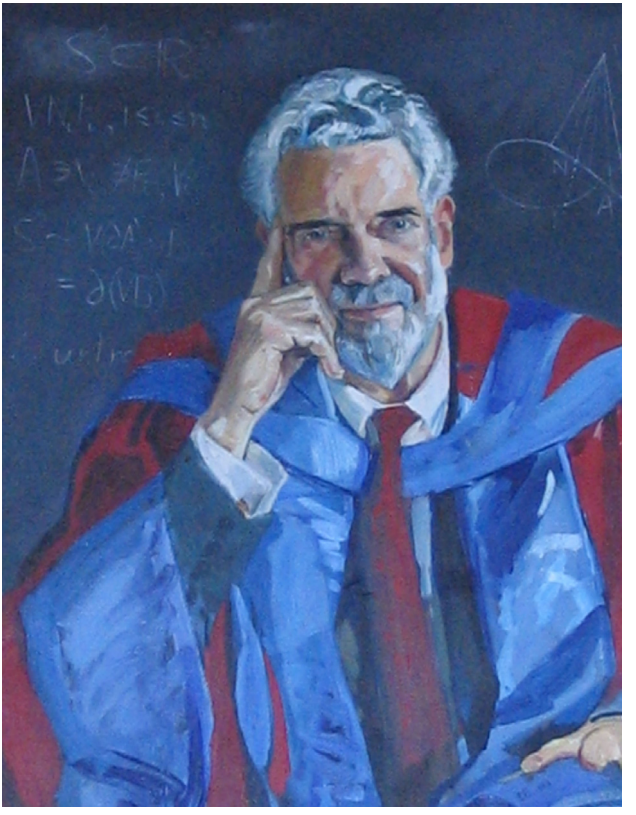
A supplement to

The Mathematical Gazette



Three-dimensional theorems for schools

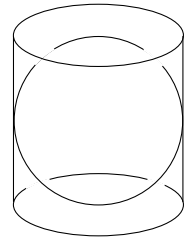
Sir Christopher Zeeman

THE MATHEMATICAL ASSOCIATION
March 2005

https://doi.org/10.1017/S0025557200590391 Published online by Cambridge University Press

Three-dimensional theorems for schools

Sir Christopher Zeeman



https://doi.org/10.1017/S0025557200590391 Published online by Cambridge University Press

This volume was published
by The Mathematical Association as a
supplement to the March 2005 issue of
The Mathematical Gazette.

Address: The Mathematical Association,
259 London Road,
Leicester
LE2 3BE
United Kingdom

Telephone: (+44) 0116 221 0013

Website: www.m-a.org.uk

The cover shows a photograph of the portrait of Sir Christopher which hangs in the Hall of Hertford College, Oxford, UK, where he was Principal 1988–1995. It was painted by Peter Edwards in 1993. The gown is an honorary DSc of the University of Warwick. The proof on the blackboard is of the unknotting of spheres in five dimensions [18]. The Mathematical Association is grateful for being given permission to use it.

ISSN 0025-5572

Contents

https://doi.org/10.1017/S0025557200590391 Published online by Cambridge University Press

Sir Christopher Zeeman

The biographical details which follow are taken from the 2004 MA Annual Report.

I was born in 1925 in Japan, with an English mother and a Danish father. My father died when I was one, so my mother brought me back to England and raised me as English. She ran a bed and breakfast place in London and, while she worked, I learnt to be happy playing alone, which was probably a good early training for a mathematician. I can only remember talking with her about mathematics when I was 7: I was baffled by a problem I had read in a children's newspaper and she showed me how to use x for the unknown. I was astounded and remember it vividly to this day. When I was 9 she sent me to boarding school at Christ's Hospital, where I was fairly unhappy, but had the good fortune to be taught mathematics by two excellent teachers, Bill Armistead and Arthur Humphrey, who had firsts from Cambridge, and who inspired me, laying the foundations for my future career. When I was 17, I won a scholarship to Christ's College, Cambridge. However, I had to postpone taking it up because of the war, during which time I became a navigator in the RAF.

Returning to Cambridge in 1947, I found mathematics easy: I did relatively little work and read no books, but more or less rediscovered the mathematics for myself by doing old exam papers, which enabled me to come top. I only began working hard when I stayed on to do research for a PhD in Topology under Shaun Wylie. He had been decoding at Bletchley during the war and did not give me any problems, which was probably a blessing in disguise because I then had to find my own. He diligently read and was enthusiastic about anything I wrote, and was like a father figure to me. I knew that two spheres could be linked in 5-dimensions, and so I tried to show that they could also be knotted. I failed, although I did construct elaborate algebraic invariants that turned out to be useful in other contexts – enough to get me a thesis, a Commonwealth Fellowship at Chicago, a lectureship at Cambridge and a fellowship at Caius College. I continued to try knotting spheres in 5-dimensions for 7 years, until one Saturday morning I sat down and tried to prove the opposite, to unknot them. To my astonishment by the end of the morning I had succeeded, and spent the weekend writing up a twenty-page paper. At 2am on the Sunday morning I was sitting on the lavatory contemplating my theorem, when suddenly it hit me that I use the same proof to unknot n-dimensional spheres in $(n + 3)$-dimensions, for all n. Once I had seen that, I was able to reduce the original proof in 5-dimensions down to ten lines, which I published as a one-page paper in the Bulletin of the AMS. This result initiated a new chapter in Geometric Topology, and was enough to provide PhD theses for a dozen of my research students. Subsequently, when Peter Edwards painted my portrait for Hertford College, Oxford, he wanted to paint some scribblings on a blackboard in the background, so I reduced the proof to five scribbled lines.

(Continued inside.)

https://doi.org/10.1017/S0025557200590391 Published online by Cambridge University Press

In 1963, I was invited to become the Foundation Professor of Mathematics at the new University of Warwick. I was 38 and had developed some fairly strong ideas on how to run a department and create a Mathematics Institute: I wanted to combine the flexibility of options that are common in most American universities, with the kind of tutorial care to be found in Oxford and Cambridge. We made the first six appointments in Topology (which raised some eyebrows) so that we could become competitive internationally for attracting research students, then the next six in Algebra, the next six in Analysis, and subsequently, the next six in Applied Mathematics, specialising in Dynamical Systems and Ergodic Theory. Being on the first finance committee of the university, I suggested that all subjects should have roughly the same staff-student ratio, which was helpful for mathematics, and allowed us to exchange four staff for money with which to pay the research students to help with the tutorial care of the undergraduates.

It was very exciting creating a new university, but all the administration did not leave much time for my own research in Topology, which dwindled to zero. We began running yearlong research symposia on different subjects each year, with many long-term senior visitors, and one on Dynamical Systems in 1968/69 attracted me to move into that subject. The following year I took a sabbatical at the IHES in Paris, where René Thom told me about Catastrophe Theory, which then became my main research interest for the next twenty years.

In 1988 I was invited to become Principal of Hertford College, Oxford, which did not leave much time for mathematics, but when I retired in 1995, I was able to start again working on difference equations and, with my daughter Mary Lou, who is a professor of mathematics in San Antonio, Texas, on n-dimensional Lotka-Volterra equations. Also, as befits a pensioner, I am increasingly interested in the history of mathematics!

In 1978 I was invited to give the televised Christmas Lectures at the Royal Institution, the first time that they had ever been given on mathematics, since Faraday started them in 1820. In the six lectures, I stated and proved 21 theorems, each chosen for being surprising, intriguing, noble (in the sense of capturing the quintessence of some major branch of mathematics), and for having a short rigorous proof that I could write out on a single transparency. There was such a demand for more from young persons that the RI then set up mathematics master classes for bright 13-year olds, which have now been going for 25 years, and spread to over forty centres around the country. Master classes usually last for two and a half hours on a Saturday morning for ten weeks. I find that the 13-year olds can appreciate university level material, which enhances their school experience. Demand for yet more after attending master classes, triggered the creation of the website NRICH (National Royal Institution Cambridge Homerton), which is thriving and growing. I learnt much about teaching from giving masterclasses, and became generally interested in enrichment for the more gifted. It was therefore a great pleasure for me to be elected President of The Mathematical Association.

Foreword

The Mathematical Association began life in 1871 as the *Association for the Improvement of Geometrical Teaching*. It was therefore highly appropriate that Professor Sir Christopher Zeeman, FRS, should choose three-dimensional geometry as the basis of his Presidential Address given in April 2004 at the end of his year as MA President. The text which follows is a greatly expanded version of that Address.

Much could be said about the teaching of geometry in schools nowadays. The amount of geometry currently taught in mainstream curricula is a small fraction of what was done in the past. For many pupils, Euclidean geometry provided a vehicle for developing the principles of proof and logic, as well as containing results of great beauty typified by the geometry of the triangle. The demise of geometry in schools is to be regretted and could arguably be regarded as the main cause of the deterioration in reasoning skills among mathematics undergraduates.

Sir Christopher is without doubt one of the greatest geometers and topologists of his time. The material in his text does not conform to any school curriculum. Rather it is an eclectic mix of topics in three-dimensional geometry which he hopes will prove fascinating and stimulating to abler students in the later stages of secondary education. There is a veritable cornucopia of topics that will provide enrichment either through private study by the reader or by its use in connection with Maths clubs, masterclasses and similar activities. Students and teachers at the tertiary level will also find much to interest them.

Sir Christopher has produced a text which is destined to become a valuable resource. It is to be hoped that it will help to bring about a renaissance in the teaching of geometry in schools in the first half of the 21st century, just as the founders of the *Association for the Improvement of Geometrical Teaching* wished to achieve in 1871.

Adam McBride

MA President 2004-2005

https://doi.org/10.1017/S0025557200590391 Published online by Cambridge University Press