THERMAL INSTABILITY IN A HOT PLASMA

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ABSTRACT. The nature of local thermal instability in static and dynamic radiating plasmas described by an equilibrium cooling function has been reexamined. Several new results have been found. In a plasma in both thermal and hydrostatic equilibrium, if the cooling function is not an explicit function of position, and does not display isentropic thermal instability (i.e. sound waves are thermally stable), then isobaric thermal instability by the Field criterion is present if and only if convective instability is present by the Schwarzschild criterion. In this case, thermal overstability does not occur. For the case of a dynamical plasma we present a very general Lagrangian equation for the development of nonradial thermal instability. In the limit of large cooling time to free-fall time ratio, the equation is solved analytically by WKBJ techniques. Results are directly applicable to cluster X-ray cooling flows. Such flows are surprisingly *stable* except for perturbation wavenumbers that are very nearly radial. We believe that the origin of cooling flow optical filaments is not to be found in linear thermal instability.

1. INTRODUCTION

That a diffuse hot plasma can be thermally unstable is well-known (Field 1965, Mathews and Bregman 1978). We have reexamined the nature of thermal instability in hot plasmas. Using standard fluid techniques, we have found some surprising results. They include: (a) In a static plasma characterized by a mass-specific radiative loss function $\mathcal{L}$ that depends upon density ρ and temperature T (but not position r), thermal instability by the Field criterion generally occurs if and only if convective instability by the Schwarzschild criterion is present. If the explicit spatial gradient $\partial\mathcal{L}/\partial r$ is sufficiently large and pointed opposite to the direction of gravity, then thermal instability or convective instability will *necessarily* be present. (b) Nonradial thermal perturbations ("blobs") in cooling flows are stable throughout most of the flow. Radial instabilities are present, but mode-coupling may severely restrict their nonlinear development. At cool temperatures ($< 10^7 K$) and small radii, flow convergence can make buoyant oscillations significantly overstable. This may lead to nonlinear clumping in the accreting gas at the center of cooling flows. We present here an explanation of these finding and speculate on their implications.

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2. THERMAL INSTABILITY OF A STATIC PLASMA

A one-dimensional static plasma is described by the loss function equation $\mathcal{L}(\rho, T, r) = 0$, or upon differentiation:

$$\frac{d\mathcal{L}}{dr} = \frac{\partial \mathcal{L}}{\partial r} + \frac{d\rho}{dr}\left(\frac{\partial \mathcal{L}}{\partial \rho}\right)_T + \frac{dT}{dr}\left(\frac{\partial \mathcal{L}}{\partial T}\right)_\rho = 0. \tag{2.1}$$

Using standard transformations, one can rewrite the thermodynamic partial derivatives of $\mathcal{L}$ in terms of temperature derivatives at constant pressure P and constant entropy S (Balbus and Soker 1988):

$$\frac{\partial \mathcal{L}}{\partial r} + T\left(\frac{\partial \mathcal{L}}{\partial T}\right)_P \left(\frac{3}{5}\frac{d\ln P}{dr} - \frac{d\ln \rho}{dr}\right) + \frac{2}{5}T\left(\frac{\partial \mathcal{L}}{\partial T}\right)_S \frac{d\ln P}{dr} = 0. \tag{2.2}$$

The product of the isobaric temperature derivative of $\mathcal{L}$ and the spatial entropy gradient has the same sign as the isentropic derivative of $\mathcal{L}$, unless the explicit spatial gradient of $\mathcal{L}$ is sufficiently large, in which case the product has the opposite sign. Since the isentropic derivative of $\mathcal{L}$ is generally positive for any standard astrophysical cooling function, we may conclude that *if $\mathcal{L}$ is independent of r, a medium is thermally unstable by the Field criterion if and only if it is convectively unstable by the Schwarzschild criterion.* This suggests that static models of gaseous galactic haloes will quite generally display convective instability, and that thermal instabilities will form in a dynamically active background. Equation (2.2) applied to a static cooling flow model heated by a $1/r^2$ source (say relativistic particles from an AGN) suggests that regardless of the form of $\mathcal{L}$, either thermal or convective instability *must* be present (but not both).

3. THERMAL INSTABILITY OF A DYNAMICAL PLASMA

The equilibrium fluid is considered to be a flowing, spherically symmetric, time-independent, optically thin plasma subject to bulk heating/cooling processes. Self-gravity is assumed to be negligible, but an external gravitational potential is present. Quasi-hydrostatic equilibrium need not prevail. We consider the local stability of this flow to general spheroidal perturbations. We introduce the quantity a, which measures the radial separation of two close points as a function of time t. The spheroidal perturbations of the fluid displacement vector $\boldsymbol{\xi}$ have associated radial wave number k, ($kr \gg 1$), and spherical harmonic index l. Neglecting thermal conduction the evolutionary equation for the radial displacement amplitude ξ of a comoving fluid element is found to be:

$$\left(\frac{d}{dt} + \frac{2}{5}T\Theta_{T,P}\right)\frac{1}{ag}\left(\frac{d}{dt}a^2\frac{d}{dt}\frac{\xi}{a} + \frac{d}{dt}\frac{k^2r^2}{l(l+1)}\frac{d}{dt}\frac{\xi}{a}\right) + a\left(\frac{3}{5}\frac{\partial \ln P}{\partial r} - \frac{\partial \ln \rho}{\partial r}\right)\frac{d}{dt}\frac{\xi}{a} = 0. \tag{3.1}$$

where

$$\Theta_{T,P} = \left[\frac{\partial(\mathcal{L}/T)}{\partial T}\right]_P, \qquad q^2 = \frac{k^2}{a^2} + \frac{l(l+1)}{r^2}, \qquad g = -\frac{1}{\rho}\frac{\partial P}{\partial r} \tag{3.2}$$

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and μ is the mean mass per particle and k_B is the Boltzmann constant. The quantity ξ enters into the equation only in the ratio ξ/a since a radial displacement that is "frozen" into the flow would scale proportional to a with no physical consequences.

The applications of eq. (3.1) to cooling flows are ideally suited to the use of WKBJ techniques because the cooling time is long compared to sound crossing time. We define

$$\beta^2 \equiv \left(1 + \frac{k^2 r^2}{l(l+1)a^2}\right)^{-1}, \qquad \omega_{BV}^2 \equiv g\left(\frac{3}{5}\frac{\partial \ln P}{\partial r} - \frac{\partial \ln \rho}{\partial r}\right), \tag{3.3}$$

where ω_{BV} is the effective Brunt-Väisälä frequency. The WKBJ solution to eq. (3.1) is

$$\frac{\xi}{\beta} = \left[\frac{e^{\pm i \int^t \beta\omega_{BV} dt'}}{(\beta\omega_{BV})^{1/2}}\right] \exp\left[-\int^t \frac{\frac{2}{15} T\Theta_{T,S}\frac{\partial \ln P}{\partial r} + \frac{1}{3}\frac{\partial \Theta}{\partial r}}{\frac{5}{3}\frac{\partial \ln \rho}{\partial r} - \frac{\partial \ln P}{\partial r}} dt'\right]. \tag{3.4}$$

Equation (3.4) may be interpreted as follows. The term $1/\beta$ on the left-hand-side of the equation is a geometric factor which converts ξ to $|\boldsymbol{\xi}|$ for large l. The first grouping of terms on the left-hand-side is simply the WKBJ expression for comoving Brunt-Väisälä oscillations. In the following group, there are two thermal terms. If the isentropic condition $\Theta_{T,S} < 0$ holds, then the nearly adiabatic oscillations are overstable, pumped by buoyancy forces that are aided by radiative losses. The second thermal term involves the explicit spatial gradient of the loss rate, and is generally present if there is a central heating source in the flow, as in a Compton driven wind (Begelman et al. 1983). In this case, overstability becomes possible if the heating increases on a downward displacement of the fluid element, and decreases on an upward displacement.

The physically important quantity $\delta\rho/\rho$ (relative Eulerian density perturbation) for WKBJ solutions is:

$$\frac{\delta\rho}{\rho} = -\left(\frac{3}{5}\frac{\partial \ln P}{\partial r} - \frac{\partial \ln \rho}{\partial r}\right)\xi \tag{3.5}$$

In other words, the relative Eulerian density amplitude is simply the radial displacement divided by the entropy scale height of the flow. Note that the true test of instability is the behavior of $\delta\rho/\rho$, since it is ultimately Eulerian perturbations that measure physical changes in flow quantities. Equations (3.4) and (3.5) can be used to give the rough scaling of $\delta\rho/\rho$ as a function of equilibrium Eulerian position r of an accreting fluid element, assuming power-like behavior for background flow variable. For oscillatory perturbations with $\beta \sim$ unity, on the most unstable part of the equilibrium cooling curve where $\mathcal{L}/T \sim T^{-3/2}$ (Raymond, Cox, and Smith 1976), $\Theta_{T,S}$ is very small. Then $\delta\rho/\rho \sim r^{-1/2}$ for an isothermal sphere potential, and $\sim r^{-1/4}$ for a central point mass. Under these conditions, oscillatory instabilities are generally mild away from $r = 0$. As shown in the two figures, they all but disappear in more detailed treatments of cooling flows. Deep in the core radius of the cluster potential, if $g \sim r$ then $\delta\rho/\rho \sim r^{-1}$ and an overstability of an essentially adiabatic character becomes important. Clumpy gas at the center

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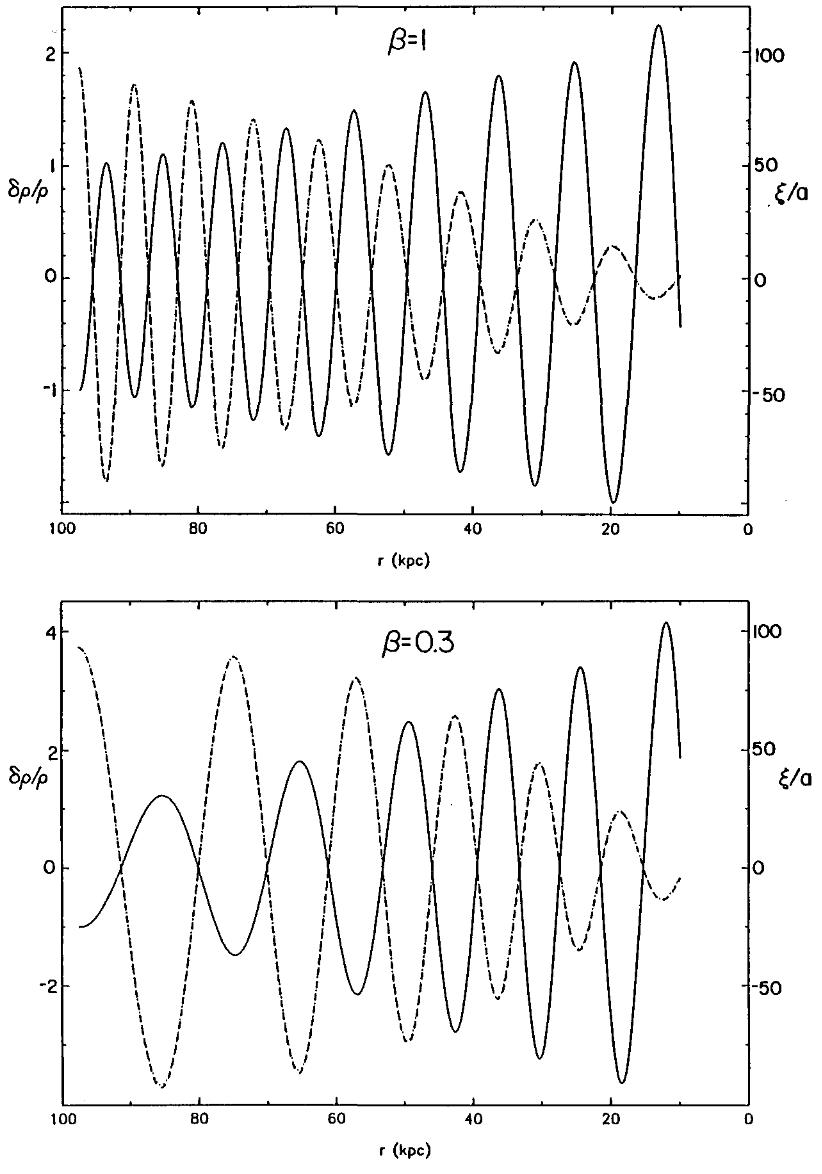

Figure 1. Evolution of two perturbations in typical cluster cooling flow: $\beta = 1$ corresponds to radially displaced perturbation $(kr \ll l(l+1))$, $\beta = 0.3$ to a more nearly azimuthal displacment $(kr \sim 3 \times i(i+1))$. Background flow is "standard, no star-formation" model of White and Sarazin (19897). Dashed line is numerical solution to eq. (3.1), dotted line is WKBJ solution eq. (3.4). (They are indistinguishable.) Solid line is $\delta\rho/\rho$; little growth is evident down to 10 kpc. Field type instability would obtain as $\beta \to 0$.

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of cooling flows is in fact seen or inferred in millimeter, optical and x-ray studies (Lazareff *et al.* 1988, Hu *et al.* 1985, Canizares *et al.* 1987).

The importance of our findings is that if significant amounts of matter are cooling and dropping out of x-ray accretion flows at large radii, highly nonlinear (and nonacoustical) disturbances are necessary *ab initio.* To both grow and avoid detection (there is no direct evidence of cooling gas at large distances from cluster or galactic center [Hu *et al.* 1985]), several constraints on the matter are necessary (e.g. Nulsen 1986, Thomas 1987): no buoyant oscillations, small blob sizes, no dynamical or conductive assimilation into diffuse flow, $\sim$ 100% efficiency of dark matter formation, etc. Alternatively, if one adopts the straightforward implication of this work that rapidly cooling, unstable blobs are not present at large flow radii, nonsteady accretion is needed to explain the x-ray observations. This conclusion is also independly supported by the recent discovery of large amounts of accreting molecular gas in the inner 5 kpc of NGC1275 (Lazareff *et al.* 1988).

5. CONCLUSIONS

[1] In gravitationally bound hot plasmas, there is an important connection between thermal instability by the Field criterion and convective instability by the Schwarzschild criterion. If the radiative loss function $\mathcal{L}$ is independent of position and is of any standard astrophysical form, the two occur simultaneously. If thermal instability is present under these conditions, it will form in a convectively unstable background. A large spatial gradient (opposite to gravity) in $\mathcal{L}$ forces either one or the other of the instabilities to be present.

[2] In slowly settling cooling flows, nonradial perturbations are essentially stable at large radii, and potentially overstable at very small radii. It is only when the oscillation frequency approaches the cooling time that Field-type thermal instability becomes important. In particular, cooling blobs must be highly nonlinear to grow and drop out of accretion flows. The alternative is young or transient cooling flows.

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