, and convective mixing). Due to frictional forces that arise in the common envelope which may be formed in Roche-lobe overflow events, the orbital separation of a binary white dwarf pair may be much smaller than the orbital separation of the progenitor main-sequence binary pair. If the white dwarfs are formed at a sufficiently small separation ($< 3R_\odot$), they will move once again into Roche lobe contact in less than the Hubble time due to the loss of orbital angular momentum by gravitational wave radiation. If the lighter white dwarf is sufficiently lighter than the heavier white dwarf, the net result may be a simple mass transfer with the ultimate dissolution of the secondary on a long time scale. GP Com may be an example of such evolution in progress (Nather, Robinson, and Stover 1981), and 40 Eridani B may be an example of the final result (Iben and Tutukov 1986). If the two white dwarfs are composed of helium and are initially of comparable and of sufficiently high mass, the immediate result of merger may be an sdO or sdB star (Webbink 1984; Iben and Tutukov 1985, 1986). If one of the dwarfs is made of carbon and oxygen and the other of helium, the immediate result of merger may be a hot helium deficient star or an R CrB star (Webbink 1984; Iben and Tutukov 1985). If both white dwarfs are composed of carbon and oxygen and the total mass of the pair exceeds the Chandrasekhar mass of $1.4M_\odot$, the net result of the merger may be a type Ia supernova (Iben and Tutukov 1984a; Webbink 1984). Finally, if the heavier white dwarf is made of oxygen and neon, the net result may be the formation of a neutron star without a very spectacular accompanying explosion (Iben 1986).

II. ROCHE-LOBE OVERFLOW, COMMON ENVELOPES, AND CONSERVATIVE MASS TRANSFER

Consider a close pair of intermediate mass ($2\text{-}8M_\odot$) main-sequence stars neither of which fills its Roche lobe until after it has exhausted hydrogen at its center. The primary evolves due to the transformation of lighter into heavier particles in its core. The core shrinks because (a) the pressure at any point is proportional to the number density of particles (which, without shrinkage, would decrease) at that point and (b) the gravitational force at any point is proportional to the mass interior to that point and, without shrinkage, this force would remain constant. Shrinkage of the core releases gravitational potential energy which is converted into heat. The increase in core temperatures causes the rate of nuclear energy production, which is proportional to a high power of the temperature, to increase. The increased flux of radiant energy through the envelope of the star causes this envelope to expand. Once hydrogen is exhausted at the center, this process of core shrinkage and envelope expansion accelerates, not to be halted until helium is ignited in the core.

If the primary fills its Roche lobe shortly after it exhausts central hydrogen and if the primary and secondary are initially of comparable mass, the time scale for mass transfer from primary to secondary will not be too different from the thermal response time scale of the envelope of the secondary and the secondary may accrete most of the matter proferred by the primary. It is thought that this may be the way that relatively massive Algols are formed. During the final phase of mass transfer, the primary is a subgiant and the secondary is a brighter and hotter main sequence star. Even though Roche-lobe filling will be maintained in any case by nuclear evolution of the subgiant, the mass-transfer rate may be augmented by a magnetic stellar wind (e.g., Iben and Tutukov 1984b).

If the primary does not fill its Roche lobe until it is about to ignite helium, the rate of envelope expansion is so large that the timescale for mass transfer will be much smaller than the thermal response time scale of the secondary, and it is expected that the matter impinging on the secondary will form an expanding envelope which itself ultimately overflows the Roche lobe of the secondary. A "common envelope" (Paczynski 1976, Meyer and Meyer-Hofmeister 1979) then forms and the matter pushed into this envelope by the primary is thereafter presumably lost from the system. One may think of the common envelope as a viscous medium within which the binary evolves and this image points up the fact that frictional forces must abstract energy and angular momentum from the orbital motion (Bodenheimer and Taam 1984; Livio and Soker 1988; Livio 1988). The initial mass transfer rate will be even larger and the propensity to form a common envelope will be even further enhanced if the initial mass of the primary is much larger than the mass of the secondary. It is thought that classical cataclysmic variable systems and close binary central stars found in several planetary nebulae may be formed in this way (e.g., Paczynski 1976; Webbink 1979; Livio, Salzman, and Shaviv 1979).

The common envelope phenomenon continues until the amount of hydrogen-rich matter that remains on the surface of the primary drops below a critical value which depends on the mass and composition of the primary. This critical value is given roughly by the thickness of the hydrogen-burning shell and thus is larger for smaller primary masses. At the surface of the remnant, which has now contracted within its Roche lobe, matter which has undergone some CNO cycling has been exposed. The remnant continues to contract until helium is ignited at its center. At this time, its position in the H-R diagram lies between the main sequence and the white dwarf sequence, near the position occupied by homogeneous models of pure helium which are burning helium quiescently in their cores (the so-called helium main sequence).

The mass of the remnant is typically much smaller than the mass of its progenitor. For example, a $10M_\odot$ main sequence model of population I composition evolves into a core helium burning star of mass $2M_\odot$ and a $5M_\odot$ model evolves into a helium star of mass $0.77M_\odot$. Note that, in a binary system, the mass of a star which will ultimately evolve into a white dwarf can be larger than the maximum mass of a single star which can evolve into a white dwarf of the same composition. The numerical values quoted here and in the following are usually from the calculations of Iben and Tutukov (1985). Other choices of composition and input physics will lead to somewhat different values.

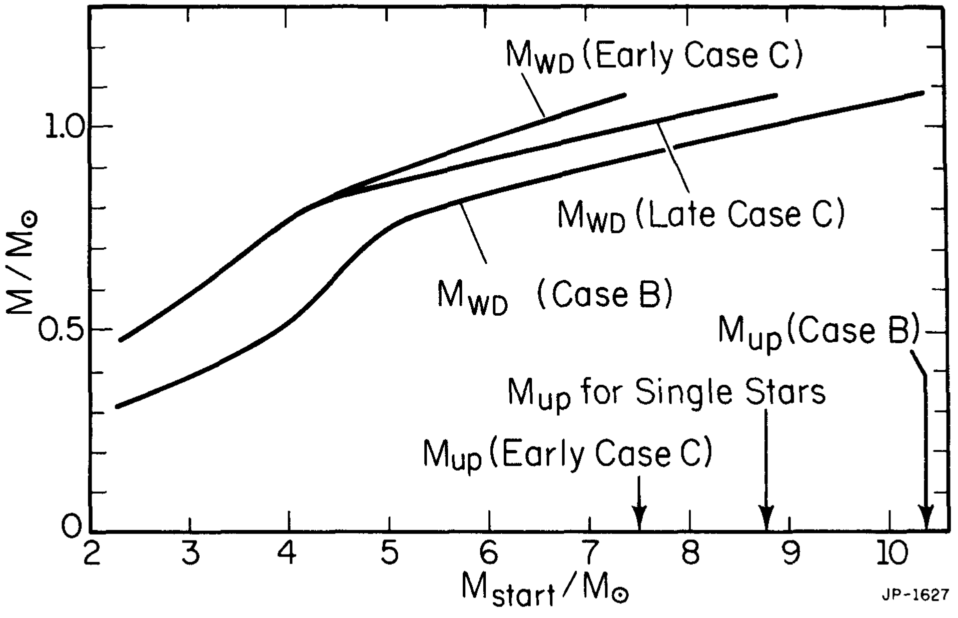

Figure 1. Transformations between progenitor mass and final white dwarf mass for case B and case C Roche-lobe overflow events.

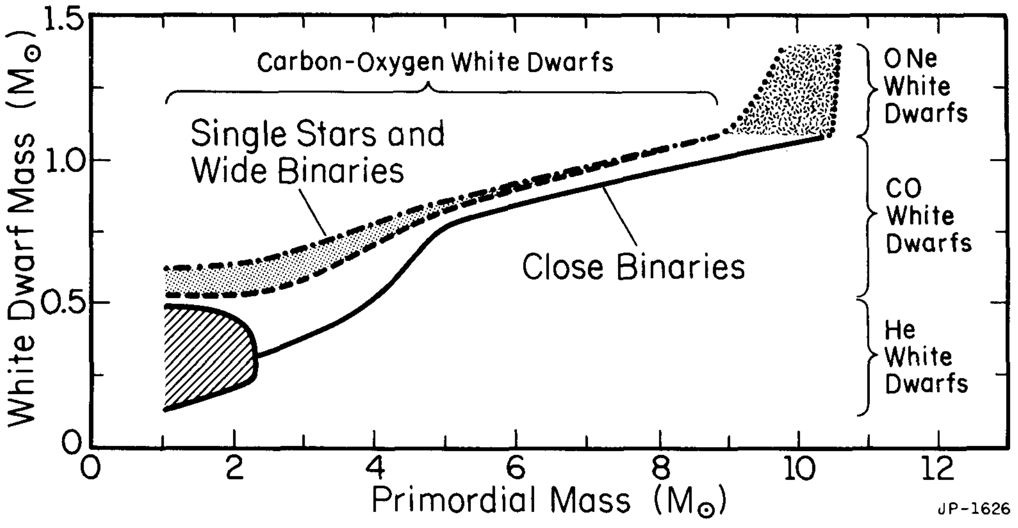

Figure 2. Transformation between initial progenitor mass and final white dwarf mass for single stars and binary stars in case B Roche-lobe overflow events.

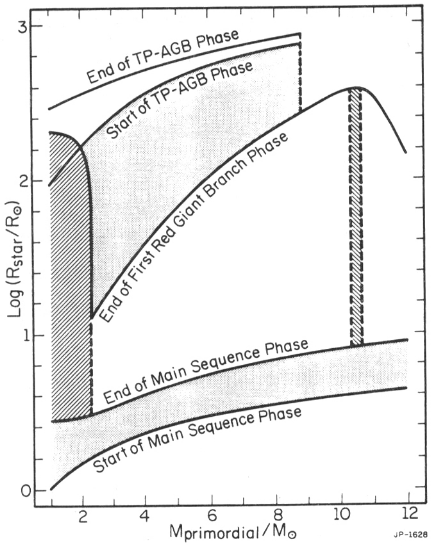

Figure 3. Radius versus initial stellar mass for various evolutionary stages of single stars.

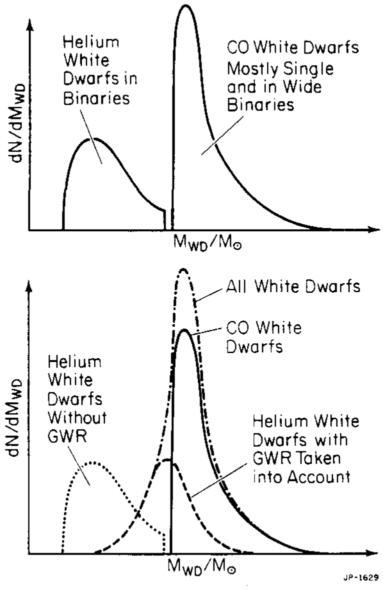

Figure 4. The top panel gives the birth rate of white dwarfs as estimated by Iben and Tutukov (1986). The lower panel is a schematic giving an impression of how the observed white dwarf mass distribution function may be built up from the estimated birth rate function when gravitational radiation is taken into account, forcing the merger of the lowest mass binary white dwarfs. The contribution of merged binaries has been overestimated by a factor of two, as we overlooked the fact that each merger reduces by one the number of white dwarfs surviving until the present.

If its mass is between ~$0.75M_\odot$ and ~$2.5M_\odot$, a helium star will expand again to fill its Roche lobe after exhausting central helium. It will begin and continue to lose mass until the mass of the helium layer remaining near its surface (its core has been converted into carbon and oxygen) has been reduced below a critical value. In this way the $2M_\odot$ remnant of the first mass exchange event becomes ultimately a $1.05M_\odot$ CO white dwarf and the $0.77M_\odot$ remnant becomes a $0.75M_\odot$ white dwarf.

If its mass is less than ~$0.75M_\odot$, the helium star remnant of the first common envelope event will not expand sufficiently on exhausting central helium to fill its Roche lobe for a second time. Instead, it will go on to become a white dwarf without losing any more mass.

If the mass of the progenitor is somewhere in the range $10\text{-}11M_\odot$, the white dwarf formed will be of the oxygen-neon variety, with a mass in the range $1.1\text{-}1.4M_\odot$. For more massive progenitors, evolution becomes more complicated, and the exact outcome of the Roche-lobe overflow episode has yet to be sorted out.

When a star fills its Roche lobe after having exhausted hydrogen at the center but before having ignited helium, we speak of a case B Roche-lobe overflow event. If the overflow event occurs before the star has crossed the Hertzsprung gap and developed a deep convective envelope (early case B event), much of the mass lost by the primary will be gained by the secondary. If orbital angular momentum is also partially conserved, the orbital separation may increase during the mass-transfer process. If the overflow event occurs after the primary has developed a deep convective envelope (late case B event), a common envelope is expected to be formed and the orbital separation is expected to be considerably reduced.

If Roche-lobe filling is delayed until the primary has exhausted helium at its center and has developed an electron-degenerate CO core, we speak of a case C event. If Roche-lobe filling occurs before thermal pulses begin (early case C event), the mass of the resultant white dwarf will be nearly the same as the CO core mass of a single star when it first begins to thermally pulse. Since mass loss can abort the "second" dredge up process (which occurs in population I stars for initial masses larger than ~$5M_\odot$) before the CO core is reduced to below $1.1M_\odot$, the maximum mass of a progenitor which can produce a CO white dwarf is reduced by ~$1M_\odot$ relative to the maximum mass for single stars. If Roche-lobe filling is delayed until thermal pulses have begun (late case C event), the mass of the white dwarf which will be formed depends on how long the progenitor has remained in the thermally pulsing phase before Roche-lobe filling occurs. For progenitor masses less than ~$2M_\odot$, the mass of the white dwarf remnant can be anywhere between the mass of the CO core of the progenitor when thermal pulses begin and something approximately $0.1M_\odot$ larger. For progenitor masses larger than this, the mass of the resultant white dwarf is essentially the same as the mass of a white dwarf formed by a single star. Because of the strong winds experienced by AGB stars, the mass of the primary could well be less than the mass of the secondary when Roche-lobe filling occurs. The result is that, in many late case C events, the initial rate of mass transfer might be considerably less than would otherwise be expected, and that may reduce the tendency toward common envelope formation.

The mapping between progenitor initial mass and final white dwarf mass for the various types of Roche-lobe overflow events, as found in one set of calculations for one set of input physics (Iben and Tutukov 1985, Iben 1986) is shown in Figure 1. It is to be emphasized that these results are not definitive; any number of variations in the input physics could alter the mapping significantly.

III. MAPPING PROGENITOR BINARIES INTO WHITE DWARF BINARIES

The mapping between the masses and orbital separation of a main-sequence binary and the masses and orbital separation of the white dwarf binary (immediately following the formation of the second white dwarf) depends on which cases of Roche-lobe overflow are appropriate and on a number of other assumptions about the nature of the several mass-loss, mass-transfer interactions; very few of these assumptions are founded on first principles.

Of crucial importance is the evolutionary state of each star when it first fills its Roche lobe. Figure 2 gives curves in the radius-(initial)mass plane which mark where significant changes in the evolutionary state of single stars occur if they are of population I composition (Iben and Tutukov 1985).

When the first Roche-lobe overflow event is of the early case B variety and component masses are comparable, it has long been the practice to suppose that both the total mass and the total angular momentum of the system are conserved. Given a mapping between initial primary mass and resultant white dwarf mass, the assumption of conservation of total angular momentum and total mass then leads to a unique determination of the masses and orbital separation of the intermediate system, now consisting of a white dwarf and a main-sequence secondary which is more massive than the initial primary.

If the first Roche-lobe overflow event is of the late case B or of the case C variety or if the initial secondary is much less massive than the initial primary, it is commonly supposed that a common envelope will be formed. A convenient way of parameterizing the degree of orbital shrinkage which occurs in a common envelope event is (Iben and Tutukov 1984a)

$$G\ M_1{}^2/A_0 \sim \alpha\ GM_{1R}M_2/A_f\,, \qquad (1)$$

where M_1 and M_2 are the masses of the primary and secondary, respectively, M_{1R} is the mass of the remnant of the primary after the common envelope event, A_f and A_0 are the final and initial orbital separation, and α is a parameter which in some cases can be estimated theoretically but, in general must be determined from the observations. The expression on the left hand side of equation (1) is a crude measure of twice the energy required to drive off the common envelope when the mass lost from the system is large compared with the mass of the primary remnant and large compared with the mass of the secondary. The expression multiplying α on the right hand side of the equation is a crude measure of twice the release in orbital binding energy for systems in which the degree of orbital shrinkage is large enough to neglect the initial orbital binding energy. When the masses of the two interacting stars are comparable, α in equation (1) should be divided by 4. With this understanding, an α of ~1 means that orbital energy is converted into the energy to drive off the common envelope with 100% efficiency. An α large compared with 1 means that some source of energy other than orbital has been tapped to drive off the common envelope. An α small compared with 1 means that only a fraction of the orbital energy has been used up in driving off the common envelope, the remaining fraction going into heating the escaping matter and supplying it with greater than escape velocities, or into radiative losses, or both.

Two and three dimensional hydrodynamical simulations of common envelope events (Livio and Soker 1988; Taam and Bodenheimer 1988) suggest an α of the order of 0.3-0.6. Livio and Soker find some evidence that, the larger the initial orbital separation, the greater the tendency for the stellar cores to spin up the common envelope, and therefore the less efficient the transfer of energy between the orbital energy and the expanding common envelope will be. In any case, one can understand the formation of cataclysmics by this kind of scenario. For example, suppose that the primary and the secondary are of initial mass $\sim 10M_{\odot}$ and $\sim 0.5M_{\odot}$, respectively, and that the primary fills its Roche lobe in a late case B event to become a white dwarf of mass $\sim 1.05M_{\odot}$. Inserting these numbers as well as $\alpha \sim 0.3$ in equation (1) gives $A_f \sim A_0/634$. For the chosen initial mass ratio, the radius of the Roche lobe of the primary is initially $R_{1L} \sim A_0/2$ and the radius of the Roche lobe of the secondary after the common envelope event is $R_{2L} \sim 0.3A_f$. Hence $R_{2L} \sim R_{1L}/1057$. But the secondary now has a radius of $\sim 0.5R_{\odot}$, and it presumably did not fill its Roche lobe until long after the common envelope event, when angular momentum loss by a magnetic stellar wind established contact. This means that $R_{1L} > 630R_{\odot}$. From Figure 2 it is obvious that our assumption of a case B event is inappropriate. A more consistent scenario requires choosing a slightly less massive primary and assuming a late case C event. Had we chosen $\alpha \sim 0.6$, the original scenario of a case B Roche-lobe overflow event would have been viable.

The main point of the exercise is to show that quite dramatic orbital shrinkages are implied by the assumption that orbital energy is the only source of energy for driving off the common envelope and that, the smaller the efficiency of conversion, the more dramatic is the shrinkage. We shall return to a discussion of this point in section V.

After the first white dwarf has been formed, if the secondary is massive enough to evolve off the main sequence in a Hubble time and if it fits well within its Roche lobe during its main-sequence lifetime, a second common envelope event will usually follow when the secondary exhausts a nuclear fuel at its center and expands to fill its Roche lobe. This is because the large ratio of donor mass to receiver mass ensures a mass transfer time scale much shorter than the thermal time scale of the donor and because, not only is the thermal response time scale of the white dwarf much larger than the mass-transfer time scale, but hydrogen-burning may be ignited near the surface of the accreting white dwarf, causing the regeneration of a red giant (common) envelope, much as in the case of novae. The role of hydrogen burning is not absolutely certain since the mean densities in the common envelope may be much less than in the red giant envelope of a single star. In any case, due to frictional dissipation, orbital shrinkage will again occur and, when the two white dwarf remnants emerge from the common envelope, the orbital separation may be quite small. If the separation is less than several solar radii, gravitational wave radiation will lead to a merger of the pair in less than a Hubble time.

IV. LOW MASS BINARIES AND HELIUM WHITE DWARF PAIRS

Conventional theory suggests that a single star which is initially massive enough to evolve off the main sequence in less than a Hubble time can make a white dwarf no lighter than $\sim 0.5M_\odot$ and that this white dwarf is made of carbon and oxygen (or of neon and oxygen in a small fraction of cases). On the other hand, if the primary in a low mass binary is less massive than about $2M_\odot$, it will develop an electron-degenerate core composed of helium as it leaves the main sequence, and, if it fills its Roche lobe before igniting helium, mass loss will transform it into the compact central star of a planetary nebula; after it has burned much of the hydrogen remaining in its envelope, it will evolve into a helium white dwarf capped by a very thin "skin" of hydrogen. The mass M_{He} of the white dwarf so formed is related to the effective radius R_L of the Roche-lobe when mass transfer begins by

$$R_L \sim 10^{3.5} M_{He}^{4}. \qquad (2)$$

If the primary and secondary are of comparable mass, and if the Roche-lobe radii are small enough (say, $R_L < 12R_\odot$, corresponding to $A_0 < 32.5R_\odot$ and orbital period $P < 12$ days) that the primary has not developed too deep a convective envelope before filling its Roche lobe, one might suppose that the mass transferred from the primary to the secondary will stick to the secondary. The primary will develop a deep convective envelope as it evolves to the giant branch, and an observer would see an Algol type binary. The primary will surely be tidally torqued so that it spins at nearly the orbital frequency, and one may further suppose that the agency which drives mass transfer is angular momentum loss by a magnetic stellar wind (MSW). If the time scale for mass transfer is short compared with the nuclear burning time scale $\tau_{nuc} = M_{He}/\dot{M}_{He}$, then the primary will shrink within its Roche lobe before M_{He} has grown appreciably and the Roche lobe of the primary will remain essential fixed during the entire mass-transfer phase. The orbital separation of the system after Roche-lobe detachment will be determined by solving

$$R_{iL} \sim 0.52(M_i/M_t)^{0.44} A, \qquad (3)$$

in conjunction with equation (2). In equation (3), R_{iL} is the Roche-lobe radius of the i^{th} component and M_i is its mass; M_t is the total mass of the system and A is the orbital separation. There are more accurate expressions for the relationship between R_{iL}, M_i, M_t, and A, but for our purposes, expression (3) is quite accurate enough and it has the virtue of being simple to use.

Using equations (2) and (3) with the requirement that the radius of the Roche lobe is the same before and after the mass-transfer event, we have that

$$A_f = (M_1/M_{1R})^{0.44} A_0, \tag{4}$$

where A_0 and A_f are the orbital separation before and after the mass-transfer event and M_{1R} is the mass of the helium white dwarf remnant of the primary. Thus, A_f is typically a few times larger than A_0.

A reverse phase of mass transfer follows when the secondary grows to fill its Roche lobe. Since it will be a larger subgiant than was the primary when it filled its Roche lobe, the secondary will have a deeper convective envelope than did the primary as a subgiant. Furthermore, the accretor will now be a very compact object with a long thermal response time scale and probably capable of igniting the accreted fuel and regenerating its own extended envelope. The second mass transfer episode is therefore likely to lead to the formation of a common envelope, with all of the mass lost by the secondary being lost from the system. We therefore use equation (1) with $M_2 = M_{2R}$, $M_1 = M_2+(M_1-M_{1R}) = M_t-M_{1R} = M_{2f}$ = mass of secondary after the first mass transfer episode, $A_0 = A_f$, and $A_f = A_{ff}$ = final orbital separation to obtain:

$$A_{ff} = \alpha\, A_f (M_{1R}/M_{2f})(M_{2R}/M_{2f}). \tag{6}$$

Equations (2) and (3) can also be used to establish a relationship between the mass M_{2R} of the helium white dwarf remnant of the secondary and M_{1R}:

$$M_{2R} = M_{1R}(M_t/M_{1R}-1)^{0.11}. \tag{5}$$

The modest increase in orbital separation during the first mass-transfer/mass-loss episode thus implies that the second white dwarf is slightly more massive than the first. This is a general property of quasiconservative mass-transfer scenarios.

As an example, let us suppose that $M_1 = 1.5$, $M_2 = 1.0$, and $M_{1R} = 0.25$. Then, $M_{2R} = 0.32$, $A_0 = 29.7$, $A_f = 2.20A_0 = 65.4$, and $A_{ff} = 1.70\alpha$. An upper limit on the time required by the white dwarf pair to move into final Roche-lobe contact comes from assuming that the only way in which orbital angular momentum is lost is by the radiation of gravitational waves. This upper limit is given by

$$\tau_{GWR}(\mathrm{yr}) = 10^{8.175} A_{ff}{}^4/(M_{1R}M_{2R}M_{tR}), \tag{7}$$

where $M_{tR} = M_{1R}+M_{2R}$. Of course, if the orbital separation is comparable to the radii of the white dwarfs, tidal torques will cause heating and serve to shorten the time scale for moving into contact. Using $A_{ff} = 1.7\alpha$ in (7), we have that the helium white dwarf pair in our example will move into Roche-lobe contact in less than $\tau_{GWR} = 2.75\mathrm{x}10^{10}\alpha^4$yr. Choosing $\alpha \sim 0.3$, $\tau_{GWR} \sim 2.2\mathrm{x}10^8$yr.

One may repeat this exercise for many different combinations of initial (low) masses, only to find similar results: if the first-formed white dwarf is less massive than $\sim 0.3M_\odot$, the ultimate result is two helium white dwarfs which merge in less than a Hubble time. If the total mass of the two white dwarfs is less than some critical value of the order of $0.4-0.5M_\odot$, merger will lead to a white dwarf of mass equal to the sum of the masses of the white dwarfs prior to merger. If the total mass of the merging pair is larger than this critical value, the immediate result of merging may well be an sdO or sdB star. But this star will ultimately evolve into a CO white dwarf.

If the initial orbital separation is large enough to permit the primary to reach the base of the giant branch (see Figure 2), it will have developed such a deep convective envelope that, when Roche-lobe filling first occurs, mass transfer takes place on a dynamical time scale, and a common envelope will be formed also in the first mass-transfer episode. If this is the case, then orbital shrinkage also occurs, with the consequence that the effective radius of the Roche lobe of the secondary becomes smaller than the effective radius of the Roche lobe of the primary (when the first mass-transfer episode begins). This means that the mass of the helium white dwarf formed by the secondary will be less than the mass of the first-formed helium white dwarf.

Using equations (1), (2), and (3) for each assumed common envelope event, we obtain:

$$M_{2R} = \alpha_1^{0.25}(M_2/M_1)^{0.36}(M_{1R}/M_1)^{0.25}(M_1+M_2/M_{1R}+M_2)^{0.11} \qquad (9)$$

and

$$A_{ff} = \alpha_2\alpha_1 10^{3.784}(M_1+M_2/M_1)^{0.44}M_{1R}^4(M_{1R}/M_1)^2(M_{2R}/M_2) \qquad (10)$$

for the mass of the second white dwarf and the final orbital separation in terms of the mass of the first white dwarf, respectively. Here we take into account the possibility that the parameter α could be quite different in the two common envelope events. Setting $M_1 = 1.5$, $M_2 = 1.0$, and $M_{1R} = 0.25$, we have $M_{2R} = 0.22\alpha_1^{0.25}$ ($= 0.16$ if $\alpha_1 = 0.3$) and $A_{ff} = 0.22\alpha_2\alpha_1^{0.25}R_\odot$ ($= 0.05R_\odot$ if $\alpha_1 = \alpha_2 = 0.3$) Equation (8) then gives $\tau_{GWR} \sim 1.36 \times 10^7 \alpha_2^4\alpha_1^5$yr ($= 267$ yr, if $\alpha_1 = \alpha_2 = 0.3$). The same exercise for $M_{1R} = 0.35$ gives $M_{2R} = 0.30\alpha^{0.25}$ ($= 0.22$ if $\alpha_1 = 0.3$), $A_{ff} = 2.23\alpha_2\alpha_1^{1.25}R_\odot$ ($= 0.5R_\odot$ if $\alpha_1 = \alpha_2 = 0.3$), and $\tau_{GWR} = 5.4 \times 10^{10}\alpha_2^4\alpha_1^5$yr ($= 10^6$yr if $\alpha_1 = \alpha_2 = 0.3$). Note that typically $M_{2R} \sim 0.6M_{1R}$ (because $M_{1R}/M_1 \sim 0.16$ in equation 9). Only for values of M_{1R} larger than $\sim 0.45M_\odot$ will it take longer than a Hubble time for a merger to occur.

From these considerations it is evident that only the most massive helium white dwarfs are born at sufficiently wide separation that they can avoid merger in a Hubble time. This has interesting consequences for future searches for gravitational wave radiation from space detectors (e.g., Lipunov, Postnov, and Prokhorov 1986; Evans, Iben, and Smarr 1987; Hils, Bender, Faller, and Webbink 1988). It also explains why the observed white dwarf distribution function contains so few white dwarfs less massive than $\sim 0.5M_\odot$, despite the expectation that of the order of 10 percent of all stars are born in binaries with initial masses and separations such that they should go through the sort of evolution we have just sketched.

In Figure 4a we present an estimate (Iben and Tutukov 1986) of the birth rate of white dwarfs as a function of white dwarf mass. All of the white dwarfs in the low-mass hump of the two-humped distribution are helium white dwarfs in binaries. Most of the white dwarfs in the second hump are CO white dwarfs which are single or in wide binaries. If, as our estimates suggest, all but the most massive helium white dwarf pairs merge in much less than the Hubble time, then, integrating over the lifetime of the galactic disk, the current white dwarf distribution function should look like what we have sketched in Figure 4b. That is, mergers shift all but the most massive helium white dwarf pairs made in binaries into the single star distribution and there are very few white dwarfs less massive than about $0.3M_\odot$.

V. THE CLOSE BINARY WHITE DWARF L870-2

Saffer, Liebert, and Olszewski (1988) have discovered that the star L870-2 is actually a pair of DA white dwarfs with a period of 1.6d. The two white dwarfs are very nearly of the same luminosity and surface temperature, implying that they are of very similar mass and age. However, the slight differences in global characteristics translate into finite differences in mass and cooling age which, though small, have a profound significance for unravelling the prior history of the system and for understanding the physics of mass transfer events in close binaries. We (Webbink and Iben 1988) have analysed data kindly provided by these authors in conjunction with theoretical evolutionary tracks in the HR diagram of cooling white dwarfs (Iben and MacDonald 1986) to estimate the masses of the white dwarfs to be $M_{1R} \sim 0.605$ and $M_{2R} \sim 0.54$. The mean cooling age of the white dwarfs is $\sim 10^9$yr, but the more massive white dwarf is older than the lighter white dwarf by $\sim 10^8$yr (adopting the mass-dependence of the cooling curves given by Winget et al. 1987). This means that the more massive white dwarf must be derived from the initially more massive of the main-sequence progenitor pair.

We shall explore several scenarios for the history of the system, showing how the properties of the current system place a constraint on the product of the two α's used to parameterize the degree of orbital shrinkage in common envelope events. Suppose, first, that the initial primary filled its Roche lobe after having exhausted central hydrogen, but before having ignited helium (a case B event). From Figure 1, we can estimate the mass of the primary to be $M_1 \sim 4.3M_\odot$. The main-sequence lifetime of such a star is $\tau_{MS} \sim 1.0\text{x}10^8$yr. If the case B event was of the early variety, we might expect there to have been some orbital expansion and we might guess that the secondary was able to delay Roche-lobe filling until it had developed an electron-degenerate CO core, filling its Roche lobe eventually in a case C event. From Figure 2, we see that the mass of the secondary must be $M_2 \sim 2.7M_\odot$ when it first filled its Roche lobe. The main sequence plus core helium burning lifetime of such a star is $\tau_{MS+He} \sim 3.5\text{x}10^8$yr. Since this is $2.5\text{x}10^8$yr longer than the main sequence lifetime of the primary, and since the cooling ages of the current white dwarfs differ by only $\sim 10^8$yr, we exclude this scenario. Let us suppose next that the first mass-transfer event was of the late case B variety, thus presumably leading to orbital shrinkage in a common envelope; this requires that the second mass-transfer event also be of the case B variety. Again from Figure 2, we have that $M_2 \sim 4.1M_\odot$. The main-sequence lifetime of such a star is $\tau_{MS} \sim 1.1\text{x}10^8$yr, or only 10^7yr longer than the main-sequence lifetime of the primary. We therefore exclude this scenario as well.

Next, suppose that the primary undergoes a case C Roche-lobe overflow event. Its mass must have been $M_1 \sim 3.1M_\odot$ and its main sequence plus core helium burning lifetime was $\tau_{MS+He} \sim 2.6\text{x}10^8$yr. The secondary could not have undergone a case B Roche-lobe overflow event because its mass just prior to this event must have been, at $4.1M_\odot$, larger than M_1, implying that it had gained at least $1M_\odot$ from its companion. This contradicts our assumption that the first mass transfer event led to the formation of a common envelope, with all of the mass lost by the primary being lost from the system.

The last option, then, is that both Roche-lobe overflowing events were of the case C variety. Remarkably, but perhaps fortuitously, the main-sequence plus core helium burning lifetime of the $2.7M_\odot$ secondary, $\tau_{MS+He} \sim 3.5\text{x}10^8$yr, is longer than that of the primary by precisely the difference in the cooling ages of the white dwarfs.

The period of L870-2 combined with our estimates of component masses imply that the current orbital separation is $A_{ff} \sim 5.8R_\odot$ and we may use this to make statements about the degree of orbital shrinkage in each mass-loss event. From equation (1) with $\alpha = \alpha_1$, we have $A_f = 0.17\alpha_1 A_0$. For the second mass- transfer event we rewrite equation (1) as

$$G\, M_2{}^2/A_f \sim \alpha_2\; GM_{1R}M_{2R}/A_{ff} \qquad (1')$$

and solve for $A_f = 130.5/\alpha_2$. From equation (3), we have that the Roche lobe radius about the primary <u>after</u> the first mass-loss event is $R_{1L,f} = 0.25A_f = 32R_\odot/\alpha_2$. With $\alpha_2 = 0.3\text{-}0.6$, $A_f = (435\text{-}217)R_\odot$ and $R_{1L,f} = (106\text{-}53)R_\odot$. from the two relationships involving A_f, we find $A_0 = 768/\alpha_1\alpha_2$ and from equation (3) we find the radius of the Roche lobe about the primary <u>before</u> the first mass-transfer event to be $R_{1L,0} = 0.395A_0 = 303R_\odot/\alpha_1\alpha_2$ From Figure 2 it is evident that $25 < R_{1L,0}/R_\odot < 500$ and this means that $0.6 < \alpha_1\alpha_2 < 12$. But, with $\alpha_2 \sim 0.3\text{-}0.6$, $\alpha_1 > 1\text{-}2$.

Recall that, when the component masses are comparable, as is the case prior to the first mass-loss event, GM_2M_{1R}/A_f overestimates (two times) the binding energy release by approximately a factor of 4. Hence, our scenario and the observational constraints tell us that orbital energy provided only one-eighth to one-fourth of the energy needed to drive off the common envelope. This result has profound ramifications for our understanding of the physics of the common envelope process. If one is used to thinking that the only source of energy for driving off the common envelope is orbital binding energy and that only a portion of the binding energy release is available for this purpose, then an α smaller than unity makes sense, but an α larger than unity is nonsense.

However, we know that single stars that reach the AGB eject their hydrogen-rich envelopes in only a matter of $10^4\text{-}10^5$ yr. Is it not possible that, in the case of L870-2, the primary reached the thermally pulsing AGB stage before it filled its Roche lobe, and that the main driving force which ejected its hydrogen-rich envelope was exactly the same as that which drives off the envelope of a single AGB star? It could be that the modification of the gravitational potential caused by the presence of a close companion speeds up the process of mass loss to a time scale of only few times 10^3 yr (cf. Eggleton 1985). The high densities in the wind from the primary and the differential velocities between the main-sequence companion and the wind would mean that there will be dissipation that causes orbital shrinkage, but shrinkage on a much more moderate scale than we are used to thinking of in terms of common envelopes (see Livio 1988 for a further discussion of this point).

At this conference, a poster paper by Bergeron, Wesemael, Liebert, Fontaine, and LaCombe (1988) suggests that the white dwarfs in L870-2 have masses $M_{1R} = 0.47$ and $M_{2R} = 0.42$. (The difference between these mass estimates and our own is almost wholly due to differences in the adopted mass-radius relationships for cooling white dwarfs). It is not feasible here to examine all of the possible scenarios which might produce these masses at the current separation. We explore one.

Suppose that both stars are helium white dwarfs that derive from low-mass progenitors after the fashion described in section IV. The white dwarf masses are large enough that we expect a common envelope to be formed in each mass-loss episode. If the constraint of 10^8yr as the difference between the cooling ages of the white dwarfs still holds, even approximately, we must assume that both main-sequence progenitors are almost identical in mass ($\tau_{MS} \sim 10^{10}$yr x $M_i^{-3.5}$ and $M_i < 2M_\odot$). From equations (2) and (3), we have $R_{1L,0} = 155R_\odot = 0.38A_0$, or $A_0 \sim 400R_\odot$. From equation (1) with $M_2 \sim M_1$, $A_f \sim 400R_\odot\alpha_1 M_{1R}/M_1 = 188R_\odot\alpha_1/M_1$. But, we also have, from equations (2) and (3) that $R_{2L,f} = 98R_\odot \sim 0.5A_f$, giving $A_f \sim 196R_\odot$. The two expressions for A_f yield $\alpha_1 \sim M_1$, and, since both M_1 and M_2 must be at least as large as $1M_\odot$ if the evolution to the current configuration takes place in less than a Hubble time, $\alpha_1 > 1$. Once again, since the two main sequence progenitors are of comparable mass, this means that only a quarter (if $M_1 \sim 1M_\odot$) to one eighth ($M_1 \sim 2M_\odot$) of the energy needed to drive off the common envelope can have come from the release of orbital binding energy. Unfortunately, there is no simple mechanism to which one can point to provide the extra energy.

We can also make a statement about α_2. Using equation (1′) and $A_{ff} = 4.9R_\odot$, we have $\alpha_2 = 0.13M_2^2$. Since $1 < M_2/M_\odot < 2$, $\alpha_2 \sim 0.13\text{-}0.52$, and these values are consistent with the estimates of Livio and Soker (1988) and Taam and Bodenheimer (1988).

This example points up a difficulty repeatedly encountered in trying to construct a scenario for the progenitor of L870-2. Whatever the scenario, the immediate progenitor of this system must have had an orbital separation of ~200-$400R_\odot$. At this separation, the binary cannot have avoided a preceding phase of mass exchange; yet, to have preserved such a wide system, the first mass transfer episode must have succeeded in dispelling most of the envelope of the initial primary with very little loss of angular momentum. Common envelope evolution alone does not appear capable of fulfilling this requirement -- the dissipation of enough orbital energy to eject the common envelope necessarily involves the dissipation of a great deal of orbital angular momentum as well. Nevertheless, at so large a separation, it is difficult to avoid the conditions at the first episode of mass transfer that would have led to common envelope evolution. One must mitigate this tendency by appealing to other, more benign forms of systemic mass loss, such as stellar winds, or perhaps by tapping the ionization energy of the relatively weakly bound first common envelope. To produce yet more widely separated close binary white dwarfs taxes the imagination. We therefore expect L870-2 to be among the longest period close binary white dwarfs created by any scenario.

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