INFLATION AND THE LARGE-SCALE STRUCTURE OF THE UNIVERSE

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1. Introduction

Before the development of the inflationary universe scenario many cosmological problems of the standard hot universe theory remained unsolved. In particular, the origin of primordial density perturbations remained obscure.

After the discovery of the inflationary universe scenario /1-4/ two important predictions were made, which we call "the flat adiabatic paradigm":

i) The observable part of the universe is almost exactly flat, $\Omega_{tot} = \rho/\rho_{cr} \simeq 1$.

ii) Inflation inevitably produces adiabatic perturbations with an almost scale-free (flat) spectrum /5,6,7/.

A common belief was that now we know all basic facts which are necessary to understand the large-scale structure of the universe.

However, everything is not so simple.

i) The first papers on adiabatic perturbations were not quite complete and give numerically different results. For examples some authors say that density perturbations $\delta\rho/\rho \sim 10^{-4}$ in the theory $V(\varphi)=\frac{1}{4}\lambda\varphi^4$ if $\lambda \simeq 10^{-8}$ whereas some other authors give $\lambda \simeq 10^{-16}$. The disagreement is by 8 orders of magnitude!

ii) There exist serious doubts that the cosmological

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models based on the flat adiabatic paradigm can lead to a completely consistent theory of the large-scale structure of the universe.

iii) Previous investigation did not tell us how the universe looks on a superlarge scale, which is much greater than the size of present horizon. What is its global geometry?

The main results to be reported here:

1. A more complete theory of adiabatic perturbations, including models with several stages of inflation /8,9,10/.
2. Isothermal perturbations with a nonflat spectrum /11-13/.
3. Non-perturbative inhomogeneities generated during inflation /13,14/.
4. Self-reproduction of the inflationary universe and its global structure (eternal chaotic inflation) /15-17/.

2. Standard inflationary cosmology

Let us remember, at first, the main idea of the "standard" inflationary cosmology. At the beginning the universe goes through some hypothetical pre-inflationary stage. Thereupon the universe passes through the inflationary stage, when its degree of expansion during the time interval 10^{-35}s was equal to or even much greater than its further degree of expansion during subsequent 10^{10} years. After inflation, the stage of relaxation of the inflation-driving field φ takes place. During this period all matter of the universe is created and reheated up to some temperature $T_R \lesssim 10^{15}$ Gev. Further, the universe, filled by the hot ultrarelativistic matter, develops according to the hot Friedmann universe theory.

Historically, there were several different versions of the inflationary universe scenario /1-4/. The two main versions are new inflation /2/ and chaotic inflation /3/. Despite many efforts, no realistic models of new inflation have been suggested so far because of the problem of initial conditions /18/. This problem is solved in the chaotic inflation scenario only /19/. One can say that the old inflation scenario is dead; the new scenario is old, and the chaotic inflation scenario is in a good order. For this reason we will discuss here the chaotic inflationary scenario only. Let us consider the universe filled by a single scalar field φ with a power-law potential $V(\varphi)$, for simplicity $V(\varphi) = \frac{1}{2} m^2 \varphi^2$. A scalar field obeys Klein-Glondon equation

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{dV}{d\varphi} = -m^2\varphi^2, \qquad (1)$$

which is similar to equation of ordinary oscillator with the friction term $3H\dot{\varphi}$. Here H is the Hubble parameter given by one of the Einstein equations:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_p^2} V(\varphi) = \frac{4\pi m^2}{3M_p^2}\varphi^2, \qquad (2)$$

where $M_p = 1/\sqrt{G} \simeq 10^{19} Gev$ is Plank mass. From Eqs. (1), (2) we obtain the following logical chain: large $\varphi \to$ large $V(\varphi) \to$ large $H \to$ large friction $\to$ slow motion of $\varphi \to$ almos constant $V(\varphi) \to$ exponential solution for $a(t)$ (i.e. inflation):

$$a(t) = a_0 \exp[H(\varphi)t], \quad |\dot{H}| \ll H^2. \qquad (3)$$

It occurs at $\varphi \gtrsim \frac{1}{5} M_p$. At $\varphi \lesssim \frac{1}{5} M_p$ we have another chain: small $\varphi \to$ small $H \to$ small friction $\to$ rapid oscillation near minimum of $V(\varphi) \to$ reheating of the matter filled the universe.

Now we will consider the following perturbations of density in this background:

i) adiabatic (inflaton) perturbations;

ii) isothermal, or isocurvature (isoinflaton) perturbations;

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iii) nonperturbative mechanisms (strings, bubbles ...).

3. Adiabatic perturbations

The basic mechanism of its generation is connected with production of large-scale fluctuations $\delta\varphi$ of the inflation-driving scalar field φ (inflaton field) /6,20/. These fluctuations $\delta\varphi$ obey oscillator-like equations (7) involving friction term $3H\dot{\varphi}$, see below. For short waves $\lambda \lesssim H^{-1}$ this term can be neglected and fluctuations $\delta\varphi$ oscillate, whereas long waves with $\lambda \gtrsim H^{-1}$ do not oscillate due to the friction term, and behave as a long-wave (its wavelengths grow as $\exp(Ht)$) classical scalar field. During each typical time interval $\Delta t \sim H^{-1} \sim 10^{-35}$s new classical inflaton field $\delta\varphi$ with wavelength $\lambda \gtrsim H^{-1}$ is produced with the average amplitude

$$\delta\varphi \simeq \frac{H}{2\pi}\,, \qquad \frac{\Delta(\delta\varphi)}{\Delta t} \simeq \frac{H^2}{2\pi}\,. \tag{4}$$

Before the inflationary stage the spectrum of fluctuations $\delta\varphi$ corresponds to the ordinary vacuum fluctuations. After inflation the spectrum of $\delta\varphi$ would be flat (4).

Perturbations $\delta\varphi$ of the field φ durind inflation leads to perturbations of metric which lead to adiabatic perturbations of density after inflation

$$\frac{\delta\rho}{\rho} = C(\varphi)\,\delta\varphi\,. \tag{5}$$

The main purpose of the theory of adiabatic perturbations is to determine the function $C(\varphi)$. Pioneering works in this direction are Refs. /5,6,7/. We will try to do it in a gauge-invariant way (see also /8,21/), and to obtain a more precise equation for $C(\varphi)$.

It proves useful to use the conformally-Newtonian coordinates /22/

$$ds^2 = (1+2\phi)dt^2 - (1-2\psi)a^2(t)\,dl^2. \tag{6}$$

ϕ and Ψ are some gauge-invariant functions (with small values) which in this coordinate system coincide with the gauge-invariant functions Φ_H and $-\Phi_A$ introduced by Bardeen /23/.

In the scalar field theories it can be shown that $\phi=\Psi$ and $\delta\rho/\rho = -2\phi$. An important feature of our approach is that fluctuations of scalar field $\delta\varphi$ occur to be gauge-invariant automatically /9/. Then one must solve Einstein equations and equations for the scalar field linearized in perturbations $\delta\varphi$ and ϕ :

$$\frac{1}{a^2}\Delta\phi - 3H\dot{\phi} - 3H^2\phi = \frac{4\pi}{M_p^2}\left(\dot{\varphi}\,\delta\dot{\varphi} - \dot{\varphi}^2\phi + \frac{dV}{d\varphi}\delta\varphi\right), \qquad (7)$$

$$\dot{\phi} + H\phi = \frac{4\pi}{M_p^2}\dot{\varphi}\,\delta\varphi .$$

Rather unexpectedly, it proves possible to solve these equations at all stages of the universe evolution /8/ (see also /24,9/). Analytical solution in the long-wave limit is

$$\frac{\delta\rho}{\rho} = -2\phi = \left[-\frac{2}{\dot{\varphi}}\frac{d}{dt}\ln\left(\frac{1}{a}\int dt\, a\right)\right]\delta\varphi, \qquad (8)$$

which gives the value of $c(\varphi)$ in eq. (5). Let us discuss some particular applications. The main result for inflation in the theory $V(\varphi) = \frac{1}{2}m^2\varphi^2$ is

$$\delta\rho/\rho = 1.5\,\frac{m}{M_p}\ln k^{-1}(cm);$$

and in the theory $V(\varphi) = \frac{1}{4}\lambda\varphi^4$

$$\delta\rho/\rho = 0.3\sqrt{\lambda}\,\ln^{3/2} k^{-1}(cm)$$

for wavelength k^{-1}. From restrictions on fluctuations in microwave background radiation $\Delta T/T$ we obtain constraints on parameters: $m \lesssim 10^{13}\div 10^{14}\,GeV$; $\lambda \lesssim 10^{-9}\div 10^{-11}$.

Thus, in the case of a single inflation the spectrum of density perturbations is flat. In the multiple inflation scenario, when a number of successive stages of inflation are driven by a number of classical fields φ_K (which are present during all inflationary stages) with different effective potentials $V_k(\varphi_k)$, $k = 1, \cdots, n$. There is the general rule for adiabatic perturbations /10/: The spectrum of adiabatic perturbations in a scenario with any number of different stages of inflation grows with the growth of the wavelength (see also /24-26/). However, we propose a possible exception. If the field φ_n , which drives the last inflationary stage, appears only during previous inflationary stage due to some phase transition at the moment t_c , then there may be a cut-off of the spectrum at large wavelength $\lambda > \lambda_c$.

3. Isothermal perturbations and nonperturbative effects

Isothermal perturbations with a spectrum which may differ considerably from the flat one can be generated during inflation /13/. One can obtain different model-depended spectra: the spectra which decrease at large (at small) wavelengths, which has a sharp maximum, corresponding to some phase transition, etc. Let us consider here the last case of the phase transition during inflation in a theory of a scalar field χ , which interacts with the inflation scalar field φ . The full potential is

$$V(\varphi,\chi) = \left(\frac{1}{2} m_\varphi^2 \varphi^2 + \frac{1}{4}\lambda_\varphi \varphi^4\right) + \left(\frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{4}\lambda_\chi \chi^4\right) - \frac{1}{2}\nu^2 \varphi^2 \chi^2. \qquad (9)$$

The shape of the potential $V(\varphi,\chi)$ changes during inflationary stage due to the change of inflation-driving field φ . We assume that $\lambda_\varphi \ll \lambda_\chi$ /13/. (In the opposite case $\lambda_\varphi > \lambda_\chi$ studied in /26/ double inflation occurs instead of the effect interesting for us). The field χ has a small mass $|m_\chi^2| \ll H^2$ only in some small vicinity of the critical value φ_c , so that the long-wave perturbations $\delta\chi$ are generated only at the time of the phase transition. This leads to the peak in the spectrum of isoinflaton (isothermal) perturbations of density generated by fluctuations $\delta\chi$.

Let us give a very short review of nonperturbative effects /13,14/. Phase transitions during inflation may lead to formation of exponentially large objects (strings, domains, bubbles, etc.). The composition and energy density inside different domains depends on kinetics of the phase transition. For example, during the phase transition in the $SU(5)$ model the $SU(5)$ symmetry in some domains of the universe breaks down to $G_1 = SU(3) \times SU(2) \times U(1)$ phase, in some other domains to $G_2 = SU(4) \times U(1)$ phase. There are many different possibilities.

i) G_2-phase is unstable and finally the whole universe transforms to G_1-phase. Then in domains, where earlier was G_2-phase, the energy density is larger than outside them.

ii) G_2-phase is stable, and if G_2-phase is energetically unfavourable (favourable), G_2 -domains will be collapse (explode), and finally explode after the collapse.

For phase transitions in the sector of "hidden" fields χ with a small energy density, the density inside and outside bubbles may be almost equal to each other, and still galaxies can be produced only in some particular-domains:

iii) Baryons may be produced only in the presence (in the absence) of the χ -field. In such a case luminous matter (galaxies) will exist only inside (outside) the bubbles of the field χ $(\Delta B \neq 0)$.

iiii) If density perturbations are mainly associated with isothermal perturbations of the field χ , then galaxies are created only outside the bubbles in which $\chi = 0$ $(\Delta B = 0)$.

Recently the model with cosmic strings became rather popular /27/. In the standard string scenario it is assumed that strings are produced during the high temperature phase transition at $T_S \sim 10^{16}$ Gev. We would like to note /13/, that typical reheating temperature after inflation is rather small $T_R \ll T_S$. Therefore it is not very easy to make the standard string scenario compatible with inflation. However, the phase transition with the formation of strings may occur at the last stages of inflation. But the distribution of such strings will be different from the distribution of strings in the standard scenario. Particularly, the form of a string created during inflation is connected with the growth of vacuum fluctuations on the string /28/. This effect is similar to the growth of vacuum fluctuations of any light scalar field during inflation. Thus, string formed during inflation looks like a one-dimensional fractal.

Now we give a numerical estimates. Typical length scales ℓ_c connected with phase transitions during inflation

$$\ell_c \sim \exp\left(\frac{\pi \varphi_c^2}{M_p^2}\right)(cm),$$

where the value of inflaton field $\varphi_c \sim 4M_p$ gives the upper interesting scale $\ell_c \sim \ell_H \simeq 10^{28}$cm. Thus, phase transitions may be very important if they occur at $\varphi_c \lesssim 4M_p$. An examples: $V(\varphi,\chi) = -\mu^2\chi^2 + \frac{1}{4}\lambda\varphi^4 + \eta^2\chi^2\varphi^2$. There $\varphi_c = \mu/\sqrt{\lambda}$, let us take $\eta \sim 10^{-6}$ ($\eta \lesssim 10^{-5}$, since otherwise the effective coupling constant λ acquires a contribution $\lambda \sim \eta^2 > 10^{-10}$, which leads to unappropriately large $\delta\rho/\rho$). We have $\varphi_c \sim 4M_p \sim 5\cdot10^{19}$Gev, if $\mu = \varphi_c\sqrt{\eta} \sim 5\cdot10^{16}$ Gev. Such scales naturally appear in Grand Unified Theories.

This means that the effects of phase transitions discussed above actually can occur in realistic theories.Note, that we are not finetuning the mass parameters μ in the underlying elementary particle theory. Rather we determine the numerical values of these parameters by the use of cosmological experimental data. The universe serves as a huge accelerator, which makes it possible to measure parameters of the elementary particle theory by making cosmological observations, in the same way as we do it by more conventional experiments with ordinary accelerators.

4. Super-large-scale structure of the universe

Hitherto we considered fluctuations of scalar fields, which create small perturbations of density $\delta\rho/\rho \ll 1$ in the observable part of the universe inside the present horizon $\ell_H \sim 10^{28}$ cm $\simeq 12 \cdot 10^3 Mpc$.

Let us remember a general formula $\frac{\delta\rho}{\rho} \sim \sqrt{\lambda}\, \ln^{3/2} k^{-1}$ (for $V(\varphi) = \frac{1}{4}\lambda\varphi^4$ theory, for example). On a much greater scale $k^{-1} \gg \ell_H$ these fluctuations make the universe absolutely inhomogeneous, $\frac{\delta\rho}{\rho} \gtrsim 1$ due to the factor $\ln^{3/2} k^{-1}$. As a result, the global geometry of the inflationary universe is very nontrivial and has nothing in common with the geometry of the Friedmann universe.

Let us consider again the chaotic inflation scenario, see sect. 2. Inflation occurs during the slow rolling of the field φ to the minimum of the effective potential. However, surprisingly enough, in some domains of the universe the field φ due to quantum effects never rolls down, and inflation occurs eternally.

For large φ the change of the field φ due to its classical rolling down during the typical time $\Delta t \sim H^{-1}$ can be neglected as compared with the quantum jumps $\delta\varphi$ up and down. As a result, in a half of the total volume of the universe after the typical time H^{-1} the field φ increases by $\delta\varphi \sim H$ rather than decreases. During this time the total volume of the universe grows by $e^{3H\Delta t} = e^3 \approx 20$, and in a half of this volume the field φ grows. This means that each time $\Delta t = H^{-1}$ the volume, occupied by a growing field φ, grows $\sim$ 10 times.

Therefore the volume of the inflationary part of the universe filled with a large field φ exponentially grows in time without end. Inflationary universe permanently reproduces itself, its evolution has no end and may have no beginning /15-17/.

In realistic theories of elementary particles there exist many types of scalar fields φ_k , and the effective potential $V_k(\varphi_k)$ often has many different local minima. Similarly, in Kaluza-Klein theories many different types of compactification are possible. During inflation long-wave fluctuations of all scalar fields $\delta\varphi_k$ are formed, and a considerable part of the volume of the universe, filled by the growing fields φ_k , proves to be at the density $V_k(\varphi_k) \sim M_p^4$, where quantum fluctuations of metric are also large. As a result, self-reproducing inflationary universe becomes divided into many exponentially large domains (mini-universes), in which all possible vacuum states (corresponding to all possible minima of $V_k(\varphi_k)$) and all types of compactification compatible with inflation are realized. Thus, thougth the inflationary universe locally looks like a homogeneous Friedmann universe, its global structure is extremely complicated. This result is very important for justification of anthropic principle in inflationary cosmology /15-17/.

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