RECENT WORK ON THE ORIGIN OF THE KIRKWOOD GAPS

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The frequency distribution of the asteroidal mean motions shows the well known Kirkwood gaps which are found near the commensurabilities 3/1, 5/2, 7/3, and 2/1. The number of asteroids in the gaps is significantly smaller than those in the corresponding regions around these commensurabilities. In the outer part of the asteroidal belt, the frequency distribution is reversed: The number of asteroids at the commensurabilities 3/2, 4/3, and 1/1 is much larger than in the region around these commensurabilities.

This reversal of the frequency distribution is one of the main problems of any hypothesis that tries to explain the Kirkwood gaps, since it is not at all obvious how asteroids can stay close to the commensurabilities 3/2 and 4/3 but avoid the commensurabilities 3/1, 5/2, 7/3, and 2/1. Different attempts have been made to explain the Kirkwood gaps based on statistical, gravitational, collisional or cosmogonic mechanisms.

Before looking for a sophisticated explanation for the Kirkwood gaps, one has to be sure that the gaps really exist and are not simply statistically underpopulated regions. According to the statistical hypothesis, asteroids librate through the gaps moving fastest at the center of the gap and remaining outside of the gaps over a comparatively long time span. The probability of observing an asteroid inside of a gap, is therefore very low.

This statistical hypothesis was tested by Schweizer (1969) and by Wiesel (1976). In order to find librators, Schweizer calculated the orbits of numbered minor planets which surround the gaps. He thus obtained a time-averaged frequency distribution of mean motion which did not differ strongly from the non-averaged distribution. A large majority of minor planets remained on one side of the gap whereas only a few asteroids were found to librate through the gaps. A list of the few librators is given by Williams (1977).

Franklin et al. (1975) investigated 30 orbits of unnumbered minor planets and of P-L objects which might librate about the 2 : 1 resonance.

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However, as these orbits are very uncertain, one has to wait for more observations of these objects in order to determine the number of minor planets librating about the 2 : 1 resonance.

Another statistical attempt was made by Wiesel (1976). He used an analytical method in order to investigate the evolution of an initially uniform distribution of fictitious asteroids in the Kirkwood gaps. His method is based on Poincaré's integrals for resonant motion. According to Wiesel's calculations, the statistical behaviour of the orbits does not indicate a depletion of the gaps for $t \to \infty$. The density of asteroids in the gaps never becomes significantly lower than in the surrounding regions. The emptiness of the gaps in Wiesel's model is not as pronounced as indicated by observations.

Both, Wiesel's and Schweizer's work, negate the validity of the statistical hypothesis for the Kirkwood gaps. Only if further observations yield many more minor planets librating about the commensurabilities, should this hypothesis be reexamined.

The second hypothesis, the gravitational hypothesis, has been investigated ever since the discovery of the gaps by Kirkwood (1867). A lot of effort was spent on analytical approaches, since the classical first order perturbation theory fails for cases of near resonant motion. In the classical theory, the disturbing function is expanded in trigonometric series of the osculating orbital elements. The occurence of small denominators destroys the formal convergence of these series for resonant motion and the osculating elements become infinitely large. This fact provides the basis for the following gravitational hypothesis:
An asteroid situated in a gap will suffer especially strong perturbations by Jupiter and therefore will leave the gap, because very small denominators occur in the series development for the semi-major axis.

Recently, this gravitational hypothesis was tested by analytical and numerical methods. The analytical approaches use either Poincaré's or related variables which are better suited than the classical orbital elements since these variables vary slowly with time for resonant motion. A simplification of the problem was achieved by discarding the short periods and by investigating only the long period commensurable terms. Then, for further simplification, the resonant motion of an asteroid was investigated in the restricted Sun-Jupiter-Asteroid problem. Different models are now available for the two-dimensional or three-dimensional, circular or elliptic case.

For the simplest of these gravitational models, the circular planar model, Schubart (1964) published a systematic survey of orbital types for several commensurabilities. His dynamical system has one degree of freedom. The possible orbits can be classified in three main types: periodic orbits, librators or circulators with respect to the critical argument σ. According to Schubart's survey, an asteroid which is situated in a gap does not leave the gap.

Obviously, this model represents only a rough approximation to reality as Jupiter's eccentricity is neglected and because the perturbations of the other planets are not taken into account. Therefore, Scholl and Froeschlé (1974, 1975) investigated numerically fictitious asteroidal orbits at the low order commensurabilities 3/1, 5/2, 7/3, and 2/1 in Schubart's elliptic planar Sun-Jupiter-Asteroid model. As in the circular model, no asteroid was found to leave a gap. Most of the orbits exhibit a quasi periodic behaviour with the exception of peculiar orbits which alternate between circulation and libration.

Two objections were raised against these calculations:

1.) We calculated over 100 000 years only, which is a short period compared to the age of the solar system of some 10^9 years.

2.) Schubart's model uses an averaging method in order to discard the short periodic effects, thus obtaining an integral of motion. That integral, which does not exist in the unaveraged model, might stabilize the calculations.

However, a few calculations in the unaveraged Sun-Jupiter-Saturn-Asteroid model over 100 000 years also failed to yield an orbit that left a gap. We have to concede that this result is not necessarily valid for much longer periods than 10^5 years. What happens with these orbits for $t \to \infty$, remains an open question.

From all our numerical experiments we can say that Schubart's figures obtained for the averaged circular case are a good approximation for the unaveraged Sun-Jupiter-Saturn model for 10^5 years.

Another approach to the gravitational hypothesis came from the technique of surface of sections which Hénon and Heiles (1964) applied to the restricted Three-Body-Problem. The application of this technique indicates what parts of an appropriately defined phase space an asteroid for instance can cover. Especially this method enables us to say whether the motion of an asteroid is limited to a small portion of the phase space (integrable case) or whether it can fill the whole phase space (ergodic case). Applying this technique to the Hecuba gap and the Hilda group, Giffen (1973) found an ergodic behaviour for orbits starting with low eccentricities in the Hecuba gap. On the other hand, he did not find any ergodic behaviour for fictitious Hildas. We, therefore, speculated that asteroids starting in the Hecuba gap might drift out of the gap while such a drift is not possible for the Hildas.

Froeschlé and Scholl (1976) tested this hypothesis in more detail applying the surface of section technique to the 3/1, 5/2, 7/3 and 2/1 commensurability. The calculations were based on Schubart's averaged planar elliptic model. Summarizing, we can say that we found a few quasi ergodic zones. However, these zones are surrounded by forbidden regions or invariant curves. An asteroid, therefore, situated in a quasi ergodic zone can not drift out of the gap. Here, we have to emphasize that this result is only valid for the applied model and for time spans up to 10^5 years. We cannot exclude the possibility that the invariant curves will dissolve on a much longer time scale and that an asteroid, therefore, might leave a gap following Arnold's diffusion process. This problem is still open.

A third approach to the gravitational hypothesis was made recently by Kiang (1978). He investigated the stability of orbits in the Hecuba gap and for the Hilda group. The stability of an orbit is determined by Hill's exponent. For his calculations, Kiang used Schubart's averaged planar circular model. According to his astonishing results, motion in the Hecuba gap is unstable while it is stable for the Hilda group in the sense of Hill's exponent. Therefore, Kiang supposes that an asteroid which started in the Hecuba gap will drift out of the gap. This supposition however is not supported by numerical calculations nor by theoretical considerations of Schubart's circular model.

Besides the gravitational hypothesis for the origin of the Kirkwood gaps, the collisional hypothesis has been discussed by numerous authors (Jefferys, 1967; Sinclair, 1969; Lecar and Franklin, 1973; Giffen, 1973; Scholl and Froeschlé, 1974; Wiesel, 1976). This hypothesis is very attractive as it seems to solve the problem why we observe so many Hildas at the 3/2 commensurability while we observe so few asteroids in the Kirkwood gaps. A collision between two asteroids can only occur when their orbits intersect. Therefore, the collision probability for an asteroid depends on the numbers of orbits it intersects. Thus, the collision probability depends on the variation of the orbital eccentricity. The stronger the eccentricity varies, the larger the collision probability.
Collision here means both destructive collision or a close approach which changes the semi-major axes of the orbits.

It is known, that asteroids close to a low order commensurability show strong variations in eccentricity. Therefore, these asteroids have a comparatively larger collision probability. Since there are almost no asteroids around the Hilda group, the collision probability for the Hilda family is very low.

In order to test the collisional hypothesis which needs large variations in eccentricity, Scholl and Froeschlé (1974, 1975) calculated a large number of orbits with Schubart's averaged planar elliptic model. The main purpose of these calculations was to find out how strong and how fast the orbits, i.e. their eccentricities, vary. According to these calculations the following orbits do not show strong variations in eccentricity and therefore do not support the collisional hypothesis:
a) almost circular orbits close to the 3/1, 5/2, and 2/1 commensurabilities;
b) orbits started at the borders of the corresponding observed gaps;
c) all the orbits close to the 7/3 commensurability.
These orbits behave like non-resonant orbits and therefore represent the problematic cases for the collisional hypothesis.

Severe objections to the collisional hypothesis came from more detailed calculations of the collisional probability of asteroids and from calculations of the kinetic energy required for a destructive collision. Recently, Ip (1977) investigated the probability of collision for main belt asteroids. According to his results, the concept of collisional probability has to be revised. The probability of colliding with another asteroid is not only a function of orbits intersected but it is also a function of the time the asteroid remains outside the main belt. Therefore, an asteroid with a low eccentricity may have a higher collision probability than an asteroid with a large eccentricity. A realistic model which takes into account the variation of orbits and the structure of the asteroidal belt might answer the question if an asteroid in a gap has a significantly higher collision probability than an asteroid outside of a gap.

Heppenheimer (1975) calculated the kinetic energy gained by an asteroid which increases its eccentricity due to resonant motion. According to his results, this energy is not sufficient to destroy the asteroid by a collision or by a sequence of collisions. He therefore concludes that the collision theory has to be abandoned. Especially, large asteroids with some 100 kms in diameter cannot be destroyed completely within the age of the solar system. In addition, it seems to be difficult to remove such large objects from the gaps by close approaches.

Because of these objections, the number of adherents of the collisional hypothesis is decreasing. All the three hypotheses mentioned above, the statistical, gravitational, and collisional assume that the Kirkwood gaps formed after the formation of the solar system and that the gaps were originally filled with asteroids.

Recently, two different hypotheses were presented which propose the formation of the Kirkwood gaps at the same time when the planetary system formed. Greenberg (1978) proposes a mechanism which produces the Kirkwood gaps within a few thousand years by a drag effect. The larger objects of 1 km size which formed at the commensurabilities collide frequently with the smaller particles. These small particles form the drag effect which removes the larger objects to the inner edge of the gap. Because of the drag effect, the eccentricities of the larger objects are dampened and as a result their semi-major axes experience a secular decrease, and the orbital energy is dissipated. This mechanism works especially well for resonant motion because the eccentricity is more strongly increased than for nonresonant motion. This energy dissipating hypothesis is very appealing as it seems to explain also other resonance phenomena in the solar system. Those orbits which do not vary their eccentricities strongly are the same problematic cases for the energy dissipating hypothesis as for the collisional hypothesis. According to the energy dissipating hypothesis, the members of the Hilda group formed close to the 3/2 commensurability and were captured in the 3/2 resonance by the same drag effect.

Another hypothesis which will be called the cosmogonic hypothesis has been proposed by Heppenheimer (1978). He shows that planetesimals could not have formed in the gaps. The strong variations in eccentricity of a growing particle produce strong variations of the particles orbital velocity which in turn prevents the planetesimal's formation. The cosmogonic hypothesis is a completely different approach to the problem of the Kirkwood gaps because it does not presuppose asteroids or planetesimals in the gaps like the other hypotheses.

The cosmogonic hypothesis has at the moment three weaknesses:

1.) It depends strongly on the formation mechanism of planetesimals.
2.) It cannot produce the Hilda family which must be formed somewhere else in the solar system and captured into resonance, because the observed Hildas show strong variations in eccentricity.
3.) It does not explain the absence of objects in the gaps which do not vary their eccentricities strongly.

More extended investigations of the cosmogonic hypothesis might solve these problems.

Of all the extant hypotheses for the origin of the Kirkwood gaps, the statistical hypothesis appears to have been sufficiently refuted. The gaps are not a statistical phenomenon. The gravitational hypothesis is still open, since we do not know how the dynamical system Sun-All the planets-Asteroid does evolve for $t \to \infty$. If the gravitational hypothesis is correct, then the formation of the Kirkwood gaps needed much longer than 10^5 years. The collisional hypothesis can not explain why we do not observe large asteroids of 100 km size in the gaps, as it is a problem to destroy or remove them because of energetic reasons. The collisional, energy dissipating and cosmogonic hypotheses still have to explain the absence of objects with almost circular orbits and the formation of the gap at the 7/3 commensurability.

REFERENCES

Franklin,F.A., Marsden,B.G., Williams,J.G., Bardwell,C.M. : 1975, "Astron.J." 80, pp.729-746
Froeschlé,C., Scholl,H. : 1976, "Astron.Astrophys." 48, 389-393
Giffen,R. : 1973, "Astron.Astrophys." 23, pp.387-403
Greenberg,R. : 1978, "Icarus" 33, pp.62-73
Hénon,M., Heiles,C. : 1964, "Astron.J." 69, pp.73-79
Heppenheimer,T.A. : 1975, "Icarus" 26, pp.367-376
Heppenheimer,T.A. : 1978, "Astron.Astrophys." to be published
Ip,W.-H. : 1977, "Icarus" 32, pp.378-381
Jefferys,W.H. : 1967, "Astron.J." 72, pp.872-875
Kiang,T. : 1978, "Nature" to be published
Kirkwood,D. : 1867, "Meteoric Astron." Ch.13
Lecar,M., Franklin,F.A. : 1973, "Icarus" 20, pp.422-436
Scholl,H., Froeschlé,C. : 1974, "Astron.Astrophys." 33, 455-458
Scholl,H., Froeschlé,C. : 1975, "Astron.Astrophys." 42, 457-463
Schubart,J. : 1964, "SAO Special Report" 149
Schweizer,F. : 1969, "Astron.J." 74, pp.779-788
Sinclair,A.T. : 1969, "Mon.Not.R.Astron.Soc." 142, pp.289-294
Wiesel,W.E. : 1976, "Celestial Mechanics" 13, pp.3-37
Williams,J.G. : 1977, "IAU Comm. No.20" Circular No.6