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# THE CREEP OF ICE, GEOTHERMAL HEAT FLOW, AND ROOSEVELT ISLAND, ANTARCTICA*

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ABSTRACT. Measurements of ice velocity, thickness, and surface topography on the large ice rise known as Roosevelt Island are consistent with Glen's flow law, $\dot{\epsilon} = (\tau/B)^n$, for values of $\tau$ between $5 \times 10^4$ N m$^{-2}$ and $1.4 \times 10^5$ N m$^{-2}$, and there is no indication of a reduction in $n$ at low stresses. If $n = 3$ there must be progressive softening of the ice towards the edge of the ice rise, and this probably represents the combined effects of warming and recrystallization leading to a fabric favoring shear. Assuming that near the centre of the ice rise, where the effects of recrystallization are probably negligible, the ice behaves in the same way as randomly-oriented polycrystalline ice, then the geothermal flux $G$ in this area is approximately 0.06 W m$^{-2}$. In the absence of measurements of deep-ice temperatures, the distribution of $G$ across the ice rise cannot be determined. However, the simplest interpretation of the movement data requires:

(1) a linear increase in $G$ from 0.05 W m$^{-2}$ on the north-east side of Roosevelt Island to 0.07 W m$^{-2}$ in the south-west, and

(2) strain-rate enhancement, due to recrystallization, that increases outward from the centre of the ice rise to reach a maximum value of approximately two near the edges.

The calculated values of $G$ are larger than the world average, but this is consistent with the probably granitic core beneath Roosevelt Island. An increase in $G$ of 0.02 W m$^{-2}$ in a distance of 60 km would require an increase in granite thickness of about 5 km.

RÉSUMÉ. *L'écoulement de la glace, le flux géothermique et Roosevelt Island, Antarctique.* Des mesures de vitesse de la glace, d'épaisseur et de topographie de surface sur un grand dôme de glace connu sous le nom de Roosevelt Island confirment la loi d'écoulement de Glen $\dot{\epsilon} = (\tau/B)^n$ pour des valeurs de $\tau$ comprises entre $5 \times 10^4$ N m$^{-2}$ et $1{,}4 \times 10^5$ N m$^{-2}$, et il ne semble pas y avoir de réduction de $n$ pour les faibles contraintes. Si $n = 3$, il doit y avoir un affaissement progressif de la glace en direction de la bordure du dôme de glace, et ceci est probablement l'effet combiné du réchauffement et de la recristallisation qui conduit à une structure favorisant le cisaillement. En supposant que, près du centre du dôme où les effets de la recristallisation sont probablement négligeables, la glace se comporte comme une glace polycristalline à orientation quelconque, alors le flux géothermique $G$ dans cette zone est approximativement de 0,06 W m$^{-2}$. En l'absence de mesures de températures de la glace en profondeur, la distribution de $G$ à travers le dôme ne peut être déterminée. Cependant, l'interprétation la plus simple des mouvements constatés implique:

(1) un accroissement linéaire de $G$ depuis 0,05 W m$^{-2}$ sur la bordure nord-est de Roosevelt Island jusqu'à 0,07 W m$^{-2}$ au sud-ouest et

(2) un accroissement de la vitesse de déformation dû à la recristallisation qui est de plus en plus sensible quand on s'éloigne du centre du dôme, pour atteindre un maximum de l'ordre du doublement près des bords.

Les valeurs calculées pour $G$ sont supérieures à la moyenne mondiale mais ceci est cohérent avec la nature probablement granitique du sous-sol sous Roosevelt Island. Un accroissement de 0,02 W m$^{-2}$ sur une distance de 60 km supposerait un accroissement de l'épaisseur du granit d'environ 5 km.

ZUSAMMENFASSUNG. *Das Kriechen von Eis, geothermischer Wärmefluss auf Roosevelt Island, Antarktis.* Messungen der Eisgeschwindigkeit, der Eisdicke und der Oberflächentopographie auf der grossen Eisaufwölbung, die als Roosevelt Island bekannt ist, sind konsistent mit Glen's Fliessgesetz $\dot{\epsilon} = (\tau/B)^n$, für Werte von $\tau$ zwischen $5 \times 10^4$ N m$^{-2}$ und $1{,}4 \times 10^5$ N m$^{-2}$; für eine Abhahme von $n$ bei niedrigem Druck gibt as keine Anzeichen. Wenn $n = 3$ ist, muss das Eis gegen den Rand der Aufwölbung hin weicher werden, vermutlich eine Folge der Erwärmung und Rekristallisation, was zu einem scherempfindlichen Gefüge führt. Unter der Annahme,

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dass sich das Eis nahe dem Scheitel der Aufwölbung, wo die Wirkung der Rekristallisation vermutlich vernachlässigbar ist, das Eis so verhält wie zufallsorientiertes polykristallines Eis, beträgt der geothermische Wärmefluss $G$ in diesem Gebiet annähernd 0,06 W m$^{-2}$. Infolge des Fehlens von Temperaturmessungen in der Tiefe des Eises kann die Verteilung von $G$ über die Aufwölbung hin nicht bestimmt werden. Doch führt die einfachste Deutung der Bewegungsdaten auf folgende Erfordernisse:

(1) eine lineare Zunahme des Wertes $G$ von 0,05 W m$^{-2}$ an der Nordostseite von Roosevelt Island auf 0,07 W m$^{-2}$ im Südwesten.

(2) Zunahme der Verformungsrate infolge von Rekristallisation vom Scheitel der Aufwölbung nach aussen bis zu einem Maximalwert von etwa zwei nahe am Rand.

Der für $G$ berechnete Wert ist grösser als das globale Mittel, doch dies kann mit dem vermutlich granitenen Kern unter Roosevelt Island erklärt werden. Eine Zunahme um 0,02 W m$^{-2}$ auf eine Entfernung von 60 km würde eine Zunahme der Granitdicke um etwa 5 km erfordern.

## Introduction

The dynamics of ice rises form the topic of a separate paper in this *Journal* (Martin and Sanderson, 1980). In their paper Martin and Sanderson use measurements of ice velocity, thickness, and surface slope from a small ice rise to calculate values of the flow-law parameters for ice. By comparing these with values that are compatible with the observed surface profiles of four ice rises they conclude that all the ice rises are approximately in equilibrium. Here, we shall use measurements from the ice rise known as Roosevelt Island to provide additional information on the ice flow law, and to investigate the flux of geothermal heat from the rock beneath the ice rise.

Roosevelt Island is a snow- and ice-covered dome within the Ross Ice Shelf located between lat. 78° 40′ S. and 80° 10′ S., and between long. 160° W. and 164° W. (Fig. 1). It is approximately oval in shape, some 120 km long and 70 km wide. The ice dome rises to

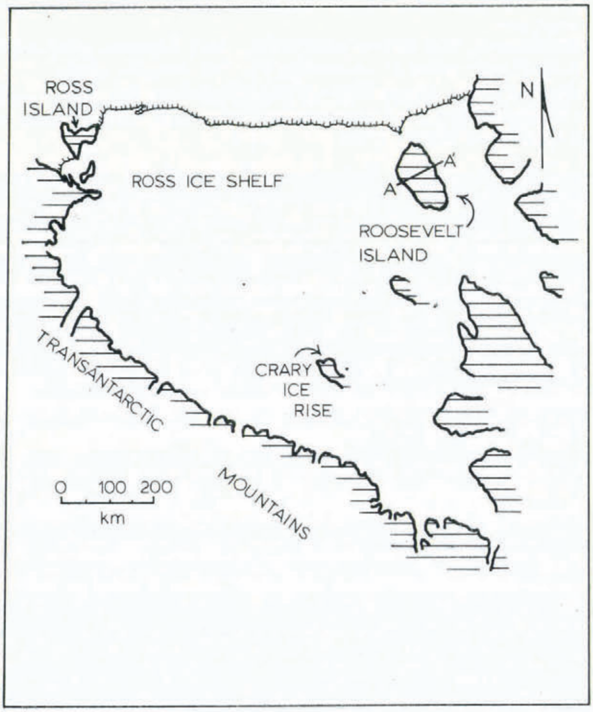


*Fig. 1. The Ross Ice Shelf, showing the position of Roosevelt Island and the section AA′, along which measurements of ice thickness, surface topography, and ice velocity were made.*

an elevation 450 m above the surrounding ice shelf. In 1961–62 a field program was inaugurated on Roosevelt Island. The purpose of the program was to determine the mass budget, strain-rates, movement rates, and thickness of the ice dome, and to measure the physical characteristics of the ice–bedrock interface and the underlying rock. The initial field work was carried out by a team of glaciologists, geophysicists, and engineers in 1961–62 and 1962–63. In 1964–65 a three-man party returned for a limited season to Roosevelt Island to make daylight star observations and snow-accumulation measurements, to replace stakes in the survey network, and to conduct experiments in radar sounding of the ice. The final season of the project was in 1967–68 when the resurvey of the strain network and additional accumulation measurements were carried out.

The control network for the determination of surface deformation consisted of a chain of braced quadrilaterals extending 70 km across the centre of the island (Fig. 1). Surface elevation and ice thickness along this network are shown in Figure 2a. A short chain of four quadrilaterals extended along the crest of the ice cap (perpendicular to the main line). In addition, two short lines were laid out, one at either end of the island. The overall network comprised 60 stations between which 150 distances were measured. Relative elevations of most of the stations were obtained by optical levelling. The geophysical measurements conducted during the 1961–62 and 1962–63 seasons included (1) determination of the surface topography by altimetry to provide approximate elevations in regions where optical levelling was not carried out, (2) measurement of temperatures in 10 m bore holes, (3) investigations of bedrock topography and geology by seismic, magnetic, and gravity measurements along the lines of the survey network, and (4) estimation of temperatures through the ice by means of d.c. resistivity observations. The results of the first two seasons' topographic and strain-network survey have been published by Clapp (1965). The results of the resurvey and of the geophysical measurements are still in the form of unpublished reports by Clapp, Hochstein, and Bentley.

## The flow law

Glen's flow law for ice, generalized to three dimensions, can be written:

$$\dot{\epsilon} = (\tau/B)^n, \tag{1}$$

where $2\dot{\epsilon}^2$ and $2\tau^2$ are the second invariants of the strain-rate and stress-deviator tensors:

$$2\dot{\epsilon}^2 = \dot{\epsilon}_{ij}\dot{\epsilon}_{ij},$$
$$2\tau^2 = \sigma_{ij}'\sigma_{ij}',$$

$B$ is determined by ice fabrics, temperature, impurities, etc., $n$ is approximately 3 for $\tau > 10^5$ N m$^{-2}$, but $n$ may decrease at lower stresses.

Assuming that $n$ is constant within a vertical column from an ice rise, and that the ice is frozen to bedrock Martin and Sanderson (1980) showed that the surface horizontal velocity can be approximated by the expression:

$$U = \frac{2}{n+1}\left\{\frac{\rho g \alpha}{B'}\right\}^n H^{n+1}, \tag{2}$$

where $\rho$ is the average density of the ice column, $g$ the acceleration due to gravity, $\alpha$ the surface slope, $H$ the ice thickness, and

$$B' = \left\{\frac{n+1}{H^{n+1}}\int_0^H \left(\frac{z}{B}\right)^n \mathrm{d}z\right\}^{-1/n}, \tag{3}$$

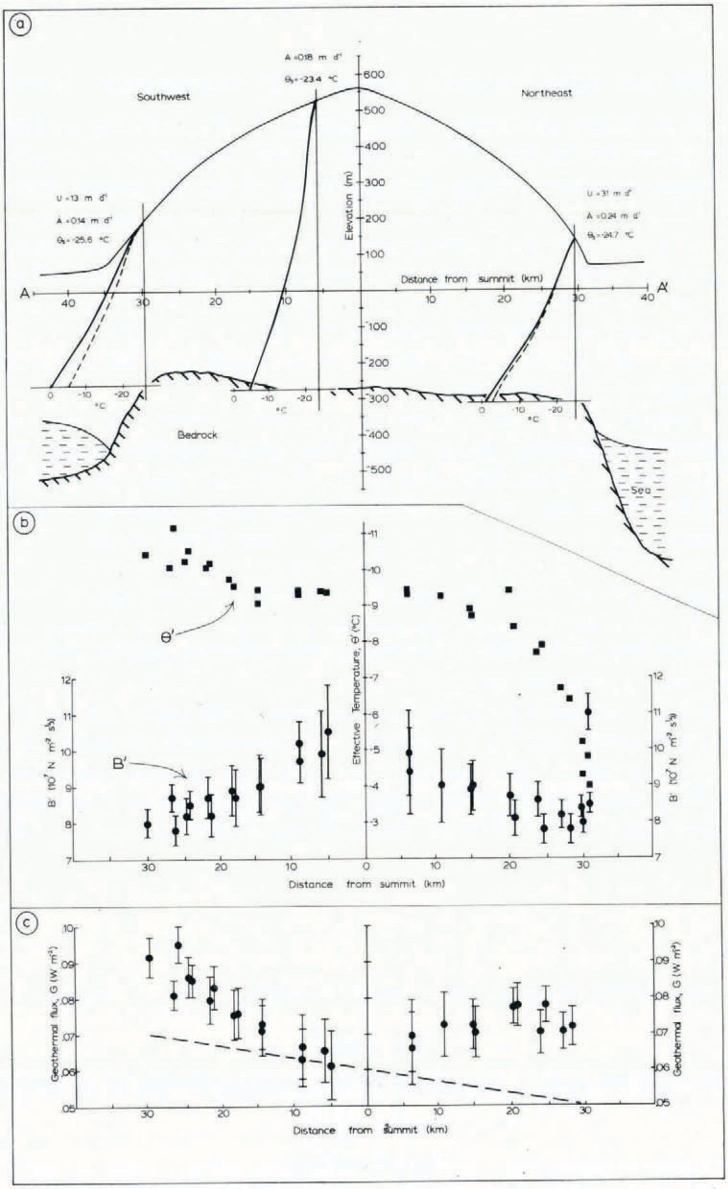

*Fig. 2*

(*a*) *Surface and bottom topography of the Roosevelt Island ice dome along the section AA′ shown in Figure 1. Values of accumulation rate* $\dot{A}$, *10 m temperatures* $\theta_s$, *and surface velocity U are given for the summit and for points near the edges of the ice rise. Calculated temperature/depth profiles are also shown; the solid lines correspond to values of geothermal flux given by the circles in Figure 2c, and the broken lines correspond to a geothermal-flux distribution given by the broken line in Figure 2c.*

(*b*) *Values (with error bars) of the flow-law parameter B′ calculated from the measurements of ice velocity, thickness, and surface slope at several points along the section, and values of effective ice temperature θ′ that were calculated assuming that the ice rise is in steady state and that the geothermal flux is 0.06 W m*$^{-2}$.

(*c*) *Values (with error bars) of geothermal flux necessary to give steady-state temperature profiles (solid lines in Fig. 2a) that are consistent with the observed ice velocities, assuming that the flow properties of the ice are identical to those deduced from laboratory experiments and ice-shelf observations. If sustained shearing within the lower layers of the ice rise causes recrystallization and softening of the ice, this fabric softening will probably be negligible at the centre of the ice rise and maximum at the edges. Under these conditions the simplest distribution of geothermal flux is described by the broken line, and the corresponding temperature profiles are shown by the broken lines in Figure 2a.*

*The increase in temperature gradient in the upper 50 m of all the temperature profiles is caused by the low conductivity of firn and snow. Without this insulating layer at the surface of the ice rise basal temperatures would be up to 5 deg colder than shown here. The effect is most pronounced where the ice is thin and the temperature difference between surface and base is large.*

with $z$ the depth below the ice surface. Because ice becomes softer as the temperature increases, $B$ generally decreases with increasing depth, so that the value of $B'$ is determined principally by the ice in the lower 10% to 20% of the ice column. Then, if we assume that $B'$ is independent of position on the ice rise, we can write:

$$U/H = C(\alpha H')^n, \tag{4}$$

where $C$ is constant, and $H'$ is thickness of the ice rise expressed as an equivalent column of solid ice of density $\rho_i$; $H'$ is approximately 15 m less than $H$.

A plot of log $(U/H)$ against log $(\alpha H')$ using data from each side of Roosevelt Island is shown in Figure 3. It is significant that, both here and on Butler Island (Martin and Sanderson, 1980), the apparent value of $n$ is greater than the value ($n = 3$) that is usually assumed. Moreover, none of our data show any indication of a reduction in the value of $n$ at low stresses (down to $5 \times 10^4$ N m$^{-2}$). Weertman (1973) gives theoretical reasons for believing that $n = 3$ at stresses appropriate to glacier movement, and support for this value is provided by Barnes and others (1971) from analysis of their own results and from a survey of published laboratory experiments. Thus, before accepting our rather large apparent values for $n$, we must examine the assumptions that we made in obtaining them.

The effect of longitudinal stresses, which we have neglected, is to increase the ice velocity slightly for a given surface slope and ice thickness. The proportional effect is greatest near the summit of the ice rise where velocities and basal shear stresses are lowest. Thus, correction for the presence of longitudinal stresses would require a reduction in the observed values of ice velocity, with the maximum correction applied to the lowest velocities. This would tend to increase the value of $n$. However, even for the stations nearest to the summit of Roosevelt Island, the calculated correction is only approximately 1% of the observed ice velocities,

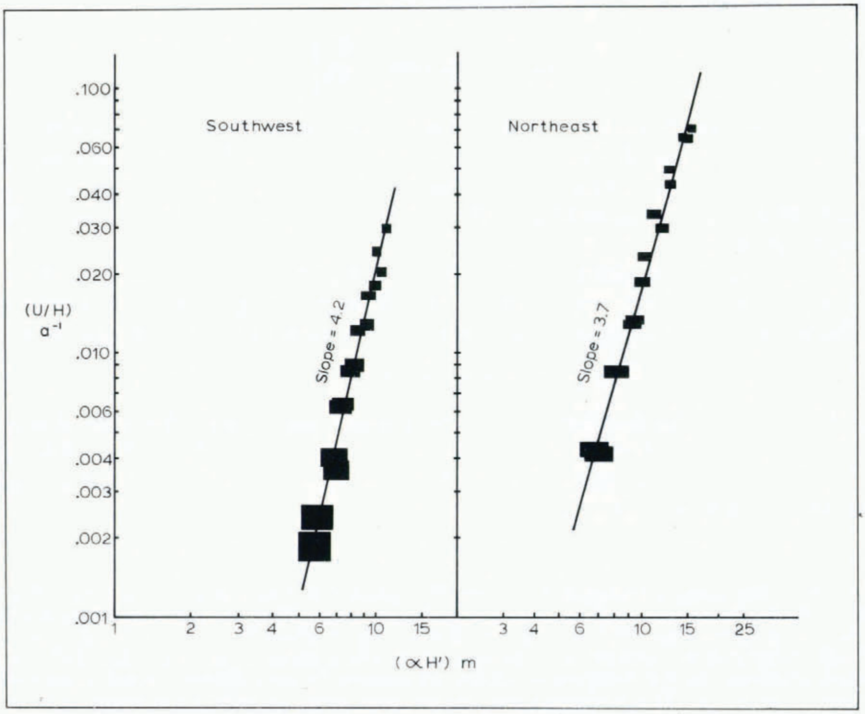


*Fig. 3. A logarithmic plot of $U/H$ against $\alpha H'$ for the two sides of Roosevelt Island. Observation errors are shown here by the error boxes.*

which is insignificant compared to the experimental errors that are illustrated by the error boxes in Figure 3. The use of Equation (2) also makes the implicit assumption that the longitudinal-stress gradient in the direction of ice movement is small compared with the shear-stress gradient in the vertical direction (Paterson, 1969). Again, errors introduced by this assumption reach a maximum near the summit of the ice rise, but even in this region the measured strain-rate gradient indicates that the shear-stress gradient is at least two orders of magnitude greater than the longitudinal-stress gradient.

The effects of spatial variations in the temperature–depth profiles were also neglected. As the ice accelerates away from the ice-rise summit, strain heating within the ice column becomes more pronounced, with the result that the basal ice may be warmer near the edge of the ice rise, despite its thinner ice cover, than it is beneath the summit. This would allow the ice to accelerate more rapidly than would be the case for no change in temperature, giving an apparently high value for $n$. This effect can be examined by assuming that $n = 3$ and using Equation (2) to calculate values of $B'$ for each station on the ice rise where velocities were measured. Values calculated in this way are plotted against distance from the ice-rise summit in Figure 2b, and they show a definite decrease, implying progressive ice softening, towards the edges of the ice rise.*

In order to see whether this apparent softening can reasonably be attributed to warming, we calculated temperature–depth curves for each station using a finite-element analysis developed by one of us (D.M.) that incorporates the effects of snow accumulation and strain heating (see Appendix). Snow accumulation-rates and surface temperatures were interpolated between measurements that were made on the summit of the ice rise and on the surrounding ice shelf. Geothermal heat flux was assumed to be 0.06 W m$^{-2}$ (1.5 heat flow units) which is consistent with the value inferred for the rock beneath Byrd Station (Rose, 1979). The variation of $B$ with temperature $\theta$ can be expressed by

$$B(\theta) = B_0 \exp \left\{ \frac{Q}{nR\theta} \right\}, \tag{5}$$

where $B_0$ is a constant, $Q$ is the activation energy for creep, $R$ is the gas constant, and $\theta$ is expressed in kelvins. Assuming a value for $Q$, Equation (3) can then be integrated numerically for each of the calculated temperature–depth curves to give corresponding values of $B'$, whence we obtain a corresponding "effective temperature", $\theta'$ defined by the equation $B' = B_0 \exp (Q/nR\theta')$.

Laboratory experiments indicate that at low temperatures $Q \approx 80$ kJ mol$^{-1}$ increasing to 120 kJ mol$^{-1}$ above about $-10$°C, when the presence of liquid water is believed to soften the ice (Barnes and others, 1971). However, for our calculations we adopted the equation

$$B(\theta) = 28 \exp (4\,000/\theta) \text{ N m}^{-2} \text{ s}^{\frac{1}{3}}, \tag{6}$$

implying that $Q \approx 100$ kJ mol$^{-1}$ for the entire temperature range. We made this assumption for three reasons: first, because it provides an adequate fit to available data (Fig. 4); second, because $\theta'$ is comparatively insensitive to errors in $Q$, and finally because it considerably simplifies the computation.

* Very close to the north-east edge of the ice rise, the values of $B'$ trend upwards, and one of the data points is anomalously high. We suspect that this does not represent a real stiffening of the ice, but rather the effects of longitudinal compressive stresses that are transmitted from the "bottleneck" of ice shelf to the east of Roosevelt Island. If this interpretation is correct, only the data points within about 2 km of the edge of the ice rise appear to be affected, implying that the longitudinal stresses are transmitted over distances equal to approximately $5H$, where $H$ is the ice thickness. This probably represents an upper limit, since the stress transition at the edge of an ice rise is more dramatic than we would expect to find within a grounded ice sheet. Consequently, we tentatively suggest that "regional" values of strain-rate and basal shear stress for grounded ice sheets should be obtained from measurements that are averaged over distances of the order of $10H$.

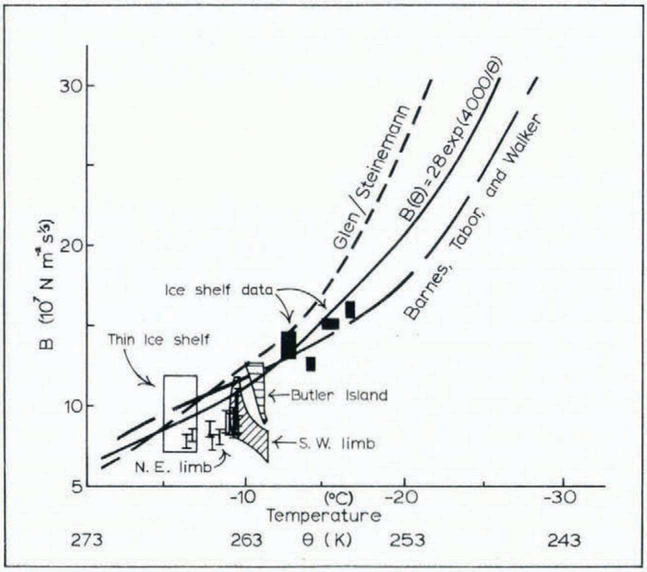


*Fig. 4. A plot of the flow-law parameter B against temperature θ. The results of laboratory investigations and ice-shelf observations are shown, together with the curve $B(\theta) = 28 \exp(4\,000/\theta)$, which provides a satisfactory fit to the various data. The temperatures for the north-east limb (error bars) and the south-west limb (diagonal shading) of Roosevelt Island and for Butler Island (horizontal shading) were calculated for a geothermal flux of 0.06 W m$^{-2}$.*

Calculated values of $\theta'$, which were found to correspond to temperatures in the ice column at heights of between 10% and 15% of the ice thickness above bedrock, are shown in Figure 2b. The values for the north-east side of the ice rise increase steadily away from the ice-rise summit as anticipated; strain heating near the edge is more than sufficient to balance the cooling effects of thinner ice and higher accumulation-rates. Along the south-west limb ice velocities are considerably lower and the ice thins more rapidly so that, despite lower accumulation-rates, $\theta'$ is effectively independent of position and shows no correlation with the pronounced softening near the edge of the ice rise.

Softening without increase in temperature could occur if the basal ice develops a progressively more anisotropic fabric as it moves away from the center of outflow. However, we would expect this to happen on each side of the ice rise so that maximum softening would occur on the north-east line, where the effects of warming would reinforce those of fabric development. In fact, there is approximately equal softening along each limb. Moreover, all of the values of $B'$ calculated from the field data lie below those calculated from Equation (6) and so are characteristic of ice that is softer than expected. This is shown in Figure 4, where both Equation (6) and values of $B'$ calculated from the field data are plotted against calculated temperatures $\theta'$. Laboratory curves of $B$ versus $\theta$ from Glen (1955), Steinemann ([1956]), and Barnes and others (1971), and estimates of $B$ obtained from ice-shelf measurements (Thomas, 1973) are also included in Figure 4.* Note that Equation (6) provides an adequate fit to all these data in the temperature range of concern ($>-25$°C).

Although the field values of $B'$ all lie below the curve from Equation (6), those from the north-east limb lie parallel to that curve, and are thus consistent with an activation energy of

* Estimates of $B$ from analysis of creep-rates on the Ward Hunt Ice Shelf (Thomas, 1973) are almost identical to those shown here for the south-west limb of Roosevelt Island. However, we believe this apparent agreement to be fortuitous since the softening of the Ward Hunt ice is probably caused by high impurity content (Lyons and Ragle, 1962) and perhaps by transient creep, so the Ward Hunt data have not been included in Figure 4.

100 kJ mol$^{-1}$. This suggests that the progressive softening of ice along this limb of the ice rise may be caused by changes in temperature. How then, do we explain the softening along the south-west limb, where $\theta'$ is approximately constant, and why is all the ice apparently softer than laboratory ice?

## Soft ice, geothermal heat, and ice fabric

Let us assume, for the moment, that creep behaviour is solely determined by the ice temperature and that there is no fabric softening of the ice. In order to calculate the effective temperature $\theta'$ we have assumed that the geothermal flux $G$ beneath Roosevelt Island is everywhere the same and that $G = 0.06$ W m$^{-2}$. An alternative approach is to reverse the problem and regard $G$ as an unknown which must be compatible with temperature profiles that give values of $B'$ (from Equations (3) and (6)) equal to those calculated from the field data. From Equation (6) we obtain values of $\theta'$ corresponding to the calculated values of $B'$, and by an iterative process values of $G$ that are associated with those of $\theta'$ can then be calculated. In doing this we assume that Equation (6) adequately describes the creep behavior of the Roosevelt Island ice. Clearly this assumption may not hold, but our purpose in making it is to obtain a distribution of geothermal flux that is consistent with the simplest interpretation of observed ice velocities. How reasonable this distribution proves to be will give us some indication of the reliability of our assumptions.

The calculated values of geothermal flux are plotted against distance from the ice-rise summit in Figure 2c, and corresponding temperature profiles near the centre and edges are shown in Figure 2a. Along the north-east limb $G$ is almost constant at 0.07 W m$^{-2}$, but with a slight increase towards the edge of the ice rise. To the south-west there is a steady increase to a maximum value of 0.095 W m$^{-2}$. Basal ice temperatures increase from $-5$°C in the central region almost to the melting point at the edges.

The apparent minimum in $G$ may be a real feature, but its position at the ice-rise centre is rather suspicious. For an ice rise with basal temperatures well below 0°C, isotherms within the underlying rock are concave upwards, so that geothermal heat tends to be focused into the ice rise from the surrounding bedrock. This would increase $G$, particularly near the edges of the ice rise. However, for Roosevelt Island the magnitude of this effect, which can be calculated using equations for the topographic correction for geothermal flux (Carslaw and Jaeger, 1959, p. 424), is very small.

Another possible explanation for the apparent minimum in $G$ is the progressive development of an anisotropic ice fabric favoring shear as the ice moves towards the sea. This is a very real possibility since, in the lower portions of the ice rise, deformation is by simple shear over an almost flat bed, and conditions should be favorable for the development of a fabric with predominantly vertical $c$-axes. Total strain is maximum near the edges of the ice rise so this is where we would expect fabric development, and its associated softening of the ice, to reach a maximum. If this is the case, Equation (6) is likely to apply only near the center of the ice rise where the ice has little or no preferred fabric. Nearer the edge, values of $\theta'$ that are calculated using Equation (6) will be too warm, and the corresponding values of geothermal flux will be too large.

If fabric softening does occur, and is negligible near the dome summit and equally pronounced on each side of the ice rise, we can reconstruct a distribution of geothermal flux by assuming that it varies linearly from north-east to south-west. The flux calculated in this way (Fig. 2c) shows an increase in $G$ from 0.05 W m$^{-2}$ in the north-east to 0.07 W m$^{-2}$ in the south-west. Temperature profiles corresponding to these values of $G$ are shown in Figure 2a, these give basal temperatures of between $-3$°C and $-5$°C at all points across the ice rise. At its maximum, near the edges of the ice rise, fabric softening consistent with a linear variation of geothermal flux is sufficient to double the strain-rates for a given stress, so that $B'$ is about 20% less than its equivalent value for randomly-oriented polycrystalline ice. Other studies (Steinemann, 1958; Paterson, 1977) indicate that fabric softening may cause strain-rate enhancement by a factor of ten or more. These studies involved strain-rates that were orders of magnitude larger than those on Roosevelt Island so that conditions may have been more favourable for recrystallization. At any event we are invoking comparatively minor fabric softening in order to obtain a simple, linear distribution of geothermal flux.

Figure 4 includes values of $B'$ that we have calculated for stations on Butler Island using data from Martin and Sanderson (1980). Here also there is a reduction in $B'$ by up to 25% towards the edge of the ice rise, with no calculated increase in temperature, and we are tempted to suggest that this is the effect of recrystallization and fabric softening. However, these values of $B'$ were calculated assuming horizontal bedrock, whereas ice thickness was measured only at the centre of the ice rise. The Butler Island results may be fully explained without invoking fabric softening if the bedrock slopes downwards in the direction of ice movement by only 15 m per km.

Clearly our conclusions are, at best, tentative. A useful test would be provided by measurement of temperature profiles within the ice rise and underlying bedrock. The temperature profile inferred by Hochstein (1967) from electrical resistivity measurements near the summit of the ice rise suggests that the ice is far colder than our calculations indicate. However, Hochstein used an activation energy for electrical conductivity of about 40 kJ mol$^{-1}$, whereas more recent measurements have indicated that a preferable figure is near 20 kJ mol$^{-1}$ (Glen and Paren, 1975; Bentley, 1979; Shabtaie and Bentley, 1979). Taking the latter value for the activation energy and using Hochstein's analysis leads to calculated temperatures in the deeper ice near the summit (−13°C at 575 m, −3°C at 750 m) in substantial agreement with those indicated by our calculation above and shown in Figure 2a. Although the details of Hochstein's analysis are open to some question in the light of more recent information (Bentley, 1977), it is clear that no major discrepancy remains.

Because these results and our calculated temperatures indicate basal temperatures close to the melting point it is important to consider the possibility of bottom sliding. Throughout our analysis we have assumed that the ice is everywhere frozen to the bed. To some extent this assumption is justified by the general agreement between our results and those from Butler Island where the basal temperature is calculated to be −13°C (Martin and Sanderson, 1980). However transition from frozen-bed conditions beneath the summit region to wet-bed conditions could explain the apparent softening of ice near the edges of Roosevelt Island. For this to be the correct explanation the transition would have to take place over a long distance rather than abruptly, otherwise there would be a slope change in the plot of $U/H$ against $\alpha H'$ (Fig. 3), corresponding to the transition. We have chosen to retain the frozen-bed assumption, but clearly this will remain an assumption until it is either confirmed or denied by field measurements.

Our temperature profiles require values of geothermal flux which are high and which increase from one side of the ice rise to the other. Is this compatible with what we know about Roosevelt Island? Unlike other shoal areas in the Ross Embayment, Roosevelt Island has a solid rock core. Three seismic refraction profiles show 50 to 100 m of sediment (P-wave velocity: 2.5 km s$^{-1}$) overlying 200–250 m of presumed sedimentary rock (P-wave velocity: 4.5 km s$^{-1}$). At Little America, Crary (1961) has interpreted a similar upper layer as partly, if not entirely, glacial marine, and an apparently equivalent second layer as Palaeozoic/Mesozoic rock, perhaps representing the Beacon Supergroup. At Little America, however, the combined thickness of these two layers is 2 km compared with only 300 m on Roosevelt Island. 20 km east of Roosevelt Island, under the Ross Ice Shelf, the combined thickness of these two layers increases to almost 1 km. Underlying these two layers on Roosevelt Island and under the neighbouring ice shelf a few kilometers to the east is rock with a P-wave velocity of 5.5 km s$^{-1}$. This material is not found at stations further away from Roosevelt

Island, and it is the cause of the topographic high. The seismic velocity, the strong negative gravity anomaly found over the northern end of the island and the shelf immediately to the north, and the indication from magnetic surveys of low-susceptibility rock in the same region (Bennett, 1964), all suggest that this rock is a granitic intrusive. It seems likely that it is similar in nature to intrusives found to the east in the Rockefeller Mountains (Wade and Wilbanks, 1972). There is also a suggestion of a magnetic edge effect produced by a boundary between the intrusive and the more basic (and higher susceptibility) crustal rock indicated by the seismic refraction results at Little America Station (Crary, 1961).

The identification of the Roosevelt Island bedrock as probably a granitic intrusive carries with it the implication that the heat flow here would probably be greater than the world average. We assume that this intrusive has an age similar to those in the Rockefeller Mountains and Edsel Ford Ranges, i.e. approximately 100 Ma (Wade and Wilbanks, 1971). According to Lee and Uyeda (1965), the average for "Mesozoic/Cenozoic orogenic areas" is about 0.08 W $m^{-2}$. Thus the heat-flow values calculated for the south-west side of Roosevelt Island (Fig. 2c) are about normal for such a region. The decrease towards the north-east could be associated with decreasing thickness of granite and therefore decreasing total heat production within the granitic layer, although from the gravity data it appears that the centre of the intrusive is further to the north. An average value for the heat productivity of granite is $4 \times 10^{-6}$ W $m^{-3}$ (Roy and others, 1968) so the 0.02 W $m^{-2}$ change in heat flow would correspond to a change in thickness of the granitic "layer" of the order of 5 km.

## Conclusions

Measurements of ice velocity, thickness, and surface topography on Roosevelt Island are consistent with Glen's flow law, $\dot{\epsilon} = (\tau/B)^n$, for values of $\tau$ between $5 \times 10^4$ and $1.4 \times 10^5$ N $m^{-2}$, and there is no indication of a reduction in $n$ at low stresses. The simplest interpretation of the data gives rather high values of $n$ (*c.* 4), but we can reconcile our results with the more generally accepted value of $n = 3$ if there is progressive softening of the ice towards the edges of the ice rise due to the combined effects of warming and recrystallization. If we assume that near the centre of the ice rise, where the effects of recrystallization are probably negligible, the ice behaves in the same way as randomly-oriented polycrystalline ice, then the geothermal flux $G$ in this area is approximately 0.06 W $m^{-2}$. A simple linear increase in $G$ from 0.05 W $m^{-2}$ on the north-east side of Roosevelt Island to 0.07 W $m^{-2}$ in the south-west requires fabric softening due to recrystallization to an equal extent on both sides of the ice rise. At maximum, the effects of recrystallization are calculated to double the strain-rate for a given stress.

In the absence of direct measurement of temperature profiles in the ice and bedrock we are unable to test our conclusions; the actual distribution of geothermal flux across the ice rise may be more complex than we have assumed and this would affect the amount of fabric softening necessary to explain the observed ice velocities. For instance, if the value of $G = 0.06$ W $m^{-2}$ at the center of the ice rise represents a minimum, there would be no fabric softening; on the other hand, if this value is a maximum, then more pronounced fabric softening than for the simple model is indicated. Our calculated values of geothermal flux are somewhat larger than the world average, but this is consistent with the probably granitic core beneath Roosevelt Island. An increase in heat flow of 0.02 W $m^{-2}$ would correspond to an increase in granite thickness of about 5 km along the 60 km profile.

In our analysis we have made no attempt to include the possible effects on ice temperatures of changes with time of ice thickness and surface temperature. The magnitudes of these changes are not known, but presumably, during the Holocene, ice thickness has decreased and surface temperature has increased. Because these two trends would have opposing effects on near-basal ice temperatures our neglect of their net influence is, to some extent, justified. Perhaps a more serious omission from our analysis is the possible existence, near the base of the ice rise, of a layer of anomalously soft Wisconsin ice (Hooke, 1973; Paterson, 1977). If such a layer exists then our calculated values of geothermal flux are all too large. However, the resulting error should be small, since the Wisconsin ice layer is probably only a few metres thick.

Further clarification must await information from bore holes near the summit and edges of Roosevelt Island. Although the acquisition of these data would be a major undertaking, we believe that it would be well worth the effort. Any attempt at describing the behaviour of ice sheets and glaciers requires a flow law that can be applied to natural ice. This flow law should take into account, preferably in a simple way, temperature variations within the ice sheet and crystal fabric that develops in response to such factors as dust concentration and ice deformation. Measurements of bore-hole closure rates (Paterson, 1977) indicate that natural ice may be considerably harder than laboratory ice. On the other hand our results from Roosevelt Island show the opposite trend, but with the strong possibility that the apparent softening of the ice is due to recrystallization and slightly high values of geothermal flux. The addition of bore-hole measurements to our data would allow us to isolate and to specify the influences of temperature and fabric on the creep properties of the ice.

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## APPENDIX

THE temperature–depth profile for a geographically fixed location on the Roosevelt Island ice dome is determined from an analysis of the steady-state heat transfer within the ice. Denoting the temperature by $\theta$ (in °C) and the vertical distance above the base of the ice dome by $h$, the following one-dimensional heat equation must be solved to obtain $\theta(h)$,

$$S = -\frac{\mathrm{d}}{\mathrm{d}h}\left(k\frac{\mathrm{d}\theta}{\mathrm{d}h}\right) + \rho c V\frac{\mathrm{d}\theta}{\mathrm{d}h}, \qquad 0 < h < H, \tag{A1}$$

subject to the following boundary conditions,

$$\theta(H) = \theta_s, \tag{A2}$$

$$-k\left.\frac{\mathrm{d}\theta}{\mathrm{d}h}\right|_{h=0} = G. \tag{A3}$$

In this notation, $S$ is the rate of strain-heat production per unit volume, $V$ is the velocity of vertical ice movement, $\rho$ is the mass density, $k$ is the thermal conductivity of ice, $c$ is the heat capacity of ice, $H$ is the ice thickness, $\theta_s$ is the surface temperature, and $G$ is the geothermal flux beneath the ice dome (specified as a positive quantity). Equation (A1) represents the principle of heat-flow continuity in the absence of horizontal heat flow. This equation is valid where surface slopes and horizontal ice velocities are low, and where surface temperatures are uniform. Such conditions exist on the Roosevelt Island ice dome except at locations near its margins. The variation of the mass density with depth is described by an expression of the form

$$\rho(h) = \rho_i - (\rho_i - \rho_s)\exp\{-D(H-h)\}, \tag{A4}$$

where $\rho_i$ is the density of solid ice, $\rho_s$ is the density of the surface snow, and $D$ is an empirical constant which accounts for the increase of density with depth. The ice-density measurements obtained from a 50 m bore hole at the summit of the Roosevelt Island ice dome have values similar to the ice densities measured on the Ross Ice Shelf at J9 (Langway, 1975; personal communication from M. M. Herron and E. Chiang, 1978). The measurements from the Roosevelt Island ice dome do not extend to sufficient depth to allow an accurate determination of the values of $\rho_s$ and $D$, so the values of $\rho_s$ and $D$ used in this study were determined from the J9 density profile. These values are 309 kg m$^{-3}$ for $\rho_s$, and 0.043 m$^{-1}$ for $D$. The velocity of vertical ice movement $V$ is determined from the condition of mass-flow continuity applied to the observed rate of snow accumulation $\dot{A}$ and the density–depth profile given by Equation (A4). With the restrictions that the ice thickness $H$ is constant with time, and that the vertical strain-rate is independent of depth, the expression for $V$ is written

$$V(h) = \frac{-1}{\rho(h)}\left\{\dot{A}\left(917h - \frac{608}{0.043}\exp\{-0.043(H-h)\}\right)\left(917H - \frac{608}{0.043}\right)^{-1}\right\}\ \mathrm{m\ a^{-1}}, \qquad \text{(A5)}$$

where $H$ and $h$ are expressed in m, $\dot{A}$ in kg m$^{-2}$ a$^{-1}$, and $\rho$ in kg m$^{-3}$. For a frozen-bed ice sheet, strain-rates must decrease to zero at the bed, and our assumption of finite strain-rates at the bed results in calculated basal temperatures that are slightly too cold (Philberth and Federer, 1971). The magnitude of this error for Roosevelt Island is probably less than 1 deg, and we have retained the assumption of constant strain-rates since nothing is known about the actual variation at depth.

The rate of strain-heat production per unit volume $S$ is a function of depth given by the sum of the products of the deviatoric-stress and strain-rate components. Since the majority of the strain heat is produced by the shear component of the strain-rate in the vertical plane containing the flow direction, $S$ can be approximated by an expression involving the surface slope $\alpha$ and the temperature-dependent flow-law parameter $B(\theta)$:

$$S(h) = 2\eta\{\rho g\alpha(H-h)\}^4\{B(\theta)\}^{-3}, \qquad \text{(A6)}$$

where $g$ is the acceleration due to gravity. Because $B(\theta)$ is not known precisely, the constant $\eta$ appears in Equation (A6) to correct for any differences between the observed horizontal surface velocity and the calculated shear strain-rate integrated over ice thickness. The flow-law parameter adopted in this study resulted in values of $\eta$ ranging between 0.8 and 1.1.

The thermal conductivity and heat capacity for ice vary with density and temperature. Empirical expressions for $k$ and $c$ determined from the experimental values listed in the review article by Weller and Schwerdtfeger (1971) are used in this study. The surface temperature $\theta_s$ and snow accumulation rate $\dot{A}$ are interpolated from measurements of 10 m temperatures (Hochstein, 1967; Thomas, 1976) and of long-term accumulation rates (Clausen and Dansgaard, 1977; personal communication from M. M. Herron and E. Chiang, 1978) on the summit of Roosevelt Island and on the surrounding ice shelf. Stake measurements made across Roosevelt Island by M. Giovinetto confirm a south-west to north-east trend of increasing accumulation rates implied by the long-term measurements. Because the stake measurements refer to only a single year's accumulation and contain a lot of "noise", the values of the accumulation-rate used in this study were smoothly interpolated from the long-term measurements. The geothermal heat flux $G$ beneath Roosevelt Island is not known. One set of temperature–depth profiles was calculated using the value of 0.06 W m$^{-2}$, which is consistent with the value of $G$ determined from measurements in the deep bore hole at Byrd Station (Rose, 1979). Another set of temperature–depth profiles was calculated with $G$ varying to obtain consistency between the calculated temperatures and observed velocities.

The technique used to solve Equation (A1) through (A3) for the temperature–depth profile is a combination of the finite-element Galerkin method and the method of successive approximation. This combined technique was chosen for its accuracy when the quantities $S$, $V$, $\rho$, $c$, and $k$ vary with temperature or depth, and for ease in its implementation by computer. The role of the finite-element Galerkin method (Fairweather, 1978) is to convert the differential equation and the associated boundary conditions expressed by Equations (A1) through (A3) into a system of algebraic equations containing the temperatures at discrete depths as unknown quantities. The method of successive approximation is then employed to solve this system of non-linear algebraic equations. The successive-approximation method requires the temperature-dependent coefficients which create the non-linearity to be held constant at values determined by an initial estimate of the unknown temperatures. The resulting linear system of equations is solved for a trial solution which is then used to recalculate the temperature-dependent coefficients. Successive trial solutions generated in this manner will converge to the actual solution of the original set of non-linear equations. The solution obtained by the combination of the finite-element Galerkin method and the method of successive approximations can be made as accurate as desired by increasing the size of the algebraic system of equations which result from the finite-element approximation. The greater size will allow the solution for the unknown temperatures at a greater number of discrete depths. The temperature profiles calculated for the Roosevelt Island ice dome resulted from the solution for the temperatures at 100 depths evenly spaced throughout the ice thickness.

The temperature–depth profiles resulting from the one-dimensional time-independent heat-transfer analysis undertaken in this study do not incorporate the effects of the climatic changes that have occurred in the past. Variations in such quantities as the surface temperature, geothermal flux, ice thickness, and accumulation-rate may have caused the actual temperatures in the ice dome to deviate from their steady-state values. The most probable changes during the Holocene are a decrease in thickness accompanied by an increase in surface temperatures. If the surface warming occurred about 10 000 years ago, then the present-day deviations from steady-state would be very small (Robin, 1970). Moreover, the effect of decreasing ice thickness would be to accelerate the decay of the deviations produced by the surface warming. Other climatic effects, such as short-period changes in surface temperature, also may have occurred throughout the Holocene, but the effects of these variations would be limited to comparatively shallow depths (Robin, 1970), and are not as important to the dynamics of ice flow as are the temperatures near the base. Because of the lack of detailed information about long-term changes in climate and ice thickness, the calculated steady-state temperature–depth profiles provide the best estimate of actual conditions. Clearly measured temperatures would be preferable, and one temperature–depth profile was measured to a depth of 50 m in a bore hole at the summit of the Roosevelt Island ice dome in 1976–77 (Langway and Herron, 1977). The measurements (personal communication from E. Chiang, 1978) were compared with the temperature–depth profiles computed for several of the locations near the summit using a geothermal flux of 0.06 W m$^{-2}$. The calculated temperatures show close agreement with the measured temperatures, but the magnitude of the temperature gradient of the measured profile is slightly larger in the upper 40 m and becomes anomalously high between 40 and 50 m. Because the measured profile extends to only 50 m, it is impossible to tell whether these differences represent a departure from long-term steady state, a response to short-period climatic oscillations, or simply the thermal disturbance produced by the drill.

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