# CORRECTIONS TO *CANONICAL EXPRESSIONS IN BOOLEAN ALGEBRA*

ARCHIE BLAKE

Two errors have appeared in my dissertation.[1] First, in the symbolic statement of Theorem (14.1), "$r_j \nsim \Lambda$" should be "$r_j \nsim \mathrm{V}$." Second, as J. C. C. McKinsey points out in his review,[2] Theorem (10.8) is not correct. The error in the proof enters in the last application of Theorem (10.7), for which the hypothesis $p^{\mathrm{syl}\,c_2}$ would be needed. The construction given at the bottom of page 35 is valid, however.

To state the correct theorem to replace Theorem (10.8), adopt the notation $\mathrm{syl}_c(p)$ for any one of the mutually congruent polynomials resulting from the adjunction to $p$ of all $\sigma$-terms informally included in $p$ which are obtainable by eliminating $c$ from pairs of terms of $p$. Then:

THEOREM. $p^{\mathrm{syl}\,c_1} \;.\!)\!\cdot\; (\mathrm{syl}_{c_2}(p))^{\mathrm{syl}\,c_1}$.

Denote $\mathrm{syl}_{c_2}(p)$ by $p_1$; evidently $p_1^{\mathrm{syl}\,c_2}$. Supposing the conclusion false, consider any two terms $m_1$ and $m_2$ of $p_1$ such that $(m_1 + m_2)^{-\mathrm{syl}}$ and $c_1^{\mathrm{fact}\ m_1}$, $c_1^{\dagger\ \mathrm{fact}\ m_2}$. Then $m_1 \nless c_1^\dagger$, $m_2 \nless c_1$, and no letter other than $c_1$ or $c_1^\dagger$ enters with opposite signs in $m_1$ and $m_2$. If $m_1$ and $m_2$ are both terms of $p$, then by hypothesis $\sigma(m_1+m_2) \ll p \ll p_1$. Next let one of $m_1$ and $m_2$—say $m_1$—be one of the new terms adjoined in forming $p_1$, so that $\exists (m_{11} \cdot m_{12})^{\mathrm{terms}\ p} \ni\cdot\ m_{11} \nless c_2^\dagger \cdot m_{12} \nless c_2$, and all co-letter factors of $m_{11}$ and $m_{12}$ except $c_2$ and $c_2^\dagger$ are factors of $m_1$.

Suppose $m_2^{\mathrm{term}\ p}$. Not both of $c_2$ and $c_2^\dagger$ are factors of $m_2$—say $c_2^{-\mathrm{fact}\ m_2}$, $m_2 \nless c_2$. Now $m_{12} \nless c_1^\dagger$ since $m_1 \nless c_1^\dagger$; also no letter other than $c_1$ or $c_1^\dagger$ enters with opposite signs in $m_{12}$ and $m_2$, because such a letter would then enter with opposite signs in $m_1$ and $m_2$. Then by Theorem (10.7), $\exists m_3^{\mathrm{term}\ p} \ni\cdot\ m_3 \nless c_1 \cdot m_3 \nless c_1^\dagger \cdot m_{12}+m_2 \sqsubset m_3$, whence $m_3 \nless c_2$, and every co-letter factor of $m_3$ except (possibly) $c_2^\dagger$ is a factor of $\sigma(m_1+m_2)$. If $c_2^\dagger$ is a factor of $m_2$, it is also a factor of $\sigma(m_1+m_2)$, so that $\sigma(m_1+m_2) \ll p \ll p_1$. But if $c_2^{\dagger\ -\mathrm{fact}\ m_2}$, then by Theorem (10.7), $\exists m_4^{\mathrm{term}\ p} \ni\cdot\ m_4 \nless c_1 \cdot m_4 \nless c_1^\dagger \cdot m_{11}+m_2 \sqsubset m_4$, whence $m_4 \nless c_2^\dagger$. Also no letter other than $c_2$ or $c_2^\dagger$ enters with opposite signs in $m_3$ and $m_4$, so that by Theorem (10.7), $\exists m_5^{\mathrm{term}\ p_1} \ni\cdot\ m_5 \nless c_2 \cdot m_5 \nless c_2^\dagger \cdot m_3+m_4 \sqsubset m_5$, whence every co-letter factor of $m_5$ is a factor of $\sigma(m_1+m_2)$, and $\sigma(m_1+m_2) \ll p_1$.

Suppose now that $m_2^{-\mathrm{term}\ p}$. Then $\exists (m_{21} \cdot m_{22})^{\mathrm{terms}\ p} \ni\cdot\ m_{21} \nless c_2^\dagger \cdot m_{22} \nless c_2$, and all co-letter factors of $m_{21}$ and $m_{22}$ except $c_2$ and $c_2^\dagger$ are factors of $m_2$. Since $m_{11} \nless c_1^\dagger$, $m_{21} \nless c_1$, and no letter other than $c_1$ or $c_1^\dagger$ enters with opposite signs in $m_{11}$ and $m_{21}$, it follows by Theorem (10.7) that $\exists m_6^{\mathrm{term}\ p} \ni\cdot\ m_6 \nless c_1 \cdot m_6 \nless c_1^\dagger \cdot m_{11}+m_{21} \sqsubset m_6$, whence $m_6 \nless c_2^\dagger$. Similarly $\exists m_7^{\mathrm{term}\ p} \ni\cdot\ m_7 \nless c_1 \cdot m_7 \nless c_1^\dagger \cdot$

---

Received July 18, 1938.

[1] ***Canonical expressions in Boolean algebra,*** The University of Chicago, 1937.

[2] ***The journal of symbolic logic,*** vol. 3 (1938), p. 93.

$m_{12}+m_{22} \sqsubset m_7 \cdot m_7 \nless c_2$. No letter except $c_2$ or $c_2^\dagger$ enters with opposite signs in $m_6$ and $m_7$, because such a letter must then enter in one of $m_{11}$ and $m_{21}$ with sign opposite to its sign in $m_{12}$ or $m_{22}$, and must therefore either enter with both signs in $m_1$ or $m_2$, or enter with opposite signs in $m_1$ and $m_2$; the latter is possible only for $c_1$ or $c_1^\dagger$, neither of which is a factor of $m_6$ or $m_7$. Then by Theorem (10.7), $\exists m_8^{\text{term } p_1} \ni \cdot\ m_8 \nless c_2 \cdot m_8 \nless c_2^\dagger \cdot m_6+m_7 \sqsubset m_8$. Then every co-letter factor of $m_8$ is a factor of $\sigma(m_1+m_2)$ so that $\sigma(m_1+m_2) \ll p_1$.

WASHINGTON, D. C.