SYSTEMATICS OF DWARF NOVAE

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ABSTRACT. From about 30 Dwarf Novae with the best determined distances the following relationships are found.

(i) a tight correlation between absolute magnitude at maximum light, M_v(max), and orbital period, P.

(ii) a correlation between M_v(min) and P showing wide scatter.

(iii) a correlation between M_v(mean), the mean absolute magnitude averaged over normal outbursts, and P, again with wide scatter. The scatter is shown to correlate strongly with outburst timescale T_n.

(iv) a strong correlation between range, M_v(min) - M_v(max), and T_n (the Kukarkin-Parenago relationship).

(v) a strong correlation between range M_v(mean) - M_v(max), and both T_n and P.

This final correlation is interpreted in terms of the disc instability model of dwarf novae and successfully predicts the observed width of outburst versus P relationship.

1. INTRODUCTION

At a conference such as this, with almost all aspects of cataclysmic variables to be critically discussed, an area of common interest is that of rates of mass transfer. Crucial parameters in the modelling of a CV are the values of $\dot{M}$: (a) from the secondary, (b) through the disc, (c) onto the white dwarf primary and (d) out of the system as a whole. All estimates of $\dot{M}$ are indirect: either from modelling of the flux distribution of an accretion disc, or from interpretation of absolute flux measurements if the distance is know. Results from the former method are very model dependent; at least an order of magnitude difference in rate

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of mass transfer emerges from use of different models (e.g. Hassall 1985). The latter method has been less used because of uncertainties in the distances to CVs. Here we will survey the CVs having the best determinations of distances and use these to find systematic trends in absolute magnitude and inter-relationships between the various subclasses of CVs. This is an extract from a much larger project in progress; only the preliminary results for dwarf novae are given here and we have omitted the tabular material from which the diagrams have been drawn.

There has been a recent tendency to de-emphasize basic absolute magnitudes in favour of the physical properties of accretion discs. Relationships such as the variation of $\dot{M}$ in the disc as a function of orbital period P (e.g. Patterson (1984)) or the connection between $\dot{M}$ and radius R_d of accretion disc (e.g. Smak 1982) use derived parameters which are highly model-dependent. Our philosophy is to retain as far as possible the observational data and empirical calibrations (although some of the latter are interpretive and model-dependent). The principle is that used in colour-magnitude diagrams of clusters: present observed data (M_v and B-V for clusters) and interpret these in terms of theory. If theories improve later, hopefully the observational diagrams will remain fairly invariant.

Two directions in which we can look for distances and hence absolute magnitudes of CVs are (i) infrared observations of the secondaries and (ii) membership of clusters, binaries, common proper motion pairs, etc.

The relative proportions of the contributions from secondary and disc in the infrared are not always easy to unravel and where it is possible to separate them it is believed that the resulting distances will be underestimates, although usually not seriously so (Berriman et al 1985). The distance to the CV, given the K magnitude of the secondary, is given by

$$\log d = \frac{K}{5} + 1 - \frac{S_k}{5} + \log \frac{R_2}{R_o}$$

where S_k is the surface intensity in the K band (Bailey 1981). S_k is constant for M stars and is a function of colour for hotter stars. It can be found as a function of CV orbital period through empirical relationships given by Patterson (1984). Berriman et al (1985) are reluctant to apply results from the Roche geometry of CVs to determine the radius R_2 of the secondary on the grounds that CVs are suspected of not following the main sequence mass-radius relationship. However, Patterson (1984) has shown that this belief arose from comparison of M_2 and R_2 in CVs with theoretical M-R relationships for dwarfs, whereas observationally there is good agreement with the empirical

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M-R relationship for the main sequence. Using Patterson's empirical relationship between R_2 and P, equation (1) becomes

$$\log d = \frac{K}{5} - 0.93 + 1.073 \log P(h) \quad P < 6.5h \qquad (2a)$$

$$= \frac{K}{5} - 2.14 + 2.56 \log P(h) \quad P > 6.5h \qquad (2b)$$

Use of these equations for CVs with K magnitudes of their secondaries, together with distances obtained from cluster membership etc., gives absolute magnitudes for about three dozen CVs.

We are interested in the absolute magnitudes of the CV accretion discs--these will be relatable to $\dot{M}$ in the disks. After allowance for interstellar absorption (where necessary - not, for example, in the DN when using their range) and the contribution of secondary or bright spot, it is still necessary to adjust the apparent absolute magnitude M'_v for inclination of the disc. We use the correction

$$\Delta M_v(i) = -2.5 \log\{(1 + \frac{3}{2} \cos i)\cos i\} \qquad (3)$$

from Paczynski & Schwarzenberg-Czerny (1980) to derive $M_v = M'_v - \Delta M_v(i)$, i.e. the absolute magnitude of the disc viewed at an inclination of 56°.7.

2. RESULTS FOR DWARF NOVAE

The results for M_v(max) and M_v(min) of dwarf novae (DN) are shown in Figure 1. Separate symbols are used for SU UMa, Z Cam and U Gem subtypes. The M_v(max) show small scatter about the relationship

$$M_v(\max) = \underset{\pm 0.13}{5.64} - \underset{\pm 0,024}{0.259}\, P(h) \qquad (4)$$

and it is seen that the stars with distances derived from the K magnitude method do not appear significantly shifted from those with distances found from independent methods. Vogt (1981) suspected that all DN have $M_v(\max) = 4.70 \pm 0.14$, based on a smaller sample than ours. In fact, the trend with P is visible for the DN with best determined distances in his Figure 10.

Application of equation (4) to DN of known P but unknown distance, together with their observed range, provides additional M_v(min) which have been included in Figure 1. They follow the linear relationship

$$M_v(\min) = \underset{\pm 0.25}{9.72} - \underset{\pm 0.056}{0.337}\, P(h) \qquad (5)$$

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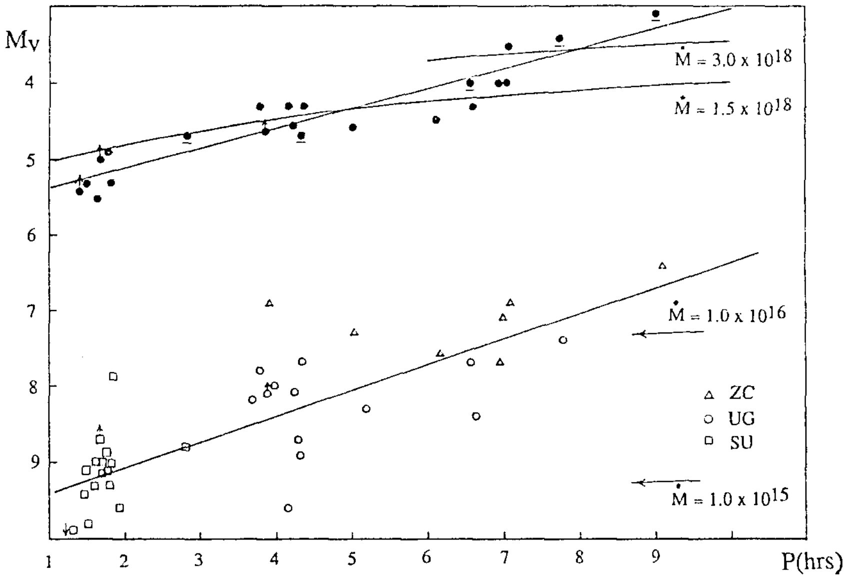

Figure 1. Absolute magnitude at maximum of outburst, M_v(max) and at minimum light, M_v(min), as a function of orbital period P. The same symbols for SU UMa stars, U Gem stars and ZC stars as defined here are used in Figures 2-6. Underlined symbols for M_v(max) indicate distances not obtained from the K magnitude method. Lines of constant rate of mass transfer $\dot{M}_d$ are indicated. The linear fits to the point are those of equations (4) and (5).

but with very much greater scatter than for M_v(max). A clear correlation is seen in that the M_v(min) for Z Cam stars are systematically brighter than the U Gem stars.

Calibration of M_v against $\dot{M}_d$ of the disc is possible from computed V fluxes of model discs. The published models do not cover the required range of disc radius R_d and M_v so we have extrapolated the Tylenda (1981) and Wade (1984) results. For R_d we adopt $R_d = 0.70\ R_L$, where R_L is the mean radius of the Roche lobe of the primary (Sulkanen et al 1981), which leads to

$$R_d = 1.14 \times 10^{10}\ (M_1/M_\odot)^{1/3}\ P^{2/3}(h) \quad \text{cm} \qquad (6)$$

We can then find an approximate function $M_v(\dot{M}_d, P)$.

In Figure 1 the M_v(max) for $P < 7$ hrs follow closely the curve for $\dot{M}_d = 1.5 \times 10^{18}$ g sec^{-1} = 2.5×10^{-8} $M_\odot$ yr^{-1}, for assumed $M_1 = 1M_\odot$. As there may be a slight systematic increase of M_1 with P (Ritter & Burkert 1986), which would brighten the discs at longer P, it is entirely possible that $\dot{M}_d$ = constant describes the discs of DN at outburst quite well. If so, this may indicate the existence of an upper limit on the viscosity that can be generated in their discs.

For M_v(min) we find that lines of constant $\dot{M}_d$ slope down only slightly towards lower orbital periods. We have indicated approximate conversions in Figure 1. Clearly at minimum light the DN show a wide range of $\dot{M}_d$ at a given P, as well as the systematic reduction with P.

Although M_v(min) gives an estimate of $\dot{M}_d$, from the point of view of secular evolution a more important parameter is the rate of mass actually lost from the secondary, $-\dot{M}_2$. In the disc instability model of DN, most of this mass is stored in the disc until outbursts transfer it to the white dwarf primary through a high viscosity disc. $\dot{M}_2$ may be estimated by defining an M_v(mean) as the absolute magnitude obtained by integrating the visual flux over many outburst cycles (Patterson 1984). We have used Patterson's estimates of m_v(mean) and our distances to obtain M_v(mean). However, for the SU UMa stars we have made some adjustments in order to integrate over only the normal outbursts: during supermaxima there is evidence of greatly increased $-\dot{M}_2$ (Osaki 1985) which, although obviously of concern to the white dwarf

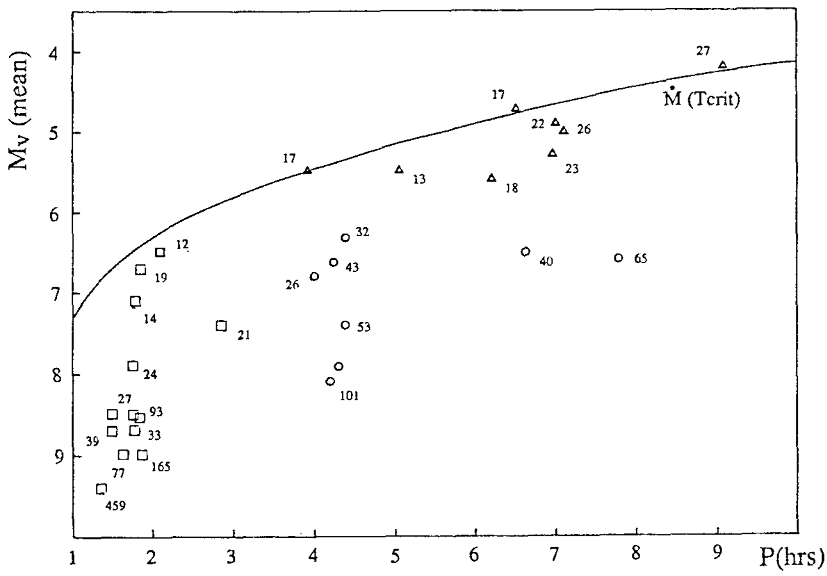

Figure 2. Absolute magnitude at mean light, M_v(mean), as a function of orbital period. The points are labelled with the recurrent times T_n of the dwarf novae normal outbursts. The curve is the theoretical line for continuous mass transfer in a stable disc.

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catcher, is not relevant to the study of normal outbursts. For the Z Cam stars we assume that m_v(mean) = m_v(standstill).

The resulting M_v(mean) are plotted in Figure 2 and show wide scatter at a given P, with the Z Cam stars positioned clearly at the top of the heap. According to the disc instability model, the Z Cam stars are able to spend part of their time in standstill because they have $\dot{M}_2$ very close to the critical value required to maintain an accretion disc in the high viscosity state. Smak (1982) has shown that $\dot{M}_d$(crit) can be found from

$$\dot{M}_d = \frac{8\pi}{3}\,\sigma\,T^4_{crit}\,\frac{R_d^3}{GM_1} \tag{7}$$

where T_{crit} is the minimum temperature for stable accretion. Taking T_{crit} from Meyer & Meyer-Hofmeister (1983) and using equations (6) and (7) we find

$$\dot{M}_d(\mathrm{crit}) = 1.60 \times 10^{16} \left(\frac{M_1}{M_\odot}\right)^{-0.10} P(h)^{1.80}\ \mathrm{g\ s^{-1}}. \tag{8}$$

Conversion of this to the M_v-P plane gives the curve shown in Figure 2. It lies about 0.5 mag above the mean region of the Z Cam stars, which is probably well within the uncertainty of the $M_v(\dot{M}_d$, P) conversion, and therefore adds considerable weight to Smak's conclusion (based on only 2 ZC and 2 DN) that the disc instability model is capable of explaining the Z Cam phenomenon.

3. CORRELATIONS WITH DWARF NOVA OUTBURST TIMESCALES

We may suspect that the wide spread in M_v(min) and M_v(mean) is well outside the range of observational error (c.f. the tight correlation for M_v(max)) which leads us to ask whether this range of $\dot{M}_d$(min) and $\dot{M}$(mean) has any other observable influence on the DN. As the mean interval T_n between outbursts of DN should be in some way related either to $\dot{M}_d$ or $\dot{M}_2$ (or both) we seek correlations between T_n and our other parameters.

The relationship between M_v(min) and T_n, seen in Figure 3, shows some correlation but with evidence that the Z Cam stars follow a different relationship from that of the SU UMa stars (indicated least squares linear fit in Figure 3). In fact, the deviation turns out to be merely dependent on P, as seen in Figure 4. Similar results are found for the correlation of M_v(mean) with T_n. We find

$$M_v(\mathrm{min}) = \underset{\pm 0.44}{7.11} + \underset{\pm 0.24}{1.44}\log T_n - \underset{\pm 0.037}{0.264}\ P \tag{9}$$

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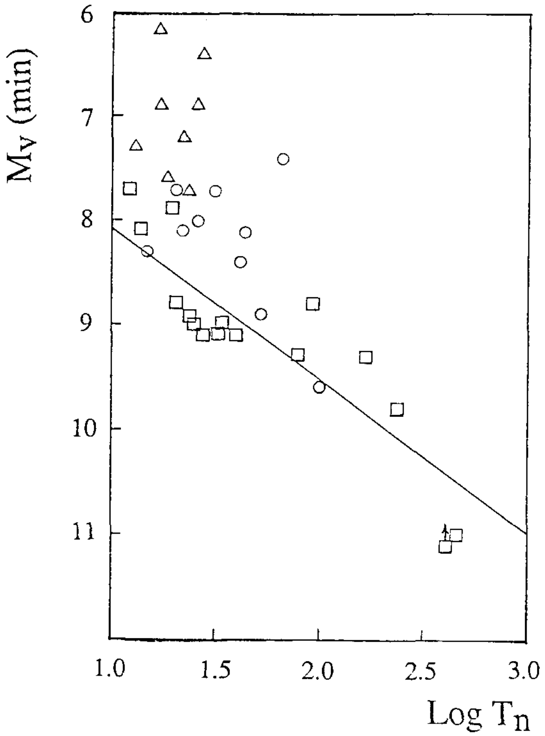

Figure 3. Correlation between M_v(min) and outburst timescale T_n. The line is the least squares linear fit to the SU UMa stars only.

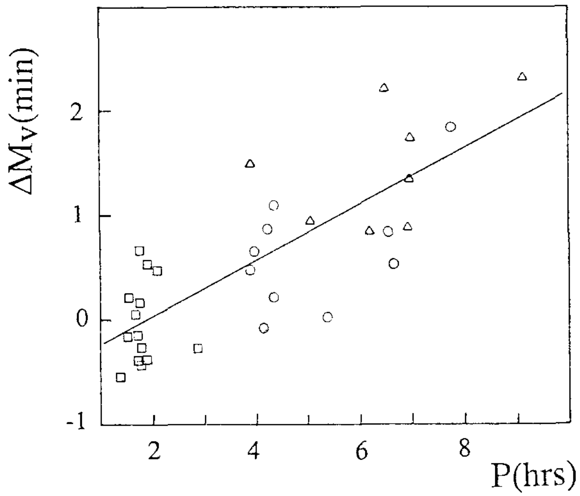

Figure 4. Deviation ΔM_v(min) from linear relationship in Figure 3 as a function of orbital period P.

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where P is in hrs and T_n in days. This is a generalized version of the result $M_v(min) = 6.55 + 1.40 \log T_n$ found by van Paridijs (1985) who based his work on the assumption $M_v(max)$ = constant obtained from Vogt (1981).

The correlation of $M_v(mean)$ with P and T_n is illustrated in Figure 2 where we have labelled the individual points with their associated T_n. If we remove the dependence on T_n by defining $M_v^*(mean) = M_v(mean) - 2.06 \log T_n$, then the remaining dependence on P is $M_v^*(mean) = 5.52 - 0.444$ P, as illustrated in Figure 5.

As the coefficients of P in equations (5) and (6) are similar, it results that there is a significant Kukarkin-Parenago relationship for the DN. Defining $A_n = M_v(min) - M_v(max)$ (which is not the same as $m_v(min) - m_v(max)$ because we have made corrections to $m_v(min)$ for contributions of the secondary and bright spot) we find, from an independent analysis to that carried out for $M_v(max)$ and $M_v(min)$:

$$A_n = \underset{\pm 0.46}{1.08} + \underset{\pm 0.24}{1.67} \log T_n \qquad (10)$$

independent of P within statistical error; and defining $A'_n = M_v(mean) - M_v(max)$, we find

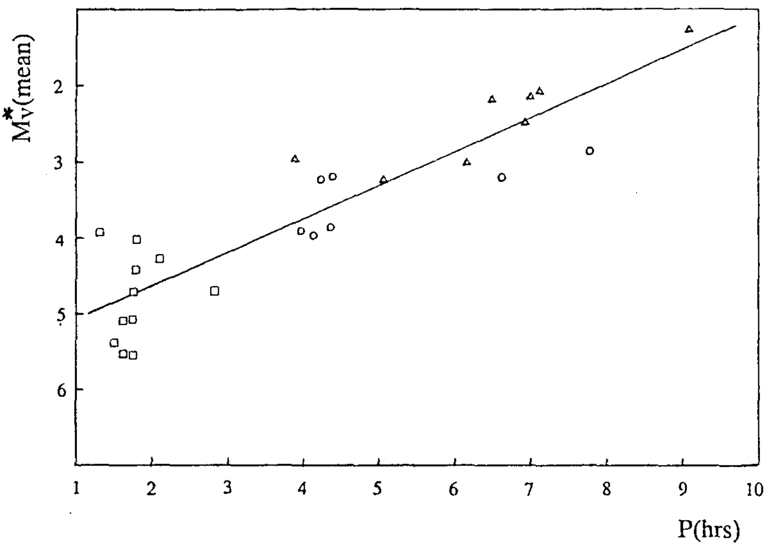

Figure 5. Correlation between $M_v^* = M_v(mean) - 2.06 \log T_n$ and orbital period P. The least squares linear fit is $M_v^*(mean) = 5.52 - 0.444P$.

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$$A'_n = -0.05 + 2.06 \log T_n - 0.20\ P(h). \quad (11)$$
$$\quad\ \ \pm 0.46 \quad \pm 0.29 \qquad\quad \pm 0.04$$

There is evidence, however, that for $T_n > 100$d the relationship may not be linear (Figure 6).

There are a number of possible interpretations of equations (9), (10) and (11). Here we draw attention to an interpretation in terms of the disc instability model. The average mass ΔM accumulated in the disc between outbursts is $\Delta M = [\dot{M}(\text{mean}) - \dot{M}_d(\text{min})]T_n$. During an outburst the DN must pass this mass through the disc, and it does so at a rate $\dot{M}_d(\text{max})$. Therefore the duration of a normal outburst is

$$\Delta T_n(\text{days}) \sim \frac{[\dot{M}(\text{mean}) - \dot{M}_d(\text{min})]}{\dot{M}_d(\text{max})} T_n. \quad (12)$$

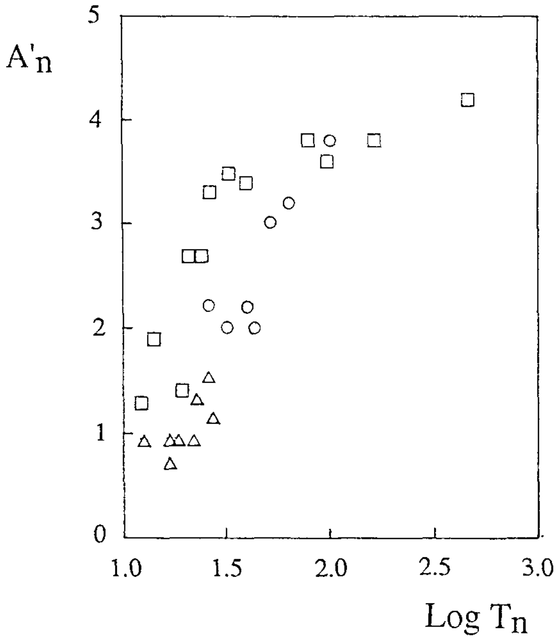

Figure 6. Correlation between amplitude $A'_n = M_v(\text{mean}) - M_v(\text{max})$ and outburst timescale T_n. The vertical scatter is partly caused by A'_n being a function of P.

From equation (11) and the approximate relationship $M_v \sim -1.85 \log \dot{M}_d$ + constant obtained from the Tylenda models, equation (12) gives

$$\log \Delta T_n(d) = 0.04 - 0.26 \log T_n(d) + 0.11\ P(h). \quad (13)$$
$$\qquad\qquad \pm 0.25 \quad \pm 0.16 \qquad\qquad \pm 0.02$$

Within observational error and the uncertainty of converting from M_v to $\dot{M}$, the coefficient of log T_n in equation (13) is not significant, but the coefficient of P certainly is. Thus

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equation (13) constitutes a prediction based on the empirical relationship equation (11) and our adopted simple model for DN outbursts.

The observed widths of outbursts of DN (van Paridijs (1983) and Szkody & Mattei (1984), using only parameters for 'narrow' outbursts to exclude supermaxima in the SU UMa stars) are plotted as a function of P in Figure 7. The least squares linear fit is

$$\log \Delta T_n(d) = \underset{\pm 0.08}{0.00} + \underset{\pm 0.016}{0.087} P(h). \qquad (14)$$

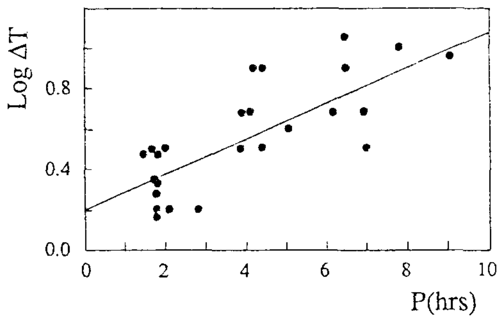

Figure 7. Correlation between outburst width ΔT_n of dwarf novae and orbital period P. The linear least squares fit is that given in equation (14).

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